Baryon Chiral Perturbation Theory in Manifestly Lorentz-Invariant Form

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1. Introduction

Effective field theory [1]

... if one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S–matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. ...

For our purposes:

Most general description of the strong interactions at low energies: $\pi\pi$, πN, NN, etc.

[1] S. Weinberg, Physica A 96, 327 (1979)

Perturbative calculations in effective field theory require two main ingredients

(1) Knowledge of the most general effective Lagrangian

(a) Mesonic ChPT [SU(3)×SU(3)] [2] (π, K, η)

$$\underbrace{2}_{\mathcal{O}(q^2)} \quad + \quad \underbrace{10+2}_{\mathcal{O}(q^4)} \quad + \quad \underbrace{90+4+23}_{\mathcal{O}(q^6)} \quad + \quad \cdots$$

– q: Small quantity such as a pion mass

– Even powers

– Two-loop level

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[2] J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985);
H. W. Fearing and S. S. , Phys. Rev. D 53, 315 (1996);
J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999);
T. Ebertshäuser, H. W. Fearing, S. S., Phys. Rev. D 65, 054033 (2002);
J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

(b) Baryonic ChPT [SU(2)×SU(2)×U(1)] [3] (π, N)

$$\underbrace{2}_{\mathcal{O}(q)} + \underbrace{7}_{\mathcal{O}(q^2)} + \underbrace{23}_{\mathcal{O}(q^3)} + \underbrace{118}_{\mathcal{O}(q^4)} + \quad \cdots$$

- **Odd and even powers (spin)**
- **One-loop level**

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[3] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988);
V. Bernard, N. Kaiser, U.-G. Meißner, Int. J. Mod. Phys. E 4, 193 (1995);
G. Ecker and M. Mojžiš, Phys. Lett. B 365, 312 (1996);
N. Fettes, U.-G. Meißner, M. Mojžiš, S. Steininger, Ann. Phys. (N.Y.) 283, 273 (2000)

(2) Expansion scheme for observables in terms of a consistent power counting

   (a) Tree-level diagrams, loop diagrams, regularization (of infinities)

   (b) Renormalization

   (c) Power counting scheme for renormalized diagrams

Commonly used methods

1. Expansion in powers of coupling constants (e. g., QED)

2. Loop expansion (expansion in $\hbar$)

3. Momentum and quark mass expansion, ChPT

Weinberg's power counting for the mesonic sector [4]

$$\mathcal{M}(tp, t^2M^2) = t^D \mathcal{M}(p, M^2) = \mathcal{O}(q^D)$$

$$\begin{aligned} D &= 2 + (n-2)N_L + \sum_{k=1}^{\infty} 2(k-1)N_{2k}^{\pi} \\ &\geq 2 \text{ in 4 dimensions} \end{aligned}$$

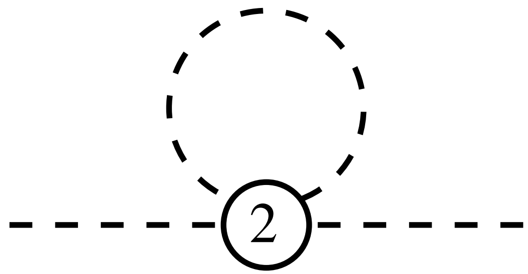
- N_L: **Number of loops**

- N_{2k}^{π}: **Number of vertices from** $\mathcal{L}_{2k}$

- **Perturbative scheme in terms of external momenta and quark masses (**$\rightarrow$ **meson masses**^{2}**) which are small compared to some scale [here:** $4\pi F = \mathcal{O}(1\text{GeV})$**]**

[4] S. Weinberg, Physica A 96, 327 (1979)

Example from the mesonic sector



$$D = n \cdot 1 - 2 \cdot 1 + 1 \cdot 2 = n$$

contains contribution $\sim$

$$\underbrace{B\hat{m}}_{\mathcal{O}(q^2)} \quad \underbrace{I(M^2, \mu^2, n)}_{\mathcal{O}(q^{n-2})} \quad \rightarrow \quad \mathcal{O}(q^4) \quad \text{for} \quad n \rightarrow 4$$

$$I(M^2, \mu^2, n) = \frac{M^2}{16\pi^2}\left[R + \ln\left(\frac{M^2}{\mu^2}\right)\right] + O(n-4)$$

$$\boxed{R} = \frac{2}{n-4} \quad - [\ln(4\pi) + \Gamma'(1)] \quad - 1 \rightarrow \boxed{\infty}$$

S. Weinberg [5]

... the cancellation of ultraviolet divergences does not really depend on renormalizability; as long as we include every one of the infinite number of interactions allowed by symmetries, the so-called non-renormalizable theories are actually just as renormalizable as renormalizable theories.

Conclusion: **Adjust (renormalize) parameters of $\mathcal{L}_4$ to cancel one-loop infinities.**

Introductory review:

- **S. S., Advances in Nuclear Physics, Vol. 27 (Kluwer Academic/Plenum Publishers, New York, 2003) p 277**

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[5] The Quantum Theory of Fields, Vol. I (Cambridge University Press, Cambridge, 1995) Chap. 12

2. Extended on-mass-shell scheme in BChPT

- **Interaction of pions and nucleons** [6]

- **Most general Lagrangian**

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots,$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left(i\gamma_\mu \partial^\mu - \boxed{m} - \frac{1}{2} \frac{\boxed{\overset{\circ}{g}_A}}{F} \gamma_\mu \gamma_5 \tau^a \partial^\mu \pi^a \right) \Psi + \cdots$$

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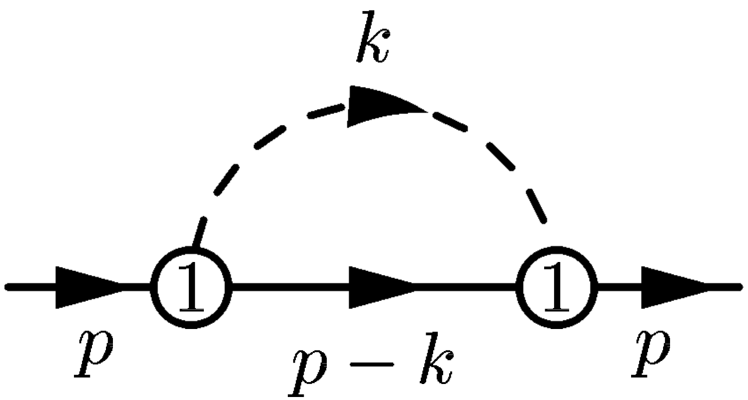
[6] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988); A. Krause, Helv. Phys. Acta 63, 3 (1990)

- **Power counting:** **Associate chiral order D with a diagram**
  - **Loop integration in n dimensions** $\sim \mathcal{O}(q^n)$
  - **Vertex from** $\mathcal{L}_{2k} \sim \mathcal{O}(q^{2k})$
  - **Vertex from** $\mathcal{L}_{\pi N}^{(k)} \sim \mathcal{O}(q^k)$
  - **Nucleon propagator** $\sim \mathcal{O}(q^{-1})$
  - **Pion propagator** $\sim \mathcal{O}(q^{-2})$

- **Example: Contribution to nucleon mass**



Goal: $D = n \cdot 1 - 2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 = n - 1$

$$\Sigma = -\frac{3g_{A0}^2}{4F_0^2}\left[(\not{p} + m)I_N + M^2(\not{p} + m)I_{N\pi}(-p, 0) + \cdots\right]$$

Apply $\widetilde{\text{MS}}$ renormalization scheme

$$\begin{aligned}\Sigma_r &= -\frac{3g_{Ar}^2}{4F_r^2}\left[M^2(\not p+m)\underbrace{I_{N\pi}^r(-p,0)}_{-\frac{1}{16\pi^2}+\cdots}+\cdots\right]\\ &= \mathcal{O}(q^2)\end{aligned}$$

GSS [7]: It turns out that loops have a much more complicated low-energy structure if baryons are included. Because the nucleon mass m_N does not vanish in the chiral limit, the mass scale m (nucleon mass in the chiral limit) occurs in the effective Lagrangian $\mathcal{L}_{\pi N}^{(1)}$ ... . This complicates life a lot.

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[7] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988)

Solutions

- Heavy-baryon chiral perturbation theory (HBChPT) [8]

- Infrared regularization [9]

  Special treatment of (the Feynman parameterization of) one-loop integrals

$$H = \int_0^1 dx \cdots = \int_0^\infty dx \cdots - \int_1^\infty dx \cdots \equiv I + R$$

  - I: Power counting o.k.

  - R: Violates power counting; regular, i.e., can be absorbed in counterterms

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[8] E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991); V. Bernard, N. Kaiser, J. Kambor, U.-G. Meißner, Nucl. Phys. B388, 315 (1992)

[9] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999)

Extended on-mass-shell (EOMS) scheme [10]

Main idea: Perform additional subtractions such that renormalized diagrams satisfy the power counting.

Motivation for this approach [11]

Terms violating the power counting are analytic in small quantities (and can thus be absorbed in a renormalization of counterterms).

- Example (chiral limit)

$$H(p^2, m^2; n) = \int \frac{d^n k}{(2\pi)^n} \frac{i}{[(k-p)^2 - m^2 + i0^+][k^2 + i0^+]}$$

[10] T. Fuchs, J. Gegelia, G. Japaridze, S. S., Phys. Rev. D 68, 056005 (2003)

[11] J. Gegelia and G. Japaridze, Phys. Rev. D 60, 114038 (1999)

Small quantity

$$\Delta = \frac{p^2 - m^2}{m^2} = \mathcal{O}(q)$$

We want the renormalized integral to be of order

$$D = n - 1 - 2 = n - 3$$

Result of integration

$$H \sim F(n, \Delta) + \Delta^{n-3} G(n, \Delta)$$

F **and** G **are hypergeometric functions; analytic in** Δ **for arbitrary** n**.**

Observation [12]

F **corresponds to first expanding the integrand in small quantities and then performing the integration.**

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[12] J. Gegelia, G. Japaridze, K. S. Turashvili, Theor. Math. Phys. 101, 1313 (1994)

$\Rightarrow$ **Algorithm: Expand integrand in small quantities and subtract those (integrated) terms whose order is smaller than suggested by the power counting.**

Here:

$$H^{\text{subtr}} = \int \frac{d^n k}{(2\pi)^n} \frac{i}{(k^2 - 2k \cdot p + i0^+)(k^2 + i0^+)}\bigg|_{p^2=m^2}$$

$$H^R = H - H^{\text{subtr}} = \mathcal{O}(q^{n-3})$$

Remarks:

- **(Axial) Vector mesons can be consistently included**

- **Extension to multi-loop level**

3. Applications

Example 1: Mass of the nucleon at $\mathcal{O}(q^3)$

- **GSS ($\widetilde{\mathrm{MS}}$)** [13]

$$m_N = m - 4c_1^r M^2 + \frac{3g_{Ar}^2 M^2}{32\pi^2 F_r^2} m \left(1 + 8c_1^r m\right) - \frac{3g_{Ar}^2 M^3}{32\pi F_r^2}.$$

$$c_1^0 = c_1^r - \frac{3\mathring{g}_A^2}{128\pi^2}$$

R

- **EOMS** [14]

$$c_1^r = c_1 + \frac{3mg_A^2}{128\pi^2 F^2}\left[1 + 8mc_1\right] + \cdots,$$

$$m_N = m - 4c_1 M^2 - \frac{3g_A^2 M^3}{32\pi F^2} + \mathcal{O}(M^4).$$

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[13] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988)

[14] T. Fuchs, J. Gegelia, G. Japaridze, S. S., Phys. Rev. D 68, 056005 (2003)

Example 2: Mass of the nucleon at $\mathcal{O}(q^4)$

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\left(\frac{M}{m}\right) + k_4 M^4 + O(M^5),$$

$$\begin{aligned}
k_1 &= -4c_1 \\
k_2 &= -\frac{3\overset{\circ}{g}_A{}^2}{32\pi F^2}, \\
k_3 &= \frac{3}{32\pi^2 F^2}\left(8c_1 - c_2 - 4c_3 - \frac{\overset{\circ}{g}_A{}^2}{m}\right), \\
k_4 &= \frac{3\overset{\circ}{g}_A{}^2}{32\pi^2 F^2 m}(1 + 4c_1 m) + \frac{3}{128\pi^2 F^2}c_2 - 16e_{38} - 2e_{115} - 2e_{116}.
\end{aligned}$$

$$m = [938.3 - 74.8 + 15.3 + 4.7 + 1.6 - 2.3]\,\text{MeV} = 882.8\,\text{MeV}$$

$$\boxed{\Delta m = 55.5\,\text{MeV}}$$

Remark: $m = m_N(m_u = 0, m_d = 0, m_s)$

Example 3: Electromagnetic form factors

Electromagnetic current operator

$$J^\mu(x) = \frac{2}{3}\,\bar{u}(x)\gamma^\mu u(x) - \frac{1}{3}\,\bar{d}(x)\gamma^\mu d(x) + \cdots = \bar{q}\,Q\,q + \cdots$$

Definition of Dirac and Pauli form factors

$$\langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu F_1(Q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_N}\,F_2(Q^2)\right]u(p),$$

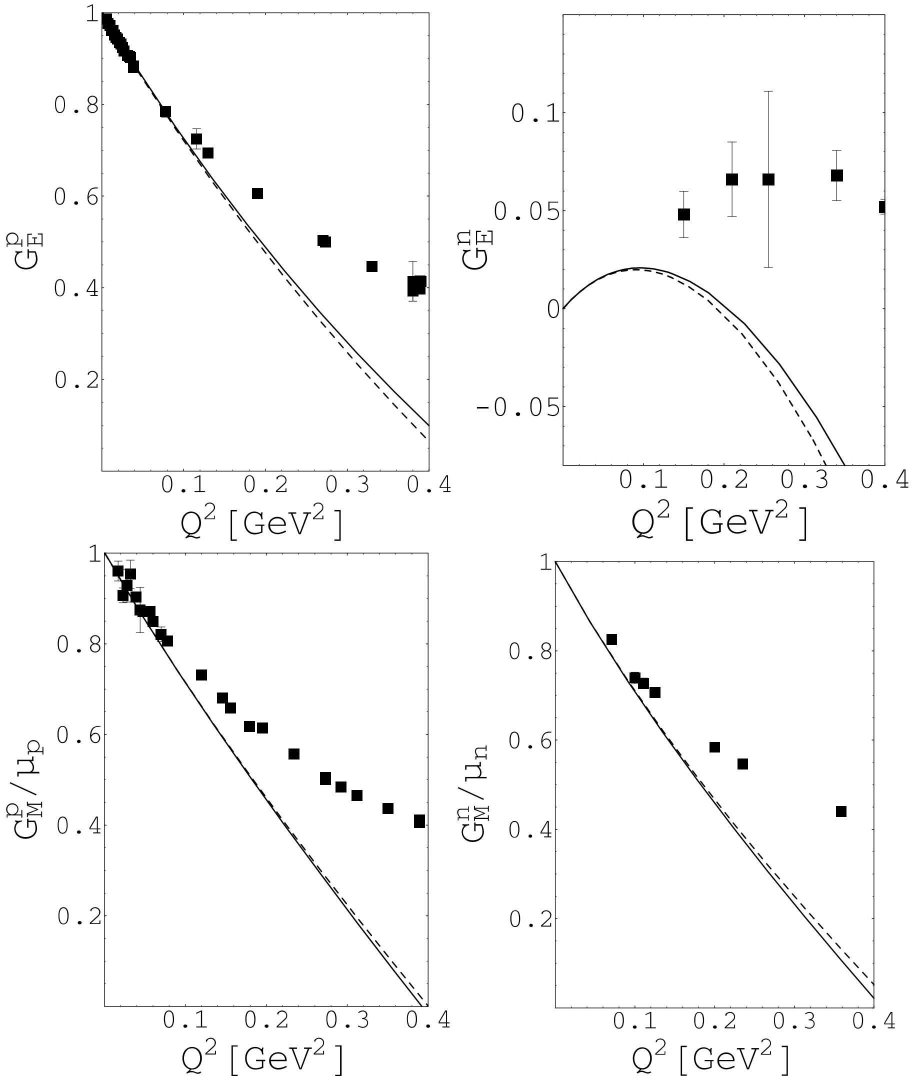
$$q^\mu = p'^\mu - p^\mu, \qquad Q^2 = -q^2$$

Sachs form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2}F_2^N(Q^2)$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)$$

Electromagnetic form factors [15]



[15] B. Kubis and U.-G. Meißner, Nucl. Phys. A679, 698 (2001);
T. Fuchs, J. Gegelia, S. S., J. Phys. G 30, 1407 (2004)

Vector meson dominance model → Important contributions to the electromagnetic form factors [16]

In standard ChPT: Vector meson contributions in low-energy constants

$$\frac{1}{q^2 - M_V^2} = -\frac{1}{M_V^2}\left[1 + \frac{q^2}{M_V^2} + \left(\frac{q^2}{M_V^2}\right)^2 + \mathcal{O}(q^6)\right]$$

Inclusion of vector mesons ⇒ re-summation of higher-order contributions

Additional rules:

- **Vector meson propagator** $\sim \mathcal{O}(q^0)$

- **Vertex from** $\mathcal{L}_V^{(i)} \sim \mathcal{O}(q^i)$

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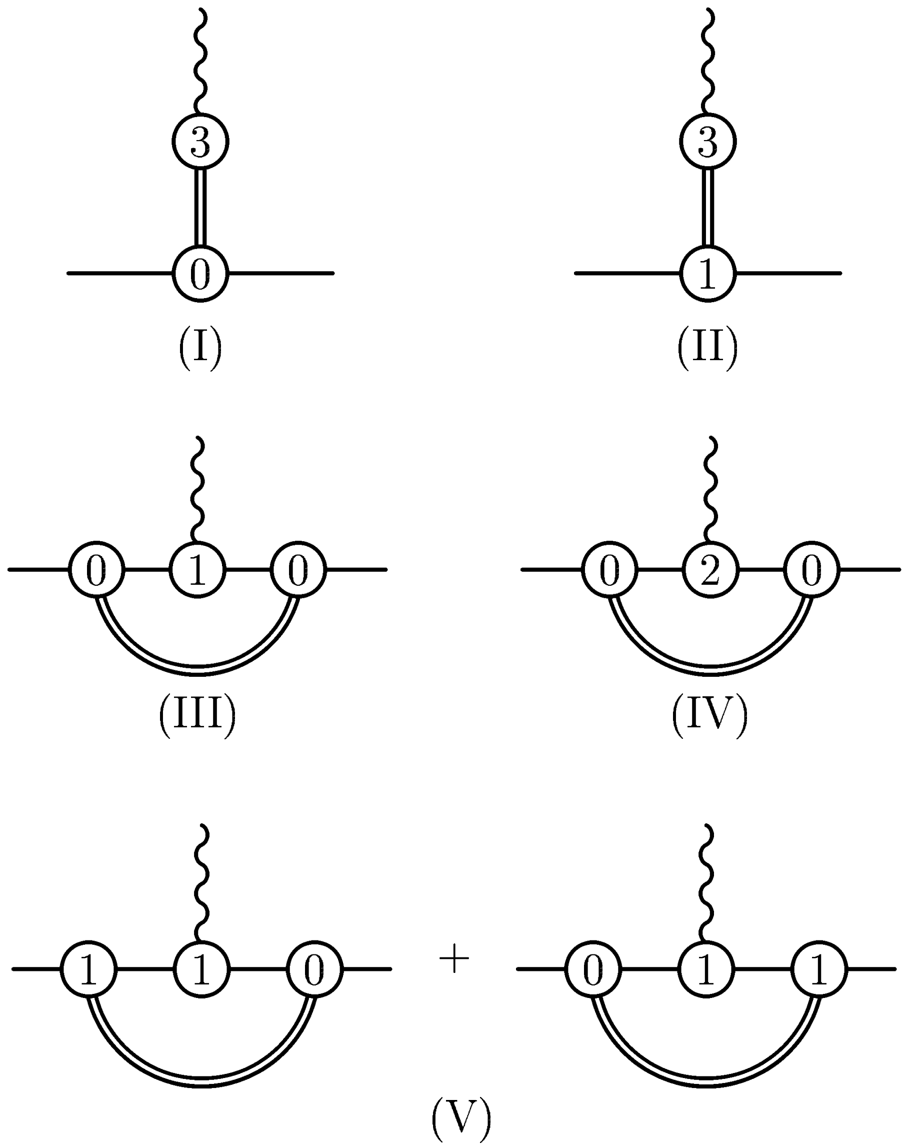
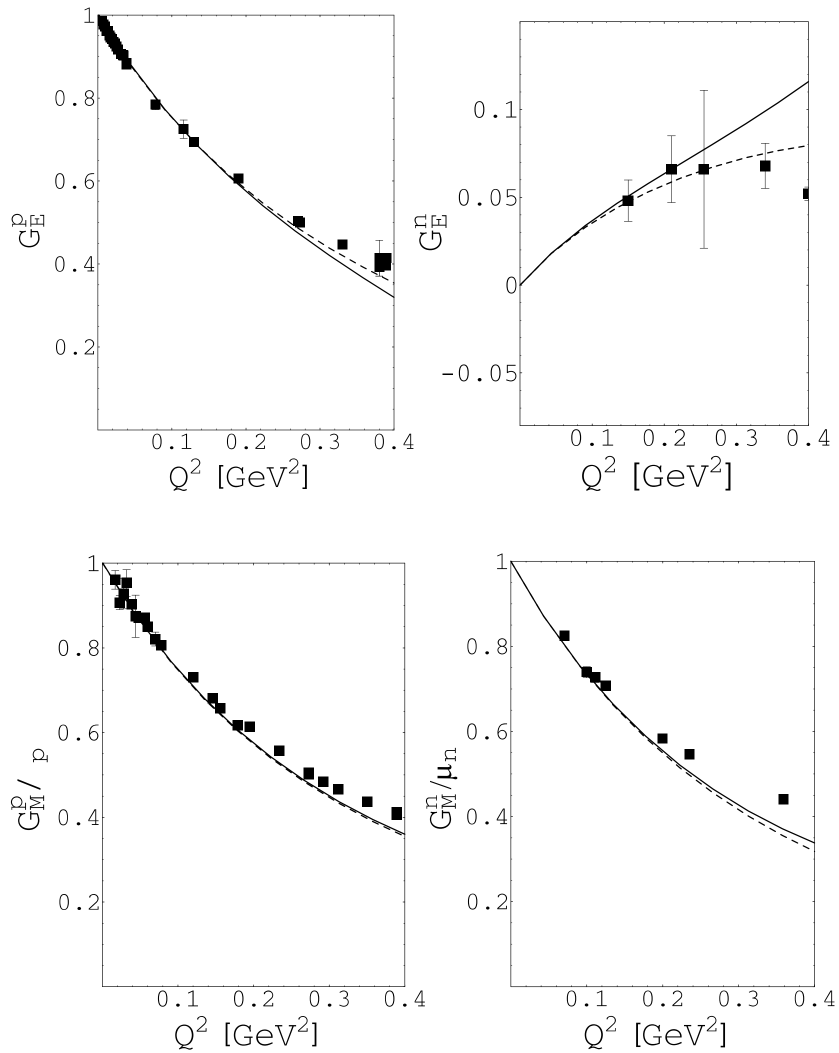
[16] B. Kubis and U.-G. Meißner, Nucl. Phys. A679, 698 (2001)



Feynman diagrams involving vector mesons contributing to the electromagnetic form factors up to and including $\mathcal{O}(q^4)$

Electromagnetic form factors including vector mesons at $\mathcal{O}(q^4)$ [17]



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[17] M. R. Schindler, J. Gegelia, S. S., (2004)

Example 4: Universality of the ρ meson coupling in EFT [18]

Chirally invariant effective Lagrangian including vector mesons

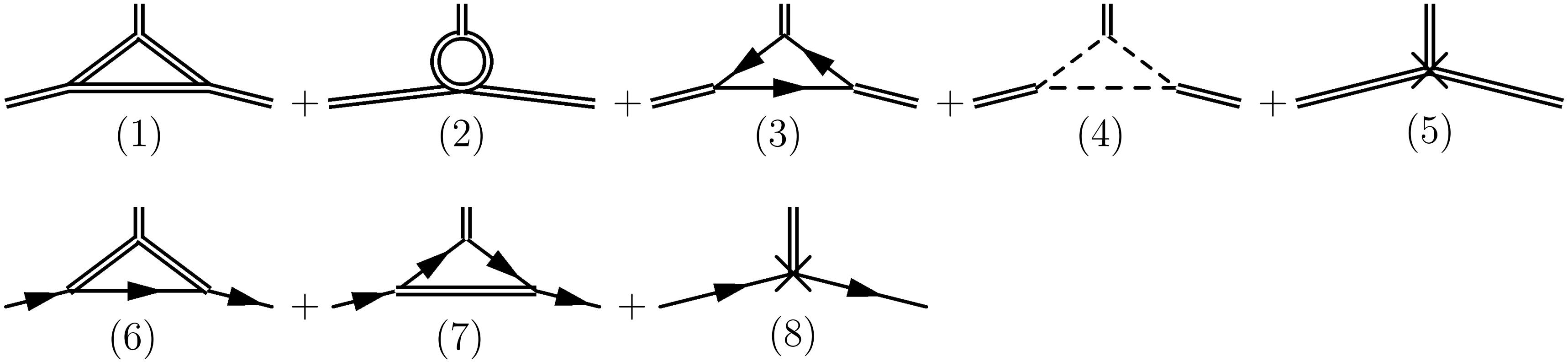
$$\mathcal{L} = \mathcal{L}_{\text{basic}} + \mathcal{L}_{\text{ct}} + \tilde{\mathcal{L}}_1$$

$$\begin{aligned}
\mathcal{L}_{\text{basic}} &= \text{free Lagrangians} \\
&\quad + \boxed{g_{\rho\pi\pi}}\ \varepsilon^{abc}\pi^a\partial_\mu\pi^b\rho^{c\mu} \\
&\quad - \boxed{g}\ \varepsilon^{abc}\partial_\mu\rho_\nu^a\rho^{b\mu}\rho^{c\nu} \\
&\quad - \frac{1}{4}\boxed{g^2}\ \varepsilon^{abc}\varepsilon^{ade}\rho_\mu^b\rho_\nu^c\rho^{d\mu}\rho^{e\nu} \\
&\quad + \boxed{g_{\rho NN}}\ \bar{\Psi}\gamma^\mu\frac{\tau^a}{2}\Psi\rho_\mu^a
\end{aligned}$$

[18] D. Djukanovic, M. R. Schindler, J. Gegelia, G. Japaridze, S. S., Phys. Rev. Lett. 93, 122002 (2004)

Chiral symmetry $\Rightarrow$ $\boxed{g_{\rho NN} = g}$ [19]

Evaluate renormalized vertex diagrams



- $\rho\rho\rho$ **vertex function** $\Rightarrow$ $\boxed{\delta g} = F(g, g_{\rho\pi\pi})$

- $\rho\bar{\Psi}\Psi$ **vertex function** $\Rightarrow$ $\boxed{\delta g} = G(g, g_{\rho\pi\pi})$

[19] S. Weinberg, Phys. Rev. 166, 1568 (1968)

$\Rightarrow$ consistency condition: $F = G$

Nontrivial solution

$$\boxed{g_{\rho\pi\pi} = g}$$ universality

Next step: Chiral symmetry [20]

$$g_{\rho\pi\pi} = \frac{M_\rho^2}{2gF^2}$$

Combine with universality $\Rightarrow$

$$g^2 = \frac{M_\rho^2}{2F^2}$$

KSRF relation [21]

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[20] S. Weinberg, Phys. Rev. 166, 1568 (1968);
G. Ecker et al., Phys. Lett. B 223, 425 (1989)

[21] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966);
Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966)

- **Kawarabayashi & Suzuki**

PCAC + current algebra ⇒

$$g g_{\rho\pi\pi} = \frac{M_\rho^2}{2F^2}$$

NB: Chiral Ward identity

$$\begin{aligned} T[\partial_\mu^x A_i^\mu(x) \partial_\nu^y A_j^\nu(y)] \;&=\; \partial_\mu^x \partial_\nu^y T[A_i^\mu(x) A_j^\nu(y)] \\ &+\partial_\mu^x \delta^4(y-x) i\varepsilon_{ijk} V_k^\mu(y) \\ &+\delta^4(x-y) i\widehat{m}\delta_{ij}\bar{q}(x)q(x) \end{aligned}$$

universality as an extra assumption ⇒ KSRF relation

- **Riazuddin & Fayyazuddin**

PCAC + current algebra (but a different Ward identity similar to the Adler-Gilman relation of pion electroproduction, makes use of the analogue of the Goldberger-Treiman relation) $\Rightarrow$

$$g g_{\rho\pi\pi} = \frac{M_\rho^2}{2F^2}$$

Dynamical assumption: $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$ **dominated by the** ρ **meson pole + CVC hypothesis $\Rightarrow$ universality $\Rightarrow$ KSRF relation**

Example 5: Quantum electrodynamics for vector mesons [22]

Inclusion of the electromagnetic interaction

$$\begin{aligned}\mathcal{L}_{\text{basic}} &= \ldots - i\,\boxed{e}\,A_\mu(\rho^{-\mu\nu}\rho_\nu^+ - \rho^{+\mu\nu}\rho^{-\nu}) \\ &+\frac{1}{2}\boxed{c}\,F_{\mu\nu}\rho^{0\mu\nu} \\ &-i\,\boxed{\kappa}\,F_{\mu\nu}\rho^{+\mu}\rho^{-\nu}+\ldots\end{aligned}$$

consistency condition $\Rightarrow$ $\boxed{\kappa = e}$, $\boxed{c = e/g}$

- **Gyromagnetic ratio of the** ρ^+**:** $g = 2$

- $M_{\rho^0} - M_{\rho^\pm} \sim 1$ MeV **using KSRF**

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[22] D. Djukanovic, M. R. Schindler, J. Gegelia, S. S., hep-ph/0505180, to be published in Phys. Rev. Lett.

4. Summary and outlook

- EFT in the description of the strong interactions
- Baryonic chiral perturbation theory and renormalization
- EOMS renormalization scheme allows for a consistent power counting of renormalized quantities in BChPT

So far

- Form factors: Electromagnetic, axial, induced pseudoscalar, scalar, and strong
- Extension to SU(3), convergence?

- **Universality of the ρ meson coupling as a consequence of consistency conditions**

- **QED for vector mesons:**
  - **Gyromagnetic ratio of the ρ^+: $g = 2$**
  - $M_{\rho^0} - M_{\rho^\pm} \sim 1\,\mathrm{MeV}$

Present and future

- **Electromagnetic processes: Real and virtual Compton scattering, pion production, etc.**

- **Two-loop calculations**

- **Inclusion of Δ resonance**

Thanks to my collaborators

- Dalibor Djukanovic
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DFG (SFB 443, SCHE 459/2-1)

σ term

$$\sigma = \sigma_1 M^2 + \sigma_2 M^3 + \sigma_3 M^4 \ln\left(\frac{M}{m}\right) + \sigma_4 M^4 + O(M^5),$$

$$\sigma_1 = -4c_1,$$

$$\sigma_2 = -\frac{9\overset{\circ}{g}_A{}^2}{64\pi F^2},$$

$$\sigma_3 = \frac{3}{16\pi^2 F^2}\left(8c_1 - c_2 - 4c_3 - \frac{\overset{\circ}{g}_A{}^2}{m}\right),$$

$$\sigma_4 = \frac{3}{8\pi^2 F^2}\left[\frac{3\overset{\circ}{g}_A{}^2}{8m} + c_1(1 + 2\overset{\circ}{g}_A{}^2) - \frac{c_3}{2}\right] + \alpha,$$

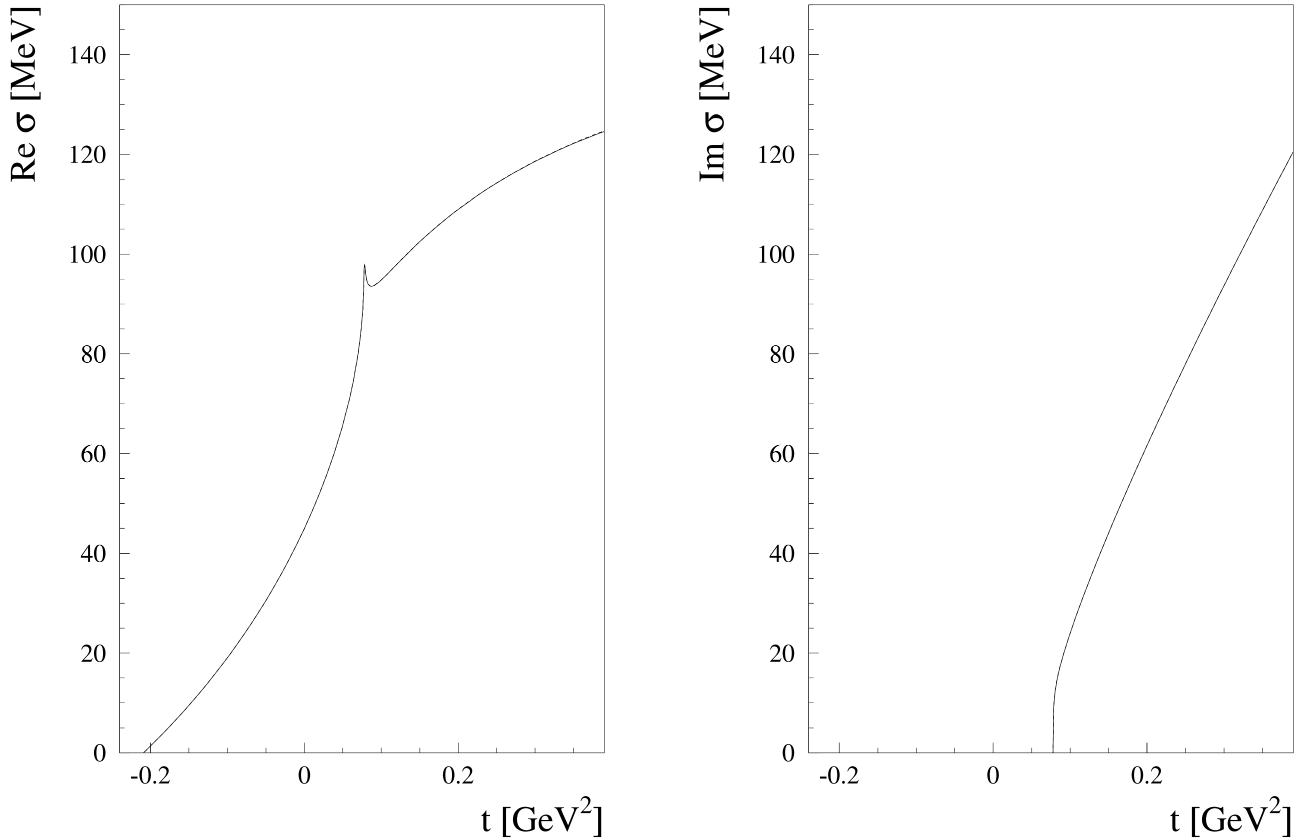
$$\sigma = 45\,\textbf{MeV} = (74.8 - 22.9 - 9.4 - 2.0 + 4.5)\,\textbf{MeV}$$

Hellmann-Feynman theorem o.k.

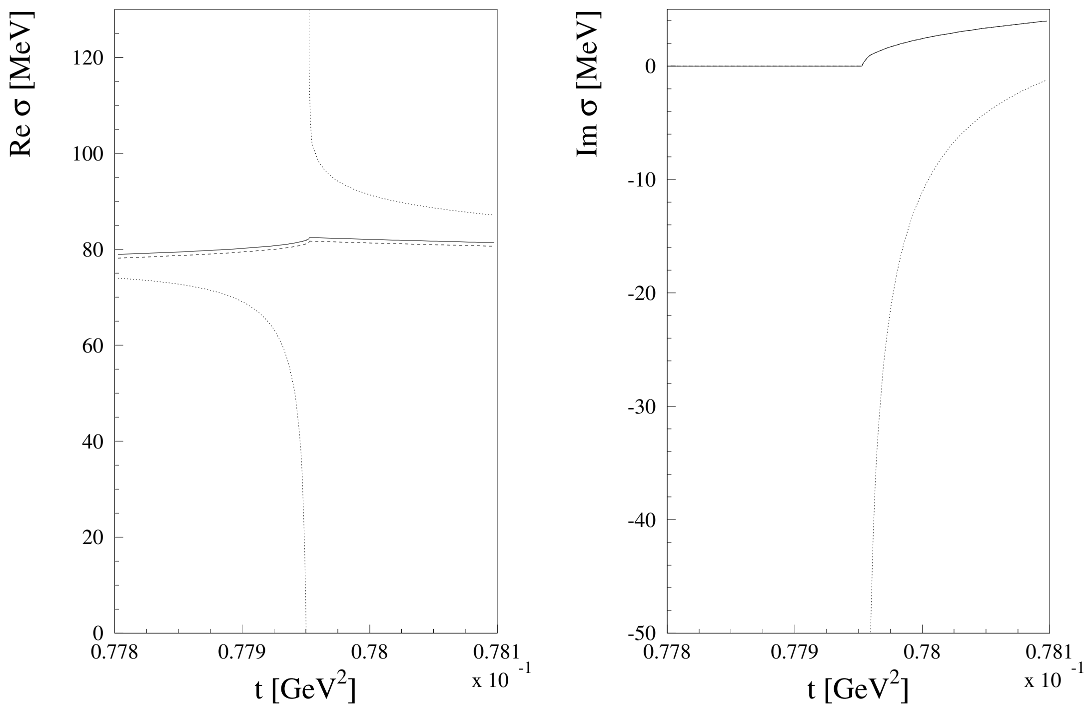
Definition of the scalar form factor

$$\langle N(p')|\hat{m}[\bar{u}(0)u(0)+\bar{d}(0)d(0)]|N(p)\rangle = \bar{u}(p')u(p)\sigma(t)$$

Form factor $\sigma(t)$ at $\mathcal{O}(q^4)$



Form factor $\sigma(t)$ at $\mathcal{O}(q^3)$



Solid line: EOMS;

dashed line: IR;

dotted line: HBChPT $\Rightarrow$ poles at $t = 4M_\pi^2$