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# Symmetric and antisymmetric Dzyaloshinskii-Moriya solitons in anisotropic ferromagnetic wires

Zai-Dong Li<sup>a,b,c,\*</sup>, Qi-Long Bao<sup>a</sup>, Peng-Bin He<sup>d</sup>, Tian-Fu Xu<sup>e</sup>, B. Wu<sup>f</sup>

<sup>a</sup> School of Science, Tianjin University of Technology, Tianjin 300384, China

<sup>b</sup> State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, Taiyuan 030006, China

<sup>c</sup> Department of Applied Physics, Hebei University of Technology, Tianjin 300401, China

<sup>d</sup> School of Physics and Electronics, Hunan University, Changsha 410082, China

<sup>e</sup> Hebei Key Laboratory of Microstructural Material Physics, School of Science, Yanshan University, Qinhuangdao 066004, China

<sup>f</sup> International Center for Quantum Materials, School of Physics, Peking University, Beijing 100871, China

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# ABSTRACT

We theoretically investigate the effect of Dzyaloshinskii-Moriya interaction (DMI) on magnetic solitons in anisotropic ferromagnetic wires driven by the adiabatic spin-transfer torque. The DMI changes the formation region of soliton solution and the corresponding phase diagram is given for different types of soliton solutions. Also, it affects the processional frequency and the internal structure distortion of solitons. We show that the DMI decreases the energy for symmetric soliton while increases the energy for antisymmetric one. It implies that the DMI favours the more steady state of magnetization for the case of strong easy-axis anisotropy.

# 1. Introduction

The antisymmetric exchange is a key contribution to the total magnetic exchange interactions between two neighboring magnetic spins,  $S_i$  and  $S_j$ . It was first postulated by Igor Dzyaloshinskii on the grounds of phenomenological considerations based on Landau theory [1]. Toru Moriya identified the spin-orbit coupling as the microscopic mechanism of the antisymmetric exchange interaction [2]. Therefore, this term is also called the Dzyaloshinskii-Moriya interaction (DMI), which favors a spin canting of otherwise (anti) parallel aligned magnetic moments in magnetically ordered systems. It can be written as  $\mathscr{H}_{DM} = \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$ , where  $\mathbf{D}_{ij}$  is the DMI vector [3,4]. Recently, the DMI has been found both for magnetic interfaces [3-5] and bulk materials such as FeGe [6] and MnSi [7]. There are also many topic researches for the effect of DMI, which arises from the spin-orbit scattering of electrons in the inversion symmetry broken system, such as metallic alloys with B20 structure [7,8,11,9,10]. The DMI can be regarded as an antisymmetric anisotropic super-exchange interaction in a metallic system [12], and it vanishes when the system is centrosymmetric [13]. The DMI also plays a crucial role in stabilizing the chiral spin textures, which tends to make the magnetization rotate around a local characteristic vector and favors a certain chirality, such as Skyrmions [14] and spin spirals [15-21,23,22] in low-dimensional metallic magnets.

It is worth to note that the DMI has brought new phenomena for domain wall dynamics driven by the magnetic fields [24–29] or charge currents [35,30-34]. The DMI changes the drift velocity of domain wall in ferromagnetic nanowires [35,37,36]. Various novel DMI related phenomena, such as the magnon Hall effect [38], molecular magnetism [39], and multiferroicity [40], have been observed. In ferromagnet, the nonlinear magnetic excitations include domain walls and solitons. A magnetic domain wall is a spatially localized configuration of magnetization in a ferromagnet, in which the magnetic moment reverses its direction gradually over a finite length. The dynamics of domain wall [41-44] is of great significance in ferromagnetic nanowires for its potential technological applications, where the external magnetic field [45] and spin-polarized current [46] can be used to manipulate and control the domain wall motion. The Walker solution analysis [47-49] has been extensively adopted to investigate the moving domain wall in response to a magnetic field [50] or spin-polarized current [51–53].

So far, most attentions have been drawn on the current-driven DWs. However, except for DW, there is another nonlinear magnetic excition, i.e., magnetic soliton. A magnetic soliton describes the localized states of magnetization which can be reduced to a uniform magnetization by continuous deformation, which is different from domain wall. Magnetic solitons also play a critically important role in nanomagnetism, and they define the typical scale for magnetic texture and enable highdensity magnetic storage [54–56]. Using spin-transfer torque the rich

\* Corresponding author.

E-mail address: lizd@email.tjut.edu.cn (Z.-D. Li).

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Fig. 1. Schematic view of ferromagnetic wire with a magnetic soliton.

dynamic properties of dissipative magnetic droplet solitons [57,58] have been observed in a magnetic thin film with perpendicular magnetic anisotropy.

In this work, we consider the bulk DMI with micromagnetic energy density  $\varepsilon_{DMI} = D\mathbf{m} \cdot (\nabla \times \mathbf{m})$ , where *D* is the DMI constant and  $\mathbf{m}$  is the normalized magnetization vector  $\mathbf{m} = \mathbf{M}/M_s$ . Here  $\mathbf{M}$  is a local magnetization vector with a saturation magnetization  $M_s$ . We investigate the nonlinear dynamics of Dzyaloshinskii-Moriya soliton solutions in anisotropic ferromagnetic wires driven by the adiabatic spin-transfer torque. Three types of solutions appear in different regions in the parameter space defined by the DMI and the magnetic anisotropies. The DMI effects can be extracted from these exact results, which can cause the precess of in-plane component of magnetization in the form of a spin wave. The DMI also affects the width and energy of the solitons. These results can open up new possibilities for the investigation of low-dimensional nonlinear magnetic dynamics.

# 2. Dzyaloshinskii-Moriya soliton solutions

We consider a uniaxial anisotropic ferromagnetic wire along *z*-axis, as shown in Fig. 1. In the continuous limit its dimensionless Hamiltonian is [35,37]

$$\mathcal{H} = \int dz \left\{ \frac{J}{2} \left( \frac{\partial \mathbf{m}}{\partial z} \right)^2 + \lambda_z m_z^2 + D_0 \mathbf{m} \cdot (\mathbf{e}_z \times \frac{\partial \mathbf{m}}{\partial z}) \right\},\tag{1}$$

where *J* is the exchange interaction constant,  $\lambda_z$  is the anisotropy constant,  $D_0$  represents the DMI constant assuming that the wire is cut or grown along the DMI vector, *z* is dimensionless space, and  $\mathbf{e}_z$  is the unit vector in the *z* direction. When a spin-polarized electric current flows along the ferromagnetic wire, its magnetization dynamics can be well described by the modified dimensionless Landau-Lifshitz-Gilbert equation [51,52,35]

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} - j \frac{\partial \mathbf{m}}{\partial z}, \qquad (2)$$

where  $\mathbf{h}_{\text{eff}} = \partial^2 \mathbf{m} / \partial z^2 - 2D_0 (\mathbf{e}_z \times \partial \mathbf{m} / \partial z) - \lambda_z m_3 \mathbf{e}_z$ , and *j* is the current term [35]. We are interested in the soliton excitation in this system. These solitons will gradually decay into a uniform magnetization due to the Gilbert damping. We will leave the life time of these solitons for future discussion and focus entirely what types of solitons exist in this system.

To this purpose, we take the following stereographic projection

$$m_1 = \frac{\Psi + \Psi^*}{1 + \Psi\Psi^*}, \ m_2 = -i\frac{\Psi - \Psi^*}{1 + \Psi\Psi^*}, \ m_3 = \frac{1 - \Psi\Psi^*}{1 + \Psi\Psi^*},$$
(3)

with which the equation of the complex function  $\Psi$  can be obtained from Eq. (2),

$$(1 + \Psi\Psi^*)\left(\frac{\partial\Psi}{\partial t} - i\frac{\partial^2\Psi}{\partial z^2} + j\frac{\partial\Psi}{\partial z}\right) + 2i\Psi^*\left(\frac{\partial\Psi}{\partial z}\right)^2 - 2D_0\left(1 - \Psi\Psi^*\right)\frac{\partial\Psi}{\partial z} - i\lambda_z\Psi(1 - \Psi\Psi^*) = 0.$$
(4)

Although the above equation is quite complex, it can be solved straightforwardly by asking for another transform  $\Psi = g/f$ , where the function g(z, t) is complex and f(z, t) is real. Applying this transform

and performing a tedious calculation, we obtain

$$g = e^{\theta + i\varphi}, \quad f = 1 + Be^{2\theta}, \tag{5}$$

where

g

$$\begin{aligned} \theta &= \kappa(z - jt), \\ \varphi &= D_0 z + \Omega t, \\ \Omega &= \lambda_z + \kappa^2 + D_0^2 - jD_0, \\ B &= -(\lambda_z + \kappa^2 + D_0^2)/(4\kappa^2). \end{aligned}$$
(6)

Here  $\kappa$  is the only free parameter in the solution and it affects the width of the soliton solution as we shall see in the following section. The final solutions are found through Eq. (3) with  $\Psi = g/f$ . From Eq. (1) we see that there are two cases, i.e., easy-axis ( $\lambda_z < 0$ ) and easy-plane ( $\lambda_z > 0$ ) anisotropic ferromagnet. It is very interesting to find the characteristic region of soliton excitation in the presence of DMI.

#### 2.1. Dzyaloshinskii-Moriya soliton solution for strong easy-axis anisotropy

In this section, we focus on the case of the strong easy-axis anisotropy. With the help of Eqs. (3) and (5) we obtain the soliton solution denoted by  $S_I$ ,

$$m_{1} = \frac{4\sqrt{B}\cosh\theta \cos\varphi}{1 + 4B\cosh^{2}\theta},$$

$$m_{2} = \frac{4\sqrt{B}\cosh\theta \sin\varphi}{1 + 4B\cosh^{2}\theta},$$

$$m_{3} = 1 - \frac{2}{1 + 4B\cosh^{2}\theta},$$
(7)

where the parameters  $\theta$  and  $\varphi$  have been given in Eq. (6). The solution in Eq. (7) demands the condition B > 0. This solution is plotted in Fig. 2, which clearly shows that it is a soliton solution. From Fig. 2 we can see that the magnetization deviates firstly from the ground state with the clockwise direction, and then returns with the anticlockwise direction. Therefore, the magnetization excitation forms a symmetrical shape relative to the maximum deviation. This soliton excitation can propagate with the speed *j*. It should be noted that Eq. (7) is a completely new form soliton solution of Landau-Lifshitz Eq. (2) with no damping term. In the absence of DMI (i.e.,  $D_0 = 0$ ), the solution is also different from the magnetic soliton in easy-axis anisotropic case [59].

The new soliton solution in Eq. (7) shows that the DMI affects many aspects of the soliton solutions. According to Eqs. (6) and (7), the propagation of this soliton is accompanied by a spin wave, in which the two in-plane components  $m_1$  and  $m_2$  of soliton precess in the form of a traveling wave with phase speed  $V_p = j - D_0 - (\lambda_z + \kappa^2)/D_0$  and group velocity  $V_g = j - 2D_0 - (\lambda_z + \kappa^2)$ . The corresponding wave number is  $D_0$ , which implies the DMI favours the space phase difference of a spin wave. Without the DMI (i.e.,  $D_0 = 0$ ), from Eq. (6) we obtain  $\varphi = \Omega t$  with  $\Omega = \lambda_z + \kappa^2$ . It shows that the components  $m_1$  and  $m_2$  precess uniformly in space and there is no spin wave. The maximum deviation is also affected by the DMI. As seen in Fig. 2 and Eq. (7), the lowest amplitude of  $m_3$  is given by 1 - 2/(1 + 4B), which decreases with



**Fig. 2.** Graphical illustration of symmetric Dzyaloshinskii-Moriya soliton solution for strong easy-axis anisotropy in Eq. (7). The magnetization excitation forms a symmetrical shape relative to the maximum deviation. The parameters are j = 0.5,  $\kappa = -0.3$ ,  $D_0 = 0.4$ , and  $\lambda_z = -0.3$ .



**Fig. 3.** The variation of the soliton width changes with the parameter  $\kappa$  and the DMI term  $D_0$  for symmetric Dzyaloshinskii-Moriya soliton solution with strong easy-axis anisotropy in Eq. (7). The soliton width is dimensionless and becomes smaller with increasing  $\kappa$  while becomes larger with the increasing DMI parameter  $|D_0|$ . Here, the uniaxial anisotropic parameter  $\lambda_{\chi} = -0.4$ .

increasing  $|D_0|$ . The width of this soliton is affected by DMI. It becomes smaller with increasing  $\kappa$  while becomes larger with the increasing DMI parameter  $|D_0|$ , as shown in Fig. 3.

It is significative to investigate the energy of soliton excitation state. To this purpose, by substituting Eq. (7) into Eq. (1), we get the dimensionless form of soliton energy

$$\varepsilon_1 = 4C_0 \operatorname{arctanh} \sqrt{\xi_1 - 4\xi_1 D_0^2 / \kappa},\tag{8}$$

where  $\xi_1 = \kappa^2/(|\lambda_z| - D_0^2)$  and  $C_0 = \sqrt{\xi_1}\lambda_z(1 - \xi_1)/\kappa$ . From Eq. (8) we clearly find that due to the non-centrosymmetric characteristic in ferromagnet the soliton excitation state takes a unique change with the DMI. The soliton energy decreases with the strength of DMI. In the absence of DMI, the soliton energy approaches the maximum value. This result implies that the DMI can favour the more steady state of magnetization.

# 2.2. Dzyaloshinskii-Moriya soliton solution for weak easy-axis anisotropy and easy-plane anisotropy

In this section, we consider the case of weak easy-axis anisotropy and easy-plane anisotropy. The parameter *B* in Eq. (9) obeys B < 0, which can be satisfied for the weak easy-axis anisotropy ( $\lambda_z < 0$ ) and easy-plane anisotropy ( $\lambda_z > 0$ ). By a tedious calculation, the system admits soliton excitation state denoted by S<sub>II</sub>,

$$m_{1} = -\frac{4\sqrt{-B}\sinh^{2}\cos\varphi}{1 - 4B\sinh^{2}\theta},$$

$$m_{2} = -\frac{4\sqrt{-B}\sinh^{2}\theta}{1 - 4B\sinh^{2}\theta},$$

$$m_{3} = 1 - \frac{2}{1 - 4B\sinh^{2}\theta},$$
(9)

where  $\theta$  and  $\varphi$  have been given in Eq. (6). This soliton is illustrated in Fig. 4. In this soliton excitation the magnetization deviates from and returns to the ground state with the clockwise direction. Therefore, the magnetization excitation forms an antisymmetric shape relative to the maximum deviation, which is different from the case S<sub>I</sub>, as shown in Fig. 4.

The soliton solution in Eq. (9) also shows that the DMI affects the soliton properties. Similar to  $S_I$ , this soliton also moves at the speed j, while its two in-plane components  $m_1$  and  $m_2$  precess as a spin wave with phase speed  $\Omega/D_0$  and wave number  $D_0$ . However, the precession pattern of the two in-plane components  $m_1$  and  $m_2$  of  $S_I$  differs from  $S_{II}$ . This difference becomes clear when one compares Eqs. (7) and (9). The soliton width of  $S_{II}$  changes with  $\kappa$  and the DMI value, i.e.,  $|D_0|$ . From Fig. 5 we observe that the soliton width decreases with increasing  $\kappa$ .



**Fig. 4.** Graphical illustration of antisymmetric Dzyaloshinskii-Moriya soliton solution for weak easy-axis anisotropy in Eq. (9). The magnetization excitation forms a antisymmetric shape relative to the maximum deviation. The parameters are j = 0.5,  $\kappa = -1$ ,  $D_0 = 0.35$ , and  $\lambda_z = -0.1$ .



**Fig. 5.** The variation of soliton width changes with the parameter  $\kappa$  and the DMI term  $D_0$  for antisymmetric Dzyaloshinskii-Moriya soliton solution with weak easy-axis anisotropy and easy-plane anisotropy in Eq. (9). The soliton width is dimensionless and decreases with increasing  $\kappa$ , while increases with increasing  $|D_0|$ . Here, the uniaxial anisotropic parameter  $\lambda_z$ , (a) easy-axis  $\lambda_z = -0.2$ , (b) easy-plane  $\lambda_z = 0.35$ .

This is different from type I, its width increases with increasing  $|D_0|$ .

In order to better understand the soliton properties (S<sub>II</sub>), we investigate the soliton energy for easy-axis and easy-plane anisotropy, respectively. For the easy-axis anisotropic case, which demand the conditions  $-1/4 \leq B < 0$ . Substituting Eq. (9) into Eq. (1), we can get the dimensionless form of soliton energy

$$\varepsilon_2 = 4\lambda_z \sqrt{\zeta_1} \left(1 - \zeta_1 \kappa^2\right) \operatorname{arccoth} \sqrt{\zeta_1 \kappa^2} - 4D_0^2 \zeta_1 \kappa, \tag{10}$$

where  $\zeta_1 = -1/(\lambda_z + D_0^2)$ . From Eq. (10) we can clearly see the effect of the non-centrosymmetric characteristic, i.e., DMI, on the soliton excitation in ferromagnet. Different from the case of S<sub>I</sub>, the soliton energy increases with the increasing DMI term  $|D_0|$  for the weak easy-axis anisotropy case. In the absence of the DMI, the soliton energy approaches the minimum value. The above results show that the DMI effects on the soliton energy are completely opposite for the cases of S<sub>I</sub> and S<sub>II</sub>. When the parameter  $B \rightarrow -1/4$ , the soliton solution can only exist in the condition  $|D_0| = \sqrt{|\lambda_z|}$ . The corresponding soliton energy takes the form

$$\varepsilon_3 = 4\kappa + 8\lambda_z / (3\kappa). \tag{11}$$

For the easy-plane anisotropic case ( $\lambda_z > 0$ ), the dimensionless soliton energy takes the form

$$\varepsilon_4 = 4\lambda_z \sqrt{\zeta_2} \left(1 + \zeta_2 \kappa^2\right) \operatorname{arccot}(\kappa \sqrt{\zeta_2}) + 4\kappa \zeta_2 D_0^2, \tag{12}$$

where  $\zeta_2 = 1/(\lambda_z + D_0^2)$ . It has been shown that the sign of DMI can be changed with deformation [60], and the strain affects the spin-wave transport in the magnetic structures [61]. From Eq. (12) we find that the soliton energy increases with DMI. It shows that the minimum value of soliton energy can be obtained in the absence of DMI. This result



**Fig. 6.** Phase diagram of symmetric Dzyaloshinskii-Moriya soliton solution for strong easy-axis anisotropy(S<sub>1</sub>), and antisymmetric Dzyaloshinskii-Moriya soliton solution for weak easy-axis anisotropy and easy-plane anisotropy (S<sub>11</sub>). S<sub>1</sub> stands for soliton type I, S<sub>11</sub> for type II. The two types of soliton solutions are separated by domain wall solutions (red dashed line). The color indicates the soliton energy variation and the soliton energy is dimensionless. Here  $D_0$  denotes the DMI term, and  $\lambda_z$  is uniaxial anisotropic parameter, and the parameter  $\kappa = 0.48$ .

implies that the presence of DMI can not favour the more steady state of magnetization in the easy-plane anisotropic ferromagnet.

With the above results, we can give a phase diagram to show how the DMI and the anisotropic parameters determine the soliton excitation, as shown in Fig. 6. It is well known that there exists the soliton excitation state in the case of easy-axis ( $\lambda_z < 0$ ) and easy-plane ( $\lambda_z > 0$ ) anisotropic ferromagnet. However, the presence of DMI can result in the characteristic region of soliton excitation for the non-centrosymmetric ferromagnet, and the corresponding graphic illustration has been denoted in Fig. 6. For the easy-axis case, two types of soliton solutions are separated by domain wall solutions (red dashed line), while there is no soliton solutions in the white region in the presence of DMI. In Fig. 6, the color indicates the dimensionless soliton energy variation in Eqs. (8), (10), and (12), respectively. It is obvious that the presence of DMI favours the more steady state of magnetization in the easy-plane anisotropic ferromagnet.

# 3. Conclusion

We have obtained the exact soliton solutions in anisotropic ferromagnetic wires with DMI. There are three different solutions, i.e.,  $S_I$  and  $S_{II}$  for easy-axis and easy-plane case. A phase diagram is drawn to show how the types of the solutions are determined by the DMI and the anisotropic parameters. We have found that the in-plane components of the magnetization precess in a spin wave fashion due to the DMI. The soliton energy is calculated analytically. The formula shows that the DMI decreases the energy in  $S_I$  while increases the energy in  $S_{II}$ .

# CRediT authorship contribution statement

Zai-Dong Li: Supervision, Methodology, Writing - review & editing. Qi-Long Bao: Methodology. Peng-Bin He: Visualization, Investigation. Tian-Fu Xu: Software. B. Wu: Writing - review & editing.

# **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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