THE HADRONIC CONTRIBUTION TO $(g-2)_\mu$

K. Maltman, Beijing, June 16, 2005

OUTLINE

- *Background*
- *The EM vs. τ discrepancy for the "LO hadronic contribution"*
- *Resolving the EM vs. τ puzzle*
- *Some comments on IB*

SOME BASICS

Current experimental situation (BNL E821)

- $a_\mu \equiv \frac{(g-2)_\mu}{2} = (11\,659\,208.0 \pm 6.0) \times 10^{-10}$ [PRL92 (2004) 161802, $\mu^\pm$ average]

- 0.5 ppm determination (BNL E969 proposal:$\rightarrow$ 0.2 ppm)

- 0.5 ppm already $<$ current SM prediction uncertainty, and $<$ 1% LO hadronic contribution

- EM-data-based SM prediction differs from experiment by $2-3\,\sigma$, τ-data-based version does not (Table)

The SM prediction c.f. experiment, units 10^{-10}

(First error: exp't; Second error: theory$\oplus$exp't)

$a_\mu^{exp} - a_\mu^{SM}$	EM or τ	Significance	Source
23.5 (6.9)(9.1)	EM	2.6σ	ICHEP04
22.1 (9.0)(10.8)	EM	2.1σ	Jeger04
23.5 (9.5)(11.3)	EM	2.1σ	ELZ05
24.5 (6.9)(9.1)	EM	2.7σ	HMNT04
22.5 (5.5)(8.1)	EM	2.8σ	TY04
6.2 (6.3)(8.7)	τ	0.7σ	DEHZ03
14.2 (5.4)(8.1)	EM+τ	1.8σ	TY04

c.f. experimental value $a_\mu \times 10^{-10} = 11659208 \pm 6$

SM EXPECTATION FOR a_μ

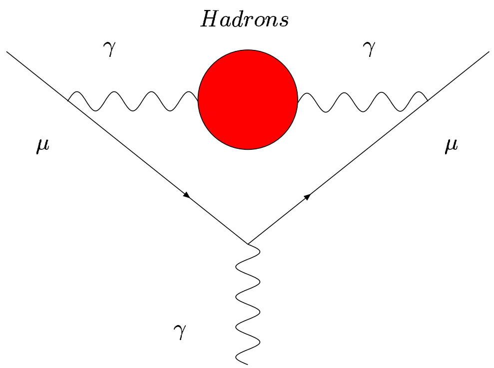
- Pure QED: dominant, known to 4-loops (plus)
- LO hadronic vac pol'n: $\sim 0.00006\ [a_\mu]_{QED}$, can be related to $\sigma[e^+e^- \rightarrow hadrons]$
- EW corrections: $\sim 0.0000013\ [a_\mu]_{QED}$,
- HO hadronic contributions ($O(\alpha^3)$):
  - HO VP contributions: $\sim 0.014\ [a_\mu]_{had,LO}$
  - "light-by-light": $\sim 0.020(4)\ [a_\mu]_{had,LO}$, NOT determinable from data

Numerical Values of SM a_μ Contributions

(from M. Passera review: hep-ph/0411168)

Source	$\delta(a_\mu) \times 10^{10}$
QED	1165847.88(3)(4)
LO had VP	$\sim 700(6 \to 8)$
EW	15.4(1)(2)
HO had LBL	13.6(2.5)
HO had VP	$-9.79(9)(3)$

LO Hadronic Contribution



- dispersive representation for $\Pi_{EM}(s)$, $\rho_{EM}(s) = \frac{1}{\pi} Im\, \Pi_{EM}(s)$ via $R(s) = \frac{3s\, \sigma[e^+ e^- \to hadrons]}{16\pi\, \alpha_{EM}^2} = 12\pi^2\, \rho_{EM}(s) \Rightarrow$

$$[a_\mu]_{had,LO} = \frac{\alpha_{EM}^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s}\, R(s)$$

with $K(s)$ known

- possibility of using hadronic τ decay input:
  - isospin relation between $I = 1$ flavor ud V current $\rho_{ud;V}(s)$ and $I = 1$ EM spectral function $\rho_{EM}^{I=1}(s)$ in isospin symmetry limit (CVC)
  - $\rho_{ud;V}(s)$ from invariant mass distribution of flavor ud V current-induced hadronic τ decays ($y_\tau = s/m_\tau^2$, $R_{V;ud} \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau \, \text{hadrons}_{V;ud}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]}$)

$$\rho_{ud;V}(s) = \frac{m_\tau^2 \, dR_{V;ud}/ds}{12\pi^2 |V_{ud}|^2 S_{EW} \, (1-y_\tau)^2 \, (1+2y_\tau)}$$

  - ALEPH, OPAL, CLEO data (plus, in principle, future B factory data)

The EM-τ Discrepancy

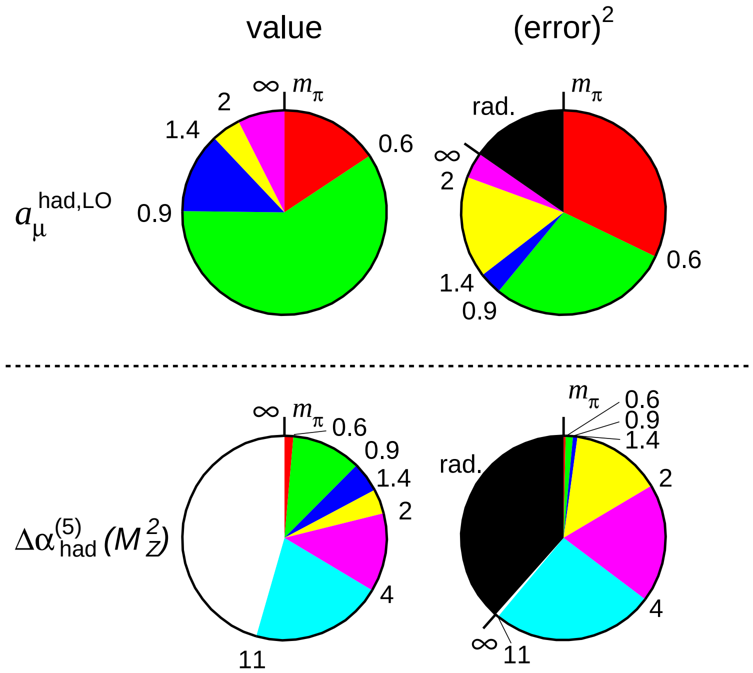
- need $[a_\mu]_{had,LO}$ to $\sim 1\%$ accuracy or better $\Rightarrow$ need to make IB corrections to use τ data/CVC relation

- $[a_\mu]_{had,LO}$ dominated by $\pi\pi$ spectral component (figure)

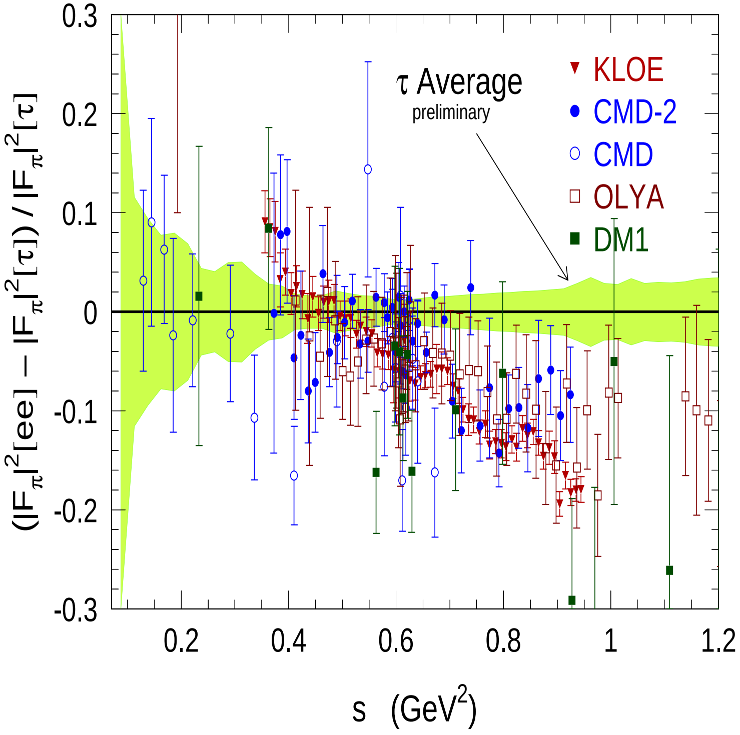
- IB for $\pi\pi$ studied in detail (CEN PLB513 (2001) 361; JHEP 0208 (2002) 002) (*plus a few additional subtleties not considered by CEN*), kinematic IB corrections for 4π

Relative contributions to $[a_\mu]_{had,LO}$

(from HMNT, PRD69 (2004) 093003)

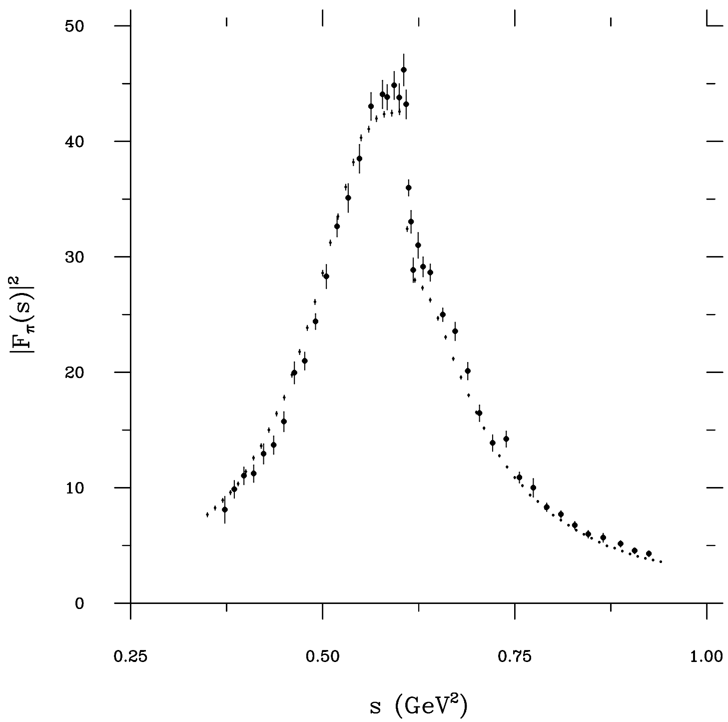


- EM and IB-corrected τ-based $\rho^{I=1}(s)$ do *not* agree
  - $\pi\pi$, 4π discrepancies (DEHZ EPJC31 (2003) 503)
  - $\pi\pi$ dominant ($|F_\pi(s)|$ data, Hocker, hep-ph/0410081)



- resulting EM and τ-based $[a_\mu]_{had,LO}$
  - differ by $\sim 2\sigma_{exp} \simeq 2\sigma_{EM}$
  - EM-based a_μ differs from exp't by $2.1 \rightarrow 2.7\sigma$
  - τ-based a_μ differs from exp't by $0.7 \rightarrow 1.3\sigma$
  - KLOE and CMD-2 $\pi\pi$ contributions to a_μ compatible within errors, so current wisdom is to work with EM data only (see, however, figure)
  - tantalizing signal for possible beyond-the-SM physics, if sensible

CMD-2 and KLOE $\pi\pi$ data



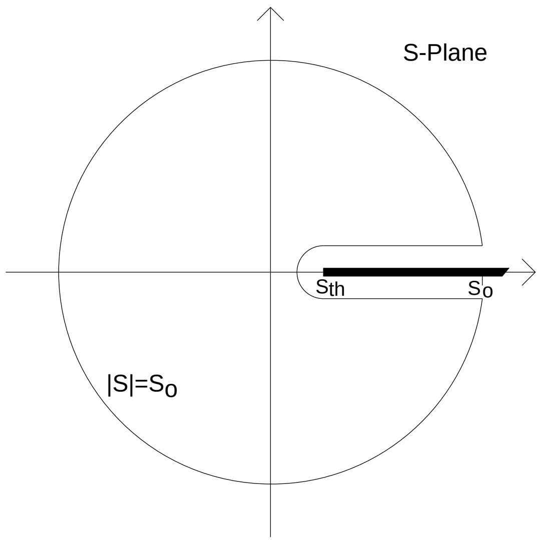
KLOE: stat errors small; 1.3% norm'n error not shown

CMD-2: small 0.6% systematic error included in error bars

RESOLVING THE EM-τ DISCREPANCY

- FESR background
  - $\Pi(s)$ (no "kinematic singularities"), spectral function $\rho(s)$, $w(s)$ analytic in $|s| < M$, $M > s_0 \Rightarrow$

$$\int_0^{s_0} w(s)\,\rho(s)\,ds = -\frac{1}{2\pi}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$

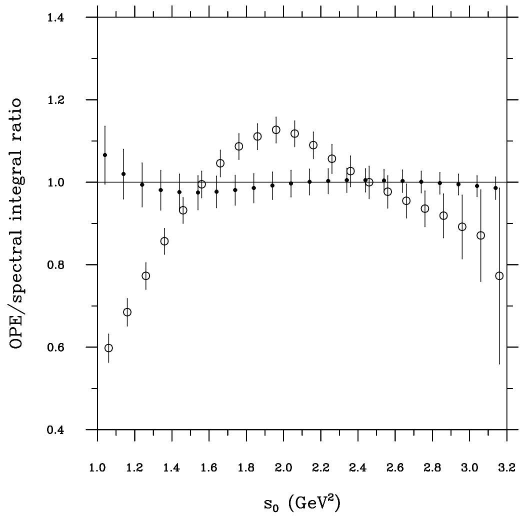


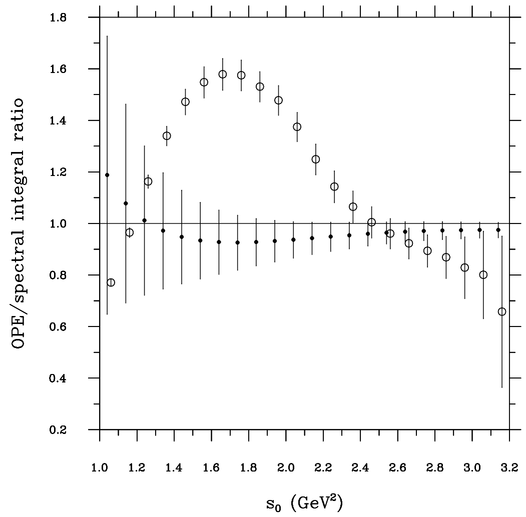
– OPE on RHS, spectral data on LHS

– for $s_0 \sim$ a few GeV^2: $w(s = s_0) = 0$ ("pinching") to suppress duality violation (OPE breakdown) near timelike point ($y = s/s_0$, $w(y)$, $w(1) = 0$)

Pinched $w(y)$ OPE/spectral integral ratios

LEFT: y, $(1 - y)$; RIGHT: y^3, $(1 - y)^2(1 + 2y)$

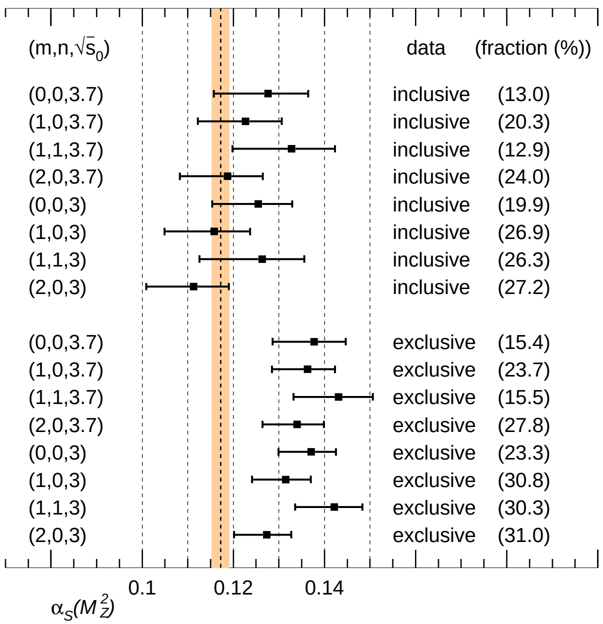




- FESR OPE features
  - V current correlators, $s_0 >\sim 2$ GeV$^2 \Rightarrow$ OPE *strongly* dominated by $D=0$ (hence e.g. from $\alpha_s(M_Z)$, 4-loop running/matching)
  - good convergence of integrated $D=0$ OPE series
  - integrated $D=2k+2$ OPE contribution scales as $1/s_0^k$, $k\geq -1$ for pinched weights
  - "unsuppressed" $D=2k+2$ contribution for $D\geq 2$, pinched $w(y)=\sum_m c_m y^m$, only if $c_k\neq 0$
  - pinching $\Rightarrow$ *strong* numerical suppression of integrated higher D $O(\alpha_s)$ OPE corrections

- hint of EM data problem/non-1γ contributions from HMNT study of inclusive vs exclusive $R(s)$ determination in 2 GeV$^2 < s_0 <$ 4 GeV^2 [PRD69 (2004) 093003: $(n, m, E) \leftrightarrow w(y) = y^m(1-y)^n$, $s_0 = E^2$ FESR]

HMNT Sum Rule Results



NEW ANALYSIS

- FESR choices: various $w(y)$, lower s_0 (enhanced contributions from region of EM-τ discrepancy)

- τ spectral input: ALEPH, CLEO, OPAL, concentrate on ALEPH

- EM spectral input:

  - large number of experiments, need for VP corrections in many old ones

  - new (2004+) data [KLOE ($\pi\pi$), CMD-2 ($\pi^0\gamma$, $\eta\gamma$, 4π), SND ($4\pi, \omega\pi$), BABAR (3π, 4π)] (only KLOE incorporated into most recent $[a_\mu]_{had,LO}$ determinations)

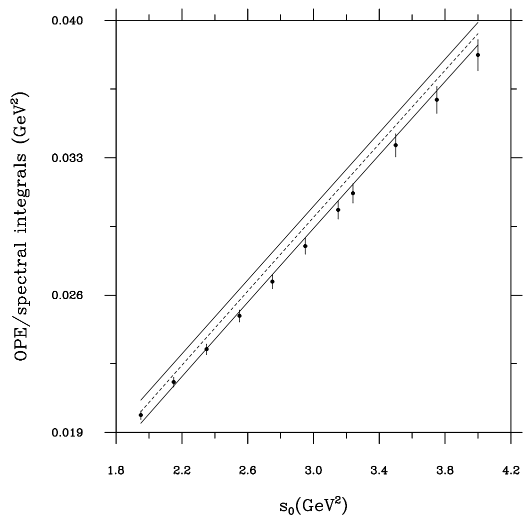
- OPE parameters (*with EM, τ data input removed*)
  - $\alpha_s(M_Z)$ from high-scale studies only (subset of input to PDG 2004 average)
  - $D = 4$ $\langle \alpha_s G^2 \rangle$ from recent re-analysis of charmonium sum rules; quark condensates from GMOR plus ChPT mass ratios
  - $w(y)$ choices: pinched, non-negative, no poorly known (VSA estimate) $D = 6$ contributions
  - integrated $D = 2N > 6$ OPE contributions scale as $1/s_0^N$ c.f. leading $D = 0$ contribution $\Rightarrow$ validity of neglect of high D contributions easily tested

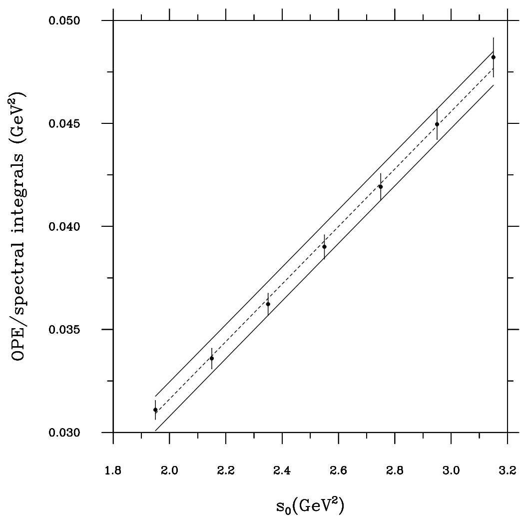
- key points for interpretation
  - $\rho^{I=1}_{EM}(s) < \rho_{V;ud}(s)$ over whole of region of discrepancy (hence choice of non-negative $w(y)$)
  - very strong correlations of OPE integrals for fixed $w(y)$, variable $s_0 \Rightarrow$ slope wrt s_0 almost independent of OPE input
  - very strong correlations of spectral integrals for fixed $w(y)$, variable s_0; variable $w(y)$, fixed s_0, especially for non-negative $w(y) \Rightarrow$ slope wrt s_0 much more accurately determined than evident from point-to-point error bars

RESULTS

- OPE/spectral integrals for $w(y) = 1 - y$

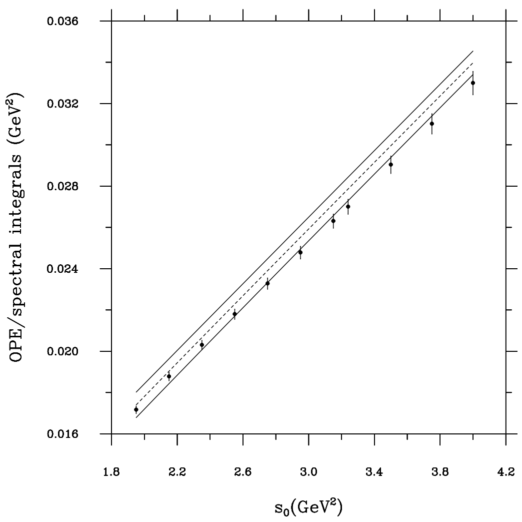
LEFT: EM, RIGHT: τ

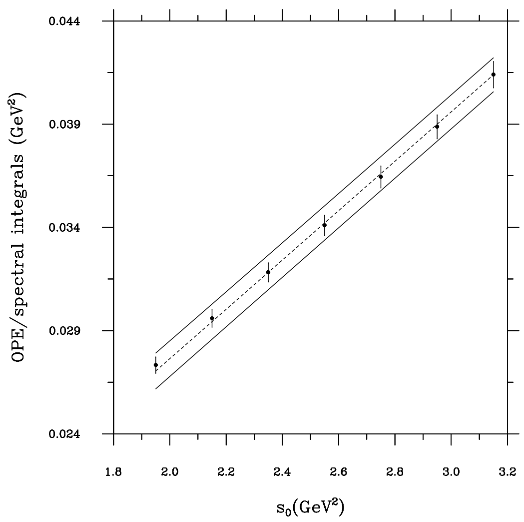




- OPE/spectral integrals for $w_6(y) = 1 - \frac{6y}{5} + \frac{y^6}{5}$

LEFT: EM, RIGHT: τ





(one of more general "doubly-pinched" weight family, $\{w_N(y)\}$, with $6 \rightarrow N$)

- key basic features (also true for other $w(y)$ choices, not shown above)
  - magnitude and slope of τ spectral and OPE integrals agree for wide range of pinched, non-negative $w(y)$, s_0
  - EM spectral integrals $<$ corresponding OPE integrals for wide range of pinched, non-negative $w(y)$, s_0
  - EM spectral integral slope similarly $<$ OPE slope for wide range of pinched, non-negative $w(y)$, s_0

- more on the EM slope problem:
  - results for OPE vs. expt slope, S
  - indep: high scale $\alpha_s(M_Z)$ input (as above)
  - fit: alternate $\alpha_s(M_Z)$ input from fit to EM spectral integral at $s_0 \sim 4$ GeV^2

Weight	S_{exp}	$\alpha_s(M_Z)$	S_{OPE}
$1-y$	$.00872 \pm .00026$	indep	$.00943 \pm .00008$
		fit	$.00934 \pm .00008$
w_6	$.00762 \pm .00017$	indep	$.00811 \pm .00009$
		fit	$.00805 \pm .00009$

  - 2.6 (2.3) σ discrepancy for $w(y) = 1 - y$ with indep (fit) input, 2.5 (2.2) σ for $w_6(y)$

- more on the EM normalization problem:

$\alpha_s(M_Z)$ values required to fit EM and τ spectral integrals for $s_0 \sim m_\tau^2$

Weight	EM or τ	$\alpha_s(M_Z)$
$1-y$	EM	$0.1138^{+0.0030}_{-0.0035}$
w_3	EM	$0.1152^{+0.0019}_{-0.0021}$
w_6	EM	$0.1150^{+0.0022}_{-0.0026}$
$1-y$	τ	$0.1218^{+0.0027}_{-0.0032}$
w_3	τ	$0.1195^{+0.0018}_{-0.0021}$
w_6	τ	$0.1201^{+0.0020}_{-0.0022}$

c.f. high-scale average: $\alpha_s(M_Z) = 0.1200 \pm 0.0020$

COMMENTS ON IB CORRECTIONS

- to use τ data for $[a_\mu]_{had,LO}$, need IB corrections

- additional IB uncertainties/corrections not discussed in literature:

  - ρ-ω interference (present for EM, must add for τ):

    * extracted flavor '38' part of ρ_{EM} depends on model for IC and IB amplitudes

    * model-dependence of integrated contribution to $[a_\mu]_{had,LO} \sim 3\times$ that associated with fitted parameter uncertainties for a given model ($\sim 2 \times 10^{-10}$ c.f. standard current CEN estimate 0.7×10^{-10})

Integrated IB $\rho - \omega$ "Mixing" Contribution

Model	χ^2/dof	$[\delta(a_\mu)]_{\rho-\omega} \times 10^{10}$
GS	35.9/38	2.0 ± 0.5
HLS	36.6/38	4.0 ± 0.6
KS	37.1/38	3.8 ± 0.6
GP/CEN†	40.6/39	2.0 ± 0.5
GP/CEN*	61.5/40	3.7 ± 0.7

- $*$ from fits to 2003 CMD-2 *bare* $\pi\pi$ cross-sections
- $*$ GP/CEN*: re-fitted GP/CEN form
- $*$ GP/CEN†: modified version with additional phase
- $*$ CEN value (pre 2003 CMD-2): 3.5 ± 0.7

- broad ρ^0-shaped IB contribution (in principle present in EM $\rho^0 \to \pi^+\pi^-$ signal due to IB coupling of isoscalar EM current to ρ^0) not taken into account in present treatments
  * unlike ρ-ω interference, *NOT* experimentally detectable
  * data errors too large to effectively extract using flavor '38' correlator sum rule analysis
  * analogous "direct" IB coupling of π^0 to $A^8_\mu \sim 0.5\%$ of IC coupling at hadronic scales (1-loop ChPT); if similar for vector mesons, would yield integrated contribution $\sim (2-3) \times 10^{-10}$
- current *total* estimated uncertainty on IB correction 2.6×10^{-10} hence significantly underestimated

CONCLUSIONS

- pFESR tests, high-scale OPE input clearly favor τ over EM data for V spectral function

- $\Rightarrow$ τ+EM-based $[a_\mu]_{had,LO}$ favored over determination based on EM data only

- with τ input, SM prediction for a_μ in good agreement with current E821 result

- uncertainties in IB correction needed for use of τ input a limiting feature of this approach, *AND* very unlikely to be reducible below $\sim 4\times 10^{-10}$ (already larger than proposed BNL E969 accuracy)

- can look forward to new experimental input:
  - BABAR, BELLE radiative return $\pi\pi$ cross-sections
  - Novosibirsk VEPP-2000 upgrade (improved luminosity, $E_{CM}^{max} \to 2$ GeV (previously 1.38 GeV), upgraded SND, CMD detectors, reduced systematic errors) $\Rightarrow$ significantly reduced errors on exclusive cross-sections (especially useful c.f. current data near threshold, above 1.38 GeV) [scheduled start-up 2005, physics results beginning 2006]
  - BABAR, BELLE should each record $\sim 100\times$ the hadronic τ decays of the combined LEP experiments, hence in principle reduce errors on the flavor ud V spectral distribution (BABAR has begun to analyze its hadronic τ decay data)