

Superfluidity of a pure spin current in ultracold Bose gases

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We study the superfluidity of a pure spin current, which is a spin current without a mass current. We examine two types of pure spin currents, planar and circular, in a spin-1 Bose gas. For the planar current, it is usually unstable but can be stabilized by the quadratic Zeeman effect. The circular current can be generated with spin-orbit coupling. When the spin-orbit coupling strength is weak, we find that the circular pure spin current is the ground state of the system and thus a superflow. We discuss the experimental schemes to realize and detect a pure spin current.

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I. INTRODUCTION

Since the experimental realization of Bose-Einstein condensation in optical traps, much effort has been devoted to the study of spinor superfluid [1–7]. With the degree of freedom of spin, spinor superfluid has much richer phases than scalar superfluid, as it has both superfluid order and spin textures. However, most spinor superfluids studied till now carry both spin current and mass current. It will be interesting to see whether these two currents can decouple and, further, whether a pure spin current which has no mass current can flow frictionlessly. Our motivation also originates from condensed matter physics, in which the concept of a spin superconductor [8,9], formed by Cooper-like pairs of electrons and holes and carrying a spin supercurrent, is proposed. It is interesting to have these ideas realized in the field of cold atoms.

In this work we focus on an unpolarized spin-1 Bose-Einstein condensate (BEC), where a pure spin current can be generated by applying a small magnetic gradient. It was found in Ref. [10] that a planar pure spin current in such a system is always unstable, as the $m = 1, -1$ components can collide into the $m = 0$ component, destroying the spin current. We find that the pure spin current can be stabilized with the quadratic Zeeman effect and become a superflow. Furthermore, we propose a scheme to create a pure spin current in the ground state, thus free from the issue of instability. Our scheme utilizes spin-orbit coupling. Specifically, we study a spin-1 BEC with Rashba spin-orbit coupling confined in a two-dimensional harmonic trap and numerically find the ground state of the system. For antiferromagnetic interactions, opposite vortices appear in the $m = 1, -1$ components with equal amplitude when the spin-orbit coupling is weak. Such a state carries a pure spin current and no mass current. This spin current is a superflow, as it is the ground state and must be stable.

We note that there has been a lot of theoretical and experimental work on the counterflow of two-species BECs [10–21] and other interesting work addressing the issue of spin superfluidity [22–25]. For two miscible BECs with counterflow, it is found that there is a critical relative speed between the two species, beyond which the state is dynamically unstable [11–19]. It is shown that the instability can lead to proliferation of solitons [14,15] and quantum turbulence [16]. This kind of counterflow is very similar to a spin current, but

it is not for two reasons: (i) Theoretically, if we regard the two species as two components of a pseudospin, this pseudospin has no SU(2) rotational symmetry, as the number of bosons in each species is conserved; and (ii) experimentally, it is hard to control the number of bosons in each component to create a spin current that has no mass current.

The paper is organized as follows. In Sec. II, we first study the stability of a spin-1 planar counterflow. We identify the mechanisms associated with the instabilities and find that the quadratic Zeeman effect can stabilize such a planar flow. We then study a similar situation in circular geometry in Sec. III. The pure spin current consists of a vortex and an antivortex in the $m = -1, 1$ components. The experimental scheme to realize the stable pure spin current is discussed in Sec. IV. Finally, we briefly summarize our main results in Sec. V.

II. PLANAR FLOW

The dynamics of a spin-1 BEC in free space is governed by the mean-field Gross-Pitaevskii equation [7],

$$i\hbar \frac{\partial}{\partial t} \psi_m = -\frac{\hbar^2 \nabla^2}{2M} \psi_m + c_0 \rho \psi_m + c_2 \sum_{n=-1}^1 \mathbf{s} \cdot \mathbf{S}_{mn} \psi_n, \quad (1)$$

where ψ_m ($m = 1, 0, -1$) are the components of the macroscopic wave function. $\rho = \sum_{m=-1}^1 |\psi_m|^2$ is the total density, $\mathbf{s}_i = \sum_{mn} \psi_m^* (S_i)_{mn} \psi_n$ is the spin density vector, and $\mathbf{S} = (S_x, S_y, S_z)$ is the spin operator vector, with S_i ($i = x, y, z$) being the three Pauli matrices in the spin-1 representation. The collisional interactions include a spin-independent part, $c_0 = 4\pi \hbar^2 (a_0 + 2a_2)/3M$, and a spin-dependent part, $c_2 = 4\pi \hbar^2 (a_2 - a_0)/3M$, with a_f ($f = 0, 2$) being the s -wave scattering length for spin-1 atoms in the symmetric channel of total spin f .

We consider a spin current state of the above Gross-Pitaevskii equation with the form

$$\psi = \sqrt{\frac{n}{2}} \begin{pmatrix} e^{i\mathbf{k}_1 \cdot \mathbf{r}} \\ 0 \\ e^{i\mathbf{k}_2 \cdot \mathbf{r}} \end{pmatrix}, \quad (2)$$

where n is the density of the uniform BEC. The wave function above describes a state with component $m = 1$ moving at speed $\hbar \mathbf{k}_1/M$, component $m = -1$ moving at speed $\hbar \mathbf{k}_2/M$,

and component $m = 0$ stationary. The requirement of equal chemical potential leads to $|\mathbf{k}_1| = |\mathbf{k}_2|$. In the case where $\mathbf{k}_1 = -\mathbf{k}_2$, this state carries a pure spin current: the total mass current is 0, as it has equal mass counterflow, while the spin current is nonzero.

It is instructive to first consider the special case where there is no counterflow, i.e., $\mathbf{k}_1 = \mathbf{k}_2 = 0$. The excitation spectra are found to be $\epsilon^0 = \sqrt{2c_2n\epsilon_q + \epsilon_q^2}$ and $\epsilon_1^{\pm 1} = \sqrt{2c_0n\epsilon_q + \epsilon_q^2}$, $\epsilon_2^{\pm 1} = \sqrt{2c_2n\epsilon_q + \epsilon_q^2}$, respectively, with $\epsilon_q = \hbar^2 q^2 / 2M$. So for antiferromagnetic interaction ($c_0 > 0$, $c_2 > 0$), all branches of the spectra are real and there is a double degeneracy in one branch of the spectra. The phonon excitations give two sound velocities, $\sqrt{nc_i/M}$ ($i = 0, 2$), corresponding to the speeds of the density wave and spin wave, respectively. However, the existence of phonon excitation does not mean that the pure spin current ($\mathbf{k}_1 = -\mathbf{k}_2 \neq 0$) is a superflow, as we cannot obtain a current with $\mathbf{k}_1 = -\mathbf{k}_2 \neq 0$ from a state with $\mathbf{k}_1 = \mathbf{k}_2 = 0$ by a Galilean transformation, nor can we infer excitation of a state $\mathbf{k}_1 = -\mathbf{k}_2 \neq 0$ from that of a state $\mathbf{k}_1 = \mathbf{k}_2 = 0$ similarly.

The stability of the spin current has been studied in Ref. [10] for the case $\mathbf{k}_1 = -\mathbf{k}_2 \neq 0$. It is found that, for the antiferromagnetic interaction case ($c_0 > 0$, $c_2 > 0$), the excitation spectrum of the $m = 0$ component always has nonzero imaginary part in the long-wavelength limit as long as there is counterflow between the two components, and the imaginary excitations in the $m = 1, -1$ components only appear for a high enough relative velocity $v_1 = 2\sqrt{nc_2/M}$. For the ferromagnetic interaction case ($c_0 > 0$, $c_2 < 0$), both excitation spectra of the $m = 0$ and $m = 1, -1$ components have nonzero imaginary parts for any relative velocity. This means that the pure spin current cannot be stable in any case.

For the general noncollinear case ($\mathbf{k} = \frac{\mathbf{k}_1 + \mathbf{k}_2}{2} \neq 0$) and antiferromagnetic interaction, the excitation spectrum for the $m = 0$ component is found to be

$$\epsilon^0 = \sqrt{\left(\epsilon_q + \frac{\hbar^2}{2M}(|\mathbf{k}|^2 - |\mathbf{k}_1|^2) + c_2n\right)^2 - c_2^2n^2}. \quad (3)$$

We see here that as long as the momenta of the two components are not exactly parallel, i.e., \mathbf{k}_1 is not exactly equal to \mathbf{k}_2 , then $|\mathbf{k}| < |\mathbf{k}_1|$, and there is always dynamical instability for the long-wavelength excitations.

Therefore, the spin current in Eq. (2) is generally unstable and not a superflow. This instability originates from the interaction process described by $\psi_0^\dagger \psi_0^\dagger \psi_1 \psi_{-1}$ in the second quantized Hamiltonian. This energetically favored process converts two particles in the $m = 1, -1$ components, respectively, into two stationary particles in the $m = 0$ component. To suppress this process and achieve a stable pure spin current, one can utilize the quadratic Zeeman effect. With the quadratic Zeeman effect of a negative coefficient, the Hamiltonian adopts an additional term, λm^2 ($\lambda < 0$ and $m = 1, 0, -1$). This term does not change the energy of the $m = 0$ component but lowers the energy of the other two components, $m = 1, -1$. As a result, there arises a barrier for two atoms in the $m = 1, -1$ components scattering to the $m = 0$ component, and the scattering process is thus suppressed.

The above intuitive argument can be made more rigorous and quantitative. Consider the case $\mathbf{k}_1 = -\mathbf{k}_2$. With the

quadratic Zeeman term, the excitation spectrum for the $m = 0$ component changes to

$$\epsilon^0 = \sqrt{\left(\epsilon_q - \frac{\hbar^2|\mathbf{k}_1|^2}{2M} + c_2n - \lambda\right)^2 - c_2^2n^2}. \quad (4)$$

So as long as $-\lambda - \hbar^2|\mathbf{k}_1|^2/2M > 0$, long-wavelength excitations will be stable for the $m = 0$ component. From the excitation of the $m = 0$ component, one can obtain a critical relative velocity of the spin current, $v_0 = 2\sqrt{-2\lambda/M}$. There is another nonzero critical velocity, $v_1 = 2\sqrt{nc_2/M}$, determined by the excitations of the $m = 1, -1$ components. The overall critical velocity of the system is the lower one of v_0 and v_1 . Therefore, below the critical relative velocity $v_c = \min\{v_0, v_1\}$, the pure spin current is stable and a superflow. The experimental scheme to realize such a Zeeman effect is discussed in Sec. IV.

III. CIRCULAR FLOW

In cylindrical geometry, we consider a pure spin current formed by two vortices with opposite circulation in the $m = 1, -1$ components. From similar arguments, one can expect that interaction will make such a current unstable. Inspired by the quadratic Zeeman effect method above, we propose to use spin-orbit coupling to stabilize it. The spin-orbit coupling can be viewed as a momentum-dependent effective magnetic field that is exerted only on the $m = 1, -1$ components. Therefore, it is possible that spin-orbit coupling lowers the energy of $m = 1, -1$ components and, consequently, suppresses the interaction process leading to the instability.

The model of a spin-1 BEC subject to Rashba spin-orbit coupling can be described by the energy functional

$$\mathcal{E}[\psi_\alpha] = \int d\mathbf{r} \left\{ \sum_\alpha \frac{\hbar^2 |\nabla \psi_\alpha|^2}{2M} + \rho V(r) + \frac{\tilde{c}_0}{2} \rho^2 + \frac{\tilde{c}_2}{2} \mathbf{s}^2 + \gamma \langle S_x p_y - S_y p_x \rangle \right\}, \quad (5)$$

where ρ is the density, $V(r) = \frac{1}{2}M\omega^2(x^2 + y^2)$ is the trapping potential, and γ is the strength of the spin-orbit coupling. $\langle \dots \rangle$ is the expectation value taken with respect to the three-component wave function $\psi = (\psi_1, \psi_0, \psi_{-1})^T$. Here we assume that the confinement in the z direction is very tight with trapping frequency $\hbar\omega_z \gg \mu, k_B T$, μ being the chemical potential and T the temperature, and have integrated the model with respect to the z direction to obtain an effective two-dimensional system. The two-dimensional coupling constants are related to the three-dimensional scattering lengths through the relations $\tilde{c}_0 = \sqrt{8\pi}\hbar^2(a_0 + 2a_2)/3Ml_z$ and $\tilde{c}_2 = \sqrt{8\pi}\hbar^2(a_2 - a_0)/3Ml_z$, with $l_z = \sqrt{\hbar/M\omega_z}$ [26]. The strength of the spin-orbit coupling γ defines a characteristic length $a_{\text{soc}} = \hbar/M\gamma$ and can be rescaled to be dimensionless with respect to the harmonic oscillator length $a_h = \sqrt{\hbar/M\omega}$. Then we characterize the strength of spin-orbit coupling with the dimensionless quantity $\kappa = a_h/a_{\text{soc}} = \gamma\sqrt{M/\hbar\omega}$. Spin-orbit coupling of the Rashba type here can be generated in various ways, which are discussed in the next section.

The above model can describe a spin-1 BEC of ^{23}Na confined in a pancake trap. Assume that the atom number is

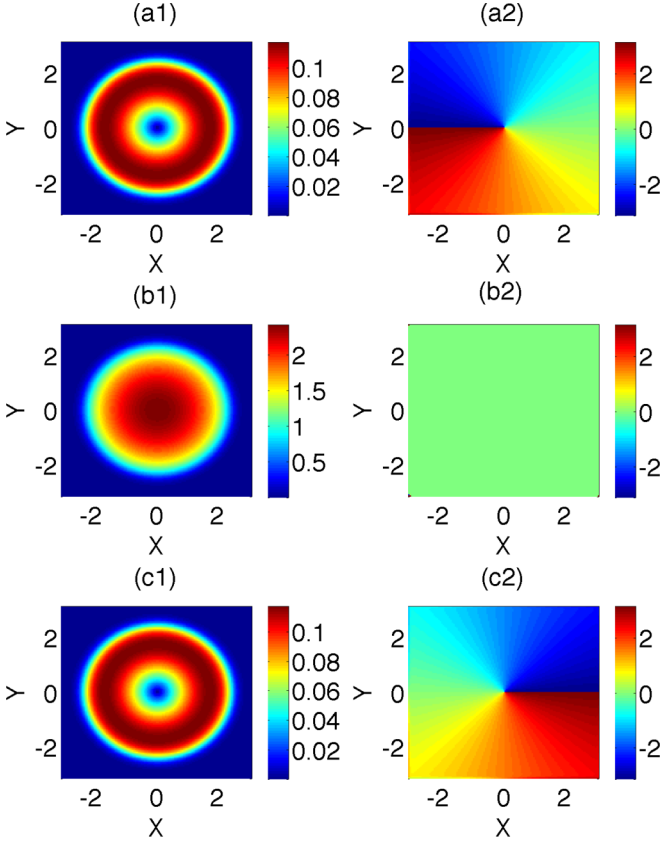


FIG. 1. (Color online) Amplitudes (a1, b1, c1) and phase angles (a2, b2, c2) of the three-component wave function $\psi = (\psi_1, \psi_0, \psi_{-1})^T$ in the ground state of Hamiltonian (5) for a BEC of ^{23}Na confined in a pancake trap. The particle number is 10^5 , the radial frequency of the trap is $2\pi \times 55$ Hz, and the dimensionless spin-orbit coupling strength is $\kappa = 0.04$. The confinement in the z direction is very tight, with $\omega_z = 2\pi \times 4200$ Hz, so the system is effectively two-dimensional. Units of the X and Y axes are a_h .

about 10^5 . Using the estimate of scattering lengths $a_0 = 50a_B$, $a_2 = 55a_B$ [27], with a_B being the Bohr radius, the ground state of spin-1 ^{23}Na should be antiferromagnetic because $\tilde{c}_0 > 0$, $\tilde{c}_2 > 0$ [5]. Previous studies of spin-1 BECs with Rashba spin-orbit coupling mostly focus on the strong spin-orbit coupling regime, where the ground state is found to be the plane-wave phase or the stripe phase, for ferromagnetic interaction and antiferromagnetic interaction, respectively [28]. Here we are interested in the antiferromagnetic interaction case and the weak spin-orbit coupling regime ($\kappa \ll 1$) and calculate the ground-state wave function of the energy functional with the method of imaginary time evolution.

We find that when the spin-orbit coupling is weak ($\kappa \ll 1$), the ground-state wave function has the form

$$\psi = \begin{pmatrix} \chi_1(r)e^{-i\phi} \\ \chi_0(r) \\ \chi_{-1}(r)e^{i\phi} \end{pmatrix}, \quad (6)$$

with $\chi_1(r) = -\chi_{-1}(r)$ and all χ_i real. The ground state is shown in Fig. 1. This ground state consists of an antivortex in the $m = 1$ component and a vortex in the $m = -1$ component.

The $m = 0$ component does not carry angular momentum. Since $|\psi_1| = |\psi_{-1}|$, the net mass current vanishes.

The wave function in Eq. (6) can be understood at the single-particle level. In terms of the ladder operators of spin and angular momentum, the spin-orbit coupling term reads

$$\mathcal{H}_{\text{soc}} = \frac{\gamma\sqrt{M\hbar\omega}}{2} [S_+(\hat{a}_R - \hat{a}_L^\dagger) + S_-(\hat{a}_R^\dagger - \hat{a}_L)], \quad (7)$$

where S_\pm is the ladder operator of spin, and $\hat{a}_{L(R)}^\dagger$ is the creation operator of the left (right) circular quanta [29]. When the spin-orbit coupling is very weak ($\kappa \ll 1$), its effect can be accounted for in a perturbative way. From the ground state $\Psi^{(0)} = |0,0\rangle$, the first-order correction to the wave function for small γ is given by

$$\begin{aligned} \Psi^{(1)} &= \frac{\gamma\sqrt{M\hbar\omega}}{2\hbar\omega} (-S_+\hat{a}_L^\dagger + S_-\hat{a}_R^\dagger)|0,0\rangle \\ &= \frac{\kappa}{2} (|-1,-1\rangle + |-1,1\rangle), \end{aligned} \quad (8)$$

where $|m_s, m_o\rangle$ denotes a state with spin quantum number m_s and orbital magnetic quantum number m_o . One immediately sees that ψ_1 has angular momentum $-\hbar$ and ψ_{-1} has angular momentum \hbar . Besides, the amplitudes of both ψ_1 and ψ_{-1} are proportional to κ .

There exists a continuity equation for the spin density and spin current, which is

$$\frac{d}{dt}(\psi^\dagger \mathbf{S}_\mu \psi) + \nabla \cdot \mathbf{J}_\mu^s = 0. \quad (9)$$

The spin current density tensor \mathbf{J}_μ^s ($\mu = x, y, z$ denotes the spin component) is defined as [30,31]

$$\begin{aligned} \mathbf{J}_\mu^s &= \frac{1}{2} \{ \psi^\dagger S_\mu \mathbf{v} \psi + \text{c.c.} \} \\ &= \frac{1}{2} \left\{ \sum_{m,n,l} \psi_m^* (S_\mu)_{mn} \mathbf{v}_{nl} \psi_l + \text{c.c.} \right\}, \end{aligned} \quad (10)$$

where

$$\mathbf{v}_{nl} = \frac{\mathbf{p}}{M} + \gamma (\hat{z} \times \mathbf{S}_{nl}), \quad (11)$$

and c.c. means the complex conjugate. The second part of \mathbf{v}_{nl} is induced by the spin-orbit coupling.

By the definition in Eq. (10), the spin current density carried by the ground state, (6), is

$$\begin{aligned} \mathbf{J}_x^s &= \gamma \sin 2\phi |\psi_1|^2 \hat{x} + \gamma (|\psi_0|^2 + 2|\psi_1|^2 \sin^2 \phi) \hat{y}, \\ \mathbf{J}_y^s &= -\gamma (|\psi_0|^2 + 2|\psi_1|^2 \cos^2 \phi) \hat{x} - \gamma \sin 2\phi |\psi_1|^2 \hat{y}, \\ \mathbf{J}_z^s &= \left(-\frac{2\hbar |\psi_1|^2}{Mr} + \sqrt{2}\gamma |\psi_1 \psi_0| \right) \hat{\phi}. \end{aligned} \quad (12)$$

From both analytical and numerical results of the wave function, $|\psi_1| \ll |\psi_0|$, so \mathbf{J}_x^s roughly points in the y direction, while \mathbf{J}_y^s almost points in the $-x$ direction. \mathbf{J}_z^s represents a flow whose amplitude has rotational symmetry. From the numerical results shown in Fig. 2, we see that \mathbf{J}_z^s is a counterclockwise flow. The amplitudes of \mathbf{J}_x^s and \mathbf{J}_y^s are of the same order, both proportional to κ , while that of \mathbf{J}_z^s , proportional to κ^2 , is much lower. It is evident that the state in Eq. (6) carries no mass

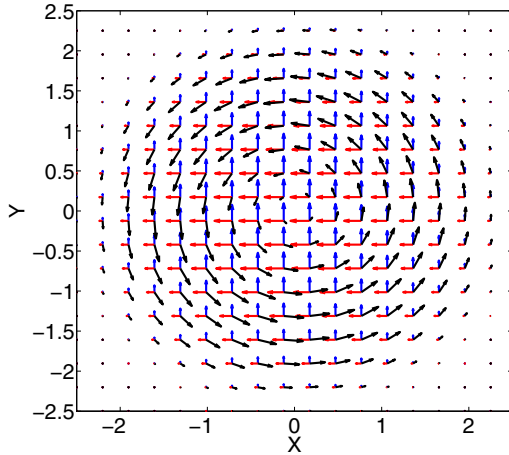


FIG. 2. (Color online) Distribution of the spin current densities \mathbf{J}_x^s (blue arrow), \mathbf{J}_y^s (red arrow), and \mathbf{J}_z^s (black arrow) of the ground state shown in Fig. 1. The length of the arrows represents the strength of the spin current. The arrow length of different colors is not to scale. $\kappa = 0.04$. Units of the X and Y axes are a_h .

current and only pure spin current. Since the spin current is in the ground state, it must be stable. In this way, we have realized a superfluid of pure spin current, or a pure spin supercurrent.

IV. EXPERIMENTAL SCHEMES

In this section, we propose the experimental schemes to generate and detect the pure spin currents discussed in Secs. II and III.

The planar pure spin current can be easily generated. By applying a magnetic-field gradient, the two components $m = 1, -1$ will be accelerated in opposite directions and a pure spin current is generated as done in Refs. [14] and [15]. To stabilize this spin current, one needs to generate the quadratic Zeeman effect. We apply an oscillating magnetic field $B \sin \omega t$, with the frequency ω being much higher than the characteristic frequency of the condensate, e.g., the chemical potential μ . The time averaging removes the linear Zeeman effect; only the quadratic Zeeman effect remains. The coefficient of the quadratic Zeeman effect from the second-order perturbation theory is given by $\lambda = (g\mu_B B)^2 / \Delta E_{\text{hf}}$, where g is the Landé g factor of the atom, μ_B is the Bohr magneton, and ΔE_{hf} is the hyperfine energy splitting [32]. For the $F = 2$ manifold of ^{87}Rb , $\Delta E_{\text{hf}} < 0$, so the coefficient of the quadratic Zeeman effect is negative.

The circular flow in Sec. III may find prospective realizations in two systems: cold atoms and exciton BEC. In cold atoms, we consider a system consisting of a BEC of ^{23}Na confined in a pancake trap, where the confinement in the z

direction is so tight that one can treat the system effectively as two-dimensional. Spin-orbit coupling can be induced by two methods. One is by the exertion of a strong external electric field \mathbf{E} in the z direction. Due to the relativistic effect, the magnetic moment of the atom will experience a weak spin-orbit coupling, where the strength $\gamma = g\mu_B |\mathbf{E}| / Mc^2$. Here M is the atomic mass and c is the speed of light. For weak spin-orbit coupling (small γ), the fraction of atoms in the $m = 1, -1$ components is proportional to γ^2 . For an experimentally observable fraction of atoms, e.g., 0.1% of 10^5 atoms, using the typical parameters of ^{23}Na BEC, the estimated electric field is of the same order of magnitude as the vacuum breakdown field. For atoms with a smaller mass or larger magnetic moment, the required electric field can be lowered. Another method of realizing spin-orbit coupling is to exploit the atom-laser interaction, where, in principle, strong spin-orbit coupling can be created [33]. In exciton BEC systems, as the effective mass of the exciton is much smaller than that of the atom, the required electric field is four to five orders of magnitude smaller, which is quite feasible in experiments [34–37].

The vortex and antivortex in the $m = 1, -1$ components can be detected by the method of time of flight. First, one can split the three spin components with the Stern-Gerlach effect. The appearance of a vortex or an antivortex in the $m = 1, -1$ components is signaled by a ring structure in the time-of-flight image. After a sufficiently long expansion time, the ring structure should be clearly visible.

V. CONCLUSION

In summary, we have studied the stability of a pure spin current of a spin-1 BEC. In planar flow, the system always suffers dynamical instability. The origin of the instability is the interaction process that converts two particles in the $m = 1, -1$ components into the $m = 0$ component. Based on this, we propose a method to stabilize the pure spin current by utilizing the quadratic Zeeman effect. In circular flow, we have proposed using spin-orbit coupling to make the pure spin current stable. For weak spin-orbit coupling, we have found that the ground state of the system is a superfluid of pure spin current. The experimental schemes to realize and detect these pure spin currents have been discussed.

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