The Renormalization of EWCL and its Application to LEP 2 TGCs measurement

Qi-Shu Yan

Theory Group, KEK, 1-1 Oho, Tsukuba, Ibaraki, Japan

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1.1 The question: How to calculate the RCs in EWCL?

1.a The notorious divergences in the massive Yang-Mills theory

$$i\Delta^{\mu\nu} = i\Delta_T^{\mu\nu} + i\Delta_L^{\mu\nu}\,, \tag{1}$$

$$\Delta_T^{\mu\nu} = \frac{1}{k^2 - m_V^2}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right)\,, \tag{2}$$

$$\Delta_L^{\mu\nu} = \frac{1}{m_V^2}\frac{k^\mu k^\nu}{k^2}\,, \tag{3}$$

Unitary gauge is not a renormalizable gauge.

M. Veltman, Nucl. Phys. B7, 637 (1968); Nucl. Phys. B21, 445 (1970); S. L. Glashow and J. Iliopoulos, Phys. Rev. D3, 1048 (1971)

1.b The gauged nonlinear sigma model

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots , \tag{4}$$

$$\mathcal{L}_2 = \frac{1}{2g^2} tr[W_{\mu\nu} W^{\mu\nu}] + \frac{v^2}{4} tr[V_\mu V^\mu] , \tag{5}$$

$$\begin{aligned} \mathcal{L}_4 = & -i\alpha_1 tr[W_{\mu\nu} V^\mu V^\nu] + \alpha_2 tr[V_\mu V_\nu] tr[V^\mu V^\nu] \\ & + \alpha_3 tr[V_\mu V^\mu] tr[V_\nu V^\nu] + \alpha_4 tr[D_\mu V^\mu] tr[D_\nu V^\nu] , \end{aligned} \tag{6}$$

$$\mathcal{L}_6 = \cdots ,$$

$$\cdots = \cdots , \tag{7}$$

Renormalization can be well-defined with the renormalizable gauge even for nonrenormalizable theories.

T. Appelquist and C. Bernard, Phys. Rev. D 22 (1980) 200; A. Longhitano, Phys. Rev. D 22 (1980) 1166; Nucl. Phys. B 188 (1981) 118; T. Appelquist, 385-432, Proceedings of the 21st Scottish Univeristies Summer School in Physics, 1981

1.c The model independent Phenomenological TGCs

$$\frac{\mathcal{L}_{\mathcal{WWN}}}{g_{WWN}} = i{g_1}^N({W^\dagger}_{\mu\nu}W^\mu N^\nu - W_{\mu\nu}{W^\dagger}^\mu N^\nu) + ik_N {W^\dagger}_\mu W_\nu N^{\mu\nu} + i\frac{\lambda_N}{M_W^2}{W^\dagger}_{\lambda\mu}{W^\mu}_\nu N^{\nu\lambda} + g_5^N \epsilon^{\mu\nu\rho\sigma}({W^\dagger}_\mu \partial_\rho W_\nu - \partial_\rho {W^\dagger}_\mu W_\nu)N_\sigma \quad (8)$$

K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B **282**, 253 (1987).

Relations with the EWCL:

$$g_A^1 = 1, g_Z^1 = 1 - (\alpha_8 + \alpha_3 + \alpha_9)g^2 + (\alpha_1 + \alpha_2){g'}^2, \quad k_A = 1 - (\alpha_1 + \alpha_8 + \alpha_2 + \alpha_3 + \alpha_9)g^2, k_Z = 1 - \alpha_3 G^2 \quad (9)$$

T. Appelquist and G. H. Wu, Phys. Rev. D **48**, 3235 (1993)

$$(\alpha_1\ \alpha_8\ \beta) \sim O(0.01),\ (\alpha_2\ \alpha_3\ \alpha_9) \sim O(0.1),\ (\alpha_4\ \alpha_5\ \alpha_6\ \alpha_7\ \alpha_{10}) \sim O(1)\,. \quad (10)$$

1.d Early attempts to calculate RCs in EWCL and puzzles

Radiative corrections of TGCs and QGCs to S-T-U, (or ϵ_1-ϵ_2-ϵ_3).

- d.1 Calculation in the U-gauge with UV cutoff: Quartic, quadratic, and logarithmic divergences are met. M. Suzuki, Phys. Lett. B **153**, 289 (1985). H. Neufeld, J. D. Stroughair and D. Schildknecht, Phys. Lett. B **198**, 563 (1987). J. J. van der Bij, Phys. Lett. B **296**, 239 (1992). P. Hernandez and F. J. Vegas, Phys. Lett. B **307**, 116 (1993) [arXiv:hep-ph/9212229].
- d.2 Calculation with Higgs as regulator K. Hagiwara, S. Ishihara, R. Szalapski and D. Zeppenfeld, Phys. Lett. B **283**, 353 (1992); K. Hagiwara, S. Ishihara, R. Szalapski and D. Zeppenfeld, Phys. Rev. D **48**, 2182 (1993).
- d.3 Calculation in the U-gauge while extracting logarithmic terms DR S. Dawson and G. Valencia, Nucl. Phys. B **439**, 3 (1995) [arXiv:hep-ph/9410364]. C. P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, Phys. Rev. D **50**, 7011 (1994) [arXiv:hep-ph/9307223].

d.4 Calculation in the nonlinear chiral Lagrangian with higher dimensional covariant operators as regulators: $S \propto \frac{1}{\Lambda^2}$ J. J. van der Bij and B. M. Kastening, Phys. Rev. D **57**, 2903 (1998)

1.e What's the correct calculation method?

e.1 What's the meaning of those quartic and quadratic divergences? P. Hernandez and F. J. Vegas, Phys. Lett. B **307**, 116 (1993) [arXiv:hep-ph/9212229].

e.2 What's the correct running beheavior of anomalous couplings in EWCL? K. G. Wilson and J. B. Kogut, Phys. Rept. **12**, 75 (1974). J. Polchinski, Nucl. Phys. B **231**, 269 (1984).

e.3 **Our RGE method:**

the non-linear gauged chiral lagrangian with dimensional regularization and in a renormalizable gauge (Feynman gauge).

e.4 What's the meaning of the β functions of anomalous couplings? M. B. Einhorn and J. Wudka, NSF-ITP-92-01; M. B. Einhorn, arXiv:hep-ph/9303323.

1.f Electroweak Chiral Lagrangian

The building block of EWCL in weak interaction includes U, W, B, and the explicit symmetry is $SU(2)_L \times U(1)_R$:

$$\mathcal{L}_{EW} = \mathcal{L}_{EW}^{p^2} + \mathcal{L}_{EW}^{p^4} + \cdots \tag{11}$$

$$\mathcal{L}_{EW}^{p^2} = \mathcal{L}_B \,, \tag{12}$$

$$\mathcal{L}_{EW}^{p^4} = \beta \mathcal{L}^{2\prime} + \sum_{i=1}^{10} \alpha_i \mathcal{L}_i \tag{13}$$

where

$$\mathcal{L}_B = -H_1 - H_2 + \mathcal{L}_{WZ} \,, \tag{14}$$

$$H_1 = \frac{1}{4g^2} W_{\mu\nu}^a W^{\mu\nu,\, a} \,, \tag{15}$$

$$H_2 = \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} \,, \tag{16}$$

$$\mathcal{L}_{WZ} = \frac{v^2}{4} tr(V \cdot V) \,, \tag{17}$$

$$\bar{\mathcal{L}}^{2\prime} = \frac{v^2}{4} [tr(\mathcal{T} V_\mu)]^2 \,, \tag{18}$$

$$\bar{\mathcal{L}}_1 = \frac{1}{2} B_{\mu\nu} tr(\mathcal{T} W^{\mu\nu}) \,, \tag{19}$$

$$\bar{\mathcal{L}}_2 = i \frac{1}{2} B_{\mu\nu} tr(\mathcal{T} [V^\mu, V^\nu]) \,, \tag{20}$$

$$\bar{\mathcal{L}}_3 = i tr(W_{\mu\nu} [V^\mu, V^\nu]) \,, \tag{21}$$

$$\bar{\mathcal{L}}_4 = [tr(V_\mu V_\nu)]^2 \,, \tag{22}$$

$$\bar{\mathcal{L}}_5 = [tr(V_\mu V^\mu)]^2 \,, \tag{23}$$

$$\bar{\mathcal{L}}_6 = tr(V_\mu V_\nu) tr(\mathcal{T} V^\mu) tr(\mathcal{T} V^\nu)\,, \tag{24}$$

$$\bar{\mathcal{L}}_7 = tr(V_\mu V^\mu)[tr(\mathcal{T} V^\nu)]^2\,, \tag{25}$$

$$\bar{\mathcal{L}}_8 = \frac{1}{4}[tr(\mathcal{T} W_{\mu\nu})]^2\,, \tag{26}$$

$$\bar{\mathcal{L}}_9 = i\frac{1}{2} tr(\mathcal{T} W_{\mu\nu}) tr(\mathcal{T}[V^\mu, V^\nu])\,, \tag{27}$$

$$\bar{\mathcal{L}}_{10} = \frac{1}{2}[tr(\mathcal{T} V_\mu) tr(\mathcal{T} V_\nu)]^2\,. \tag{28}$$

where the auxiliary variable V_μ and $\mathcal{T}$ is defined as

$$V_\mu = U^\dagger(\partial_\mu - i W_\mu^a T^a) U + i B_\mu T^3\,. \tag{29}$$

$$\mathcal{T} = 2 U^\dagger T^3 U = U^\dagger \tau^3 U\,, \tag{30}$$

The effective Lagrangian $\mathcal{L}_{EW}$ is invariant under the following local chiral transformation

$$\begin{aligned}
U &\rightarrow g_L U g_R^\dagger \,, \\
W_\mu &\rightarrow g_L W_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger \,, \\
W_{\mu\nu} &\rightarrow g_L W_{\mu\nu} g_L^\dagger \,, \\
B_\mu &\rightarrow B_\mu + i g_R \partial_\mu g_R^\dagger \,, \\
B_{\mu\nu} &\rightarrow B_{\mu\nu} \,,
\end{aligned} \tag{31}$$

where the gauge transformation factor g_L and g_R are defined as

$$g_L = \exp\left\{ i\alpha^a{}_L T^a \right\} \,, \; g_R = \exp\left\{ i\beta_R T^3 \right\} . \tag{32}$$

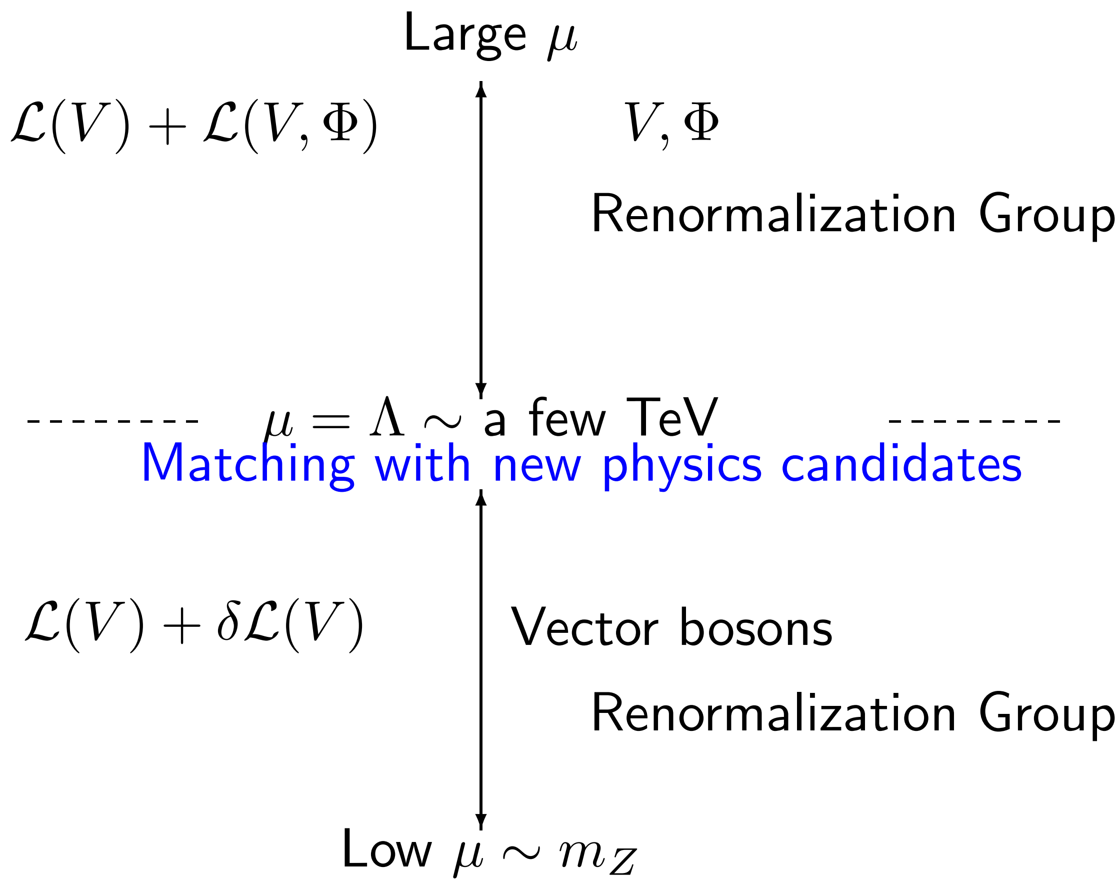


Fig. 1: EWCL in effective field theory

1.2 How to do the renormalization?

2.a The expected counter terms at one-loop level

$$\begin{aligned}\delta\Gamma(W,B,U) &= 2\frac{\delta g}{g}\bar{H}_1 + 2\frac{\delta g'}{g'}\bar{H}_2 + \frac{v\delta v}{2}tr(V\cdot V) \\ &+\frac{\delta\beta v^2 + 2\beta v\delta v}{4}[tr(\mathcal{T}V_\mu)]^2 + \sum_{i=1}^{10}\delta\alpha_i\bar{\mathcal{L}}^i \end{aligned} \quad (33)$$

C.1 In this non-canonical form of EWCL, it is not necessary to expand each operator, which might complicate the calcualtion procedure. Relation to the canonical one: $W \to gW$, and $B \to g'B$.

C.2 There is no necessity to define the wavefunction renormalization constant. The apparant reason is each operator has its efficient coupling to absorb divergence.

2.b The main concepts and techniques

M.1 Covariant background field method

A covariant quantization method. Only the quantum part needs to be quantized. B.S. DeWitt, Phys. Rev. 162 (1967), 1195, *ibid*, 1239.

M.2 Stueckelberg transformation

Combining the classic vector and Goldstone boson into a gauge invariant form. The classic gauge symmetry is guaranteed at each step. E. C. G. Stueckelberg., Helv. Phys. Acta 11 (1938) 299; 30 (1956) 209; T. Kunimasa and T. Goto, Prog. Theor. Phys. 37 (1967) 524.

M.3 Equation of motions

Eliminate terms like $(\partial \cdot Z)^2$, $(d \cdot W^+)(d \cdot W^-)$, $(\partial \cdot Z) W^+ \cdot W^-$, etc.

M.4 Gauge fixing

G.1 Eliminate some vector bilinear tensor terms and guarantee vector bilinear terms can be cast into standard form;

G.2 Make the expansion of $\ln(1 - X \Box_V^{-1} X \Box_\xi^{-1})$ stop with specified order.

M.5 Standard forms of bilinear quantum fields

$$\mathcal{L}_{quad} = -\Big\{\frac{1}{2}\widehat{V}_{\mu}^{\dagger a}\Box_V^{\mu\nu,ab}\widehat{V}_{\nu}^{b} + \frac{1}{2}\xi^{\dagger i}\Box_{\xi}^{\prime ij}\xi^{j} + \bar{c}^{a}\Box_c^{ab}c^{b} + \frac{1}{2}\widehat{V}_{\mu}^{\dagger,a}\overleftharpoon{X}^{\mu,aj}\xi^{j} + \frac{1}{2}\xi^{\dagger,i}\overrightharpoon{X}^{\nu,ib}\widehat{V}_{\nu}^{b}\Big\}, \quad (34)$$

$$\Box_V^{\mu\nu,ab} = -D^{2,ab}g^{\mu\nu} + \sigma_{0,VV}^{ab}g^{\mu\nu} + \sigma_{2,VV}^{\mu\nu,ab}, \quad (35)$$

$$\Box_{\xi}^{\prime ij} = \Box_{\xi\xi}^{ij} + X^{\alpha,ii'}d_{\alpha}^{i'j} + X^{\alpha\beta,ii'}d_{\alpha}^{i'j'}d_{\beta}^{j'j}, \quad (36)$$

$$\Box_{\xi}^{ij} = -d^{2,ij} + \sigma_{0,\xi\xi}^{ij} + \sigma_{2,\xi\xi}^{ij} + \sigma_{4,\xi\xi}^{ij}, \quad (37)$$

$$\Box_c^{ab} = -D_c^{2,ab} + \sigma_{0,cc}^{ab} + \sigma_{0,cc}^{ab}, \quad (38)$$

$$\overleftharpoon{X}^{\mu,aj} = \overleftharpoon{X}_{\alpha\beta}^{\mu,ai}d^{\alpha,ii'}d^{\beta,i'j} + \overleftharpoon{X}^{\mu\alpha,aj'}d_{\alpha}^{j'j} + \overleftharpoon{X}_{01}^{\mu,aj} + \overleftharpoon{X}_{03Z}^{\mu,aj} + \partial_{\alpha}\overleftharpoon{X}_{03Y}^{\mu\alpha,aj},$$

$$\overrightarrow{X}^{\nu,ib} = \overrightarrow{X}^{\nu,ia}_{\alpha\beta} D^{\alpha,ab'} D^{\beta,b'b} + \overrightarrow{X}^{\nu\alpha,ib'} D^{b'b}_{\alpha} + \overrightarrow{X}^{\nu,ib}_{01} + \overrightarrow{X}^{\nu,ib}_{03Z} + \partial_\alpha \overrightarrow{X}^{\nu\alpha,ib}_{03Y} ,$$

where $V^\dagger = (A, Z, W^-, W^+)$ and $\xi^\dagger = (\xi_Z, \xi^-, \xi^+)$. And the covariant differential operators $D = \partial + \Gamma_V$ and $d = \partial + \Gamma_\xi$. Q. S. Yan and D. S. Du, Phys. Rev. **D69** (2004) 085006. Sukanta Dutta, Kaoru Hagiwara, and Qi-Shu Yan, Nucl. Phys. **B704** (2005) 75

M.6 Gaussian integrals S. Dittmaier and C. Grosse-Knetter, Nucl. Phys. B **459**, 497 (1996).

$$\begin{aligned} \Gamma^{1-loop} &= -\frac{1}{2}\left\{Tr \ln \Box_V + Tr \ln \Box'_\xi - 2Tr \ln \Box_c \right. \\ &\quad \left. + Tr \ln \left(1 - \overrightarrow{X}^{\mu} \Box^{-1}_{V;\mu\nu} \overleftarrow{X}^{\nu} \Box^{'-1}_{\xi}\right)\right\} \\ &= -\frac{1}{2}\{Tr \ln \Box_V + Tr \ln \Box_\xi - 2Tr \ln \Box_c \end{aligned}$$

$$+Tr(X^{\alpha\beta} d_\alpha d_\beta \Box_\xi^{-1}) - \frac{1}{2} Tr(X^{\alpha\beta} d_\alpha d_\beta \Box_\xi^{-1} X^{\alpha'\beta'} d_{\alpha'} d_{\beta'} \Box_\xi^{-1})$$
$$-Tr(\overrightarrow{X}^{\mu} \Box^{-1}_{V;\mu\nu} \overleftarrow{X}^{\nu} \Box_\xi^{-1}) + Tr(\overrightarrow{X}^{\mu} \Box^{-1}_{V;\mu\nu} \overleftarrow{X}^{\nu} \Box_\xi^{-1} X^{\alpha\beta} d_\alpha d_\beta \Box_\xi^{-1})$$
$$-\frac{1}{2} Tr(\overrightarrow{X}^{\mu} \Box^{-1}_{V;\mu\nu} \overleftarrow{X}^{\nu} \Box_\xi^{-1} \overrightarrow{X}^{\mu'} \Box^{-1}_{V;\mu'\nu'} \overleftarrow{X}^{\nu'} \Box_\xi^{-1}) + \cdots \Big\} \,. \tag{39}$$

M.7 Trace and log

Calculation are conducted in x-coordinate space I. Jack and H. Osborn, Nucl. Phys. B207 (1982) 474; A. E. M. van de Ven, Nucl. Phys. B378 (1992) 309; J. Bijens, G. Colangelo and G. Ecker, Anns. Phys. 280 (2002) 100, [hep-ph/9907333]; J. Boernsen and A. E. M. van de Ven, Nucl. Phys. B657 (2003) 257

$$\begin{aligned} e.g. &= Tr(\overrightarrow{X}^{\mu} \Box^{-1}_{V;\mu\nu} \overleftarrow{X}^{\nu} \Box_\xi^{-1}) \\ &= \lim_{x' \to x} \int_x \langle x | \overrightarrow{X}^{\mu} \Box^{-1}_{V;\mu\nu} \overleftarrow{X}^{\nu} \Box_\xi^{-1} | x' \rangle \end{aligned}$$

$$= \lim_{x'\to x}\int_x\int_{x_i}\langle x|\overrightarrow{X}^{\mu}|x_1\rangle\langle x_1|\Box^{-1}_{V;\mu\nu}|x_2\rangle\langle x_2|\overleftarrow{X}^{\nu}|x_3\rangle\langle x_3|\Box^{-1}_{\xi}|x'\rangle$$

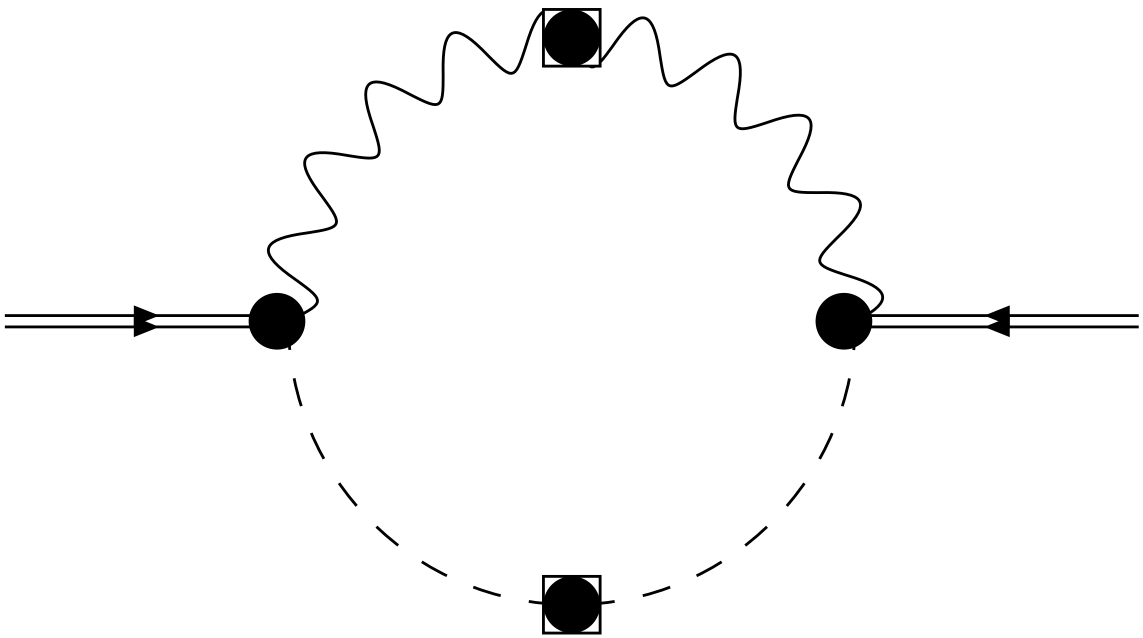
Vertices:

$$\langle x|\overrightarrow{X}^{\mu}|x_1\rangle\ ,\langle x_2|\overleftarrow{X}^{\nu}|x_3\rangle \tag{40}$$

Propagators: Schwinger, Phys. Rev. 82 (1951) 664; I. G. Avramidi, *Lecture Notes in Physics: N.s. M. Monogrph; 64* **Heat Kernel and Quantum Gravity**, Springer (Berlin), 2000; R. D. Ball, Phys. Rep. 182 (1989) 1.

$$\langle x_1|\Box^{-1}_{V;\mu\nu}|x_2\rangle\ ,\langle x_3|\Box^{-1}_{\xi}|x'\rangle \tag{41}$$

Diagram:



M.8 One-loop effective Lagrangian and RGEs

The $\overline{MS}$ renormalization scheme

$$\beta_{\alpha_i} = \frac{d\alpha_i}{d\,\ln\mu}\Big|_{\alpha_{i,bare} fixed} . \qquad (42)$$

1.3 The improved RGEs

3.a Renormalization group equations

The improved RGEs with linear term

$$\beta_g = g^2\Big(-\frac{29}{4} - \frac{\beta}{6} + \alpha_1 {g'}^2 - \frac{13\alpha_8 g^2}{6} - \frac{5\alpha_2 {g'}^2}{6} + \alpha_3 (\frac{28 g^2}{3} + \frac{{g'}^2}{2}) + \frac{13\alpha_9 g^2}{6}\Big)\,, \quad (43)$$

$$\beta_{g'} = {g'}^2\Big(\frac{1}{12} - \frac{\beta}{3} + 2\alpha_1 g^2 + \alpha_2 g^2 - \frac{5\alpha_3 g^2}{3}\Big)\,, \quad (44)$$

$$\beta_{\alpha_1} = \frac{1}{12} + 2\alpha_1 g^2 + \alpha_8 g^2 + \frac{5\alpha_2 g^2}{2} - \frac{5\alpha_3 g^2}{6} + \frac{\alpha_9 g^2}{2}\,, \quad (45)$$

$$\beta_{\alpha_2} = -\frac{1}{24} - \frac{\beta}{6} - \frac{\alpha_1 g^2}{2} + \alpha_2\left(\frac{e^2}{2} + \frac{19{g'}^2}{12}\right) + \frac{17\alpha_3 g^2}{12} + \alpha_9\left(\frac{-e^2}{2} + \frac{35 g^2}{12}\right) + \alpha_4\left(\frac{-g^2}{4} - \frac{{g'}^2}{2}\right) + \alpha_5\left(\frac{g^2}{2} - {g'}^2\right) - \frac{\alpha_6 {g'}^2}{2} - \alpha_7 {g'}^2 , \quad (46)$$

$$\beta_{\alpha_3} = -\frac{1}{24} + \frac{\beta}{6} - \frac{\alpha_1 {g'}^2}{4} + \frac{\alpha_8 g^2}{4} + \frac{2\alpha_2 {g'}^2}{3} + \frac{71\alpha_3 g^2}{12} + \frac{13\alpha_9 g^2}{12} + \alpha_4\left(\frac{-5g^2}{4} - \frac{3{g'}^2}{8}\right) + \alpha_5\left(\frac{5g^2}{2} - \frac{{g'}^2}{4}\right) + \alpha_6\left(\frac{-5g^2}{4} - \frac{3{g'}^2}{8}\right) + \alpha_7\left(\frac{5g^2}{2} - \frac{{g'}^2}{4}\right) , \quad (47)$$

$$\beta_{\alpha_4} = -\frac{1}{12} + \alpha_2\left(-e^2 + \frac{11g'^2}{6}\right) + \frac{3\alpha_3 g^2}{2} + \alpha_9\left(e^2 + \frac{7g^2}{6}\right)$$
$$+\alpha_4\left(\frac{19g^2}{4} + \frac{11g'^2}{2}\right) + \alpha_5\left(\frac{3g^2}{2} + 3g'^2\right)$$
$$+3\alpha_6 g^2 + 2\alpha_7 g'^2 \,, \tag{48}$$

$$\beta_{\alpha_5} = -\frac{1}{24} + \frac{\beta}{2} + \alpha_2\left(e^2 - \frac{4g'^2}{3}\right) - \frac{\alpha_3 g^2}{2} + \alpha_9\left(-e^2 - \frac{2g^2}{3}\right)$$
$$+\alpha_4\left(\frac{-5g^2}{4} - \frac{15g'^2}{4}\right) + \alpha_5\left(\frac{g^2}{2} - 2g'^2\right)$$
$$+\alpha_6\left(\frac{-g^2}{4} - \frac{3g'^2}{4}\right) + \alpha_7\left(2g^2 - 2g'^2\right) \,, \tag{49}$$

$$
\begin{aligned}
\beta_{\alpha_6} \;=\;& \alpha_2\left(e^2 - \frac{3g'^2}{2} + \frac{e^2 g'^2}{2G^2}\right) - \alpha_3 \frac{e^2 g'^2}{G^2} + \alpha_9\left(\frac{-3e^2}{2} - g^2 - \frac{e^2 g'^2}{2G^2}\right) \\
& + \alpha_4\left(\frac{e^2}{4} - \frac{37 g'^2}{8} - \frac{3e^2 g'^2}{G^2}\right) + \alpha_5\left(\frac{-e^2}{2} - \frac{9g'^2}{4} - \frac{2e^2 g'^2}{G^2}\right) \\
& + \alpha_6\left(\frac{-11e^2}{4} + \frac{g^2}{4} - \frac{g'^2}{8}\right) + \alpha_7\left(\frac{-5e^2}{2} + \frac{3g^2}{2} - \frac{9g'^2}{4}\right) \\
& + \alpha_{10}\left(6g^2 - 2g'^2\right), \qquad (50) \\
\beta_{\alpha_7} \;=\;& -\frac{3\beta}{4} + \alpha_2\left(-e^2 + \frac{5g'^2}{4}\right) + \alpha_9\left(\frac{3e^2}{2} + \frac{3g^2}{4}\right) \\
& + \alpha_4\left(\frac{3e^2}{4} + \frac{19g'^2}{4} - \frac{e^2 g'^2}{G^2}\right) + \alpha_5\left(\frac{3e^2}{2} + \frac{11g'^2}{4} - \frac{2e^2 g'^2}{G^2}\right)
\end{aligned}
$$

$$+\alpha_6\left(\frac{-e^2}{4}-\frac{7g^2}{8}+2g'^2\right)+\alpha_7\left(\frac{-e^2}{2}-\frac{5g^2}{2}+3g'^2\right)$$

$$+\alpha_{10}\left(\frac{3g^2}{2}+\frac{g'^2}{2}\right), \tag{51}$$

$$\beta_{\alpha_8} = \frac{\beta}{2}-\alpha_1 g'^2+\frac{37\alpha_8 g^2}{6}+\frac{5\alpha_2 g'^2}{6}-\frac{\alpha_3 g'^2}{2}+\frac{17\alpha_9 g^2}{6}, \tag{52}$$

$$\beta_{\alpha_9} = -\frac{\beta}{2}+\frac{\alpha_1 g'^2}{4}-\frac{11\alpha_8 g^2}{12}+\alpha_2\left(\frac{-e^2}{2}-\frac{g'^2}{12}\right)+\alpha_9\left(\frac{e^2}{2}+\frac{8g^2}{3}+\frac{3g'^2}{2}\right)$$

$$+\frac{7\alpha_4 g'^2}{8}+\frac{5\alpha_5 g'^2}{4}+\alpha_6\left(\frac{5g^2}{4}+\frac{7g'^2}{8}\right)+\alpha_7\left(\frac{-5g^2}{2}+\frac{5g'^2}{4}\right), \tag{53}$$

$$\beta_{\alpha_{10}} = -\alpha_2\frac{e^2 g'^2}{2G^2}+\alpha_3\frac{e^2 g'^2}{G^2}+\alpha_9\frac{e^2 g'^2}{2G^2}$$

$$
\begin{aligned}
& +\alpha_4\left(e^2-g'^2+\frac{2e^2g'^2}{G^2}\right)+\alpha_5\left(e^2-g'^2+\frac{2e^2g'^2}{G^2}\right) \\
& +\alpha_6\left(5e^2-\frac{3g'^2}{2}-\frac{2e^2g'^2}{G^2}\right)+\alpha_7\left(5e^2-\frac{3g'^2}{4}-\frac{2e^2g'^2}{G^2}\right) \\
& +\alpha_{10}\left(\frac{-19g^2}{2}+\frac{3g'^2}{2}\right), \qquad (54) \\
\beta_v \;=\; & \frac{3g^2}{4}+\frac{3g'^2}{8}+\frac{5\beta g^2}{4} \\
& +\alpha_1\left(\frac{-9g^2g'^2}{4}-\frac{g'^4}{2}+\frac{g'^6}{2G^2}\right)+\alpha_8\left(\frac{-g^4}{8}+\frac{g^2g'^2}{4}-\frac{g'^4}{4}+\frac{g'^6}{4G^2}\right) \\
& +\alpha_2\left(\frac{-5g^2g'^2}{2}+\frac{3g'^4}{4}\right)+\alpha_3\left(g^4-\frac{5g^2g'^2}{2}\right)+\alpha_9\left(\frac{g^4}{2}-\frac{3g^2g'^2}{4}\right)
\end{aligned}
$$

$$
-\alpha_4\left(\frac{21g^4}{8}+\frac{3G^4}{8}\right)-\alpha_5\left(\frac{15g^4}{4}+\frac{3G^4}{2}\right)
$$

$$
-\alpha_6\left(\frac{3G^4}{8}\right)-\alpha_7\left(\frac{3G^4}{2}\right), \tag{55}
$$

$$
\begin{aligned}
\beta_\beta \quad = \quad & \frac{-3{g'}^2}{8}-\frac{15\beta g^2}{4}-\frac{3\beta {g'}^2}{4} \\
& +\alpha_1\left(\frac{9g^2{g'}^2}{4}+\frac{{g'}^4}{2}-\frac{{g'}^6}{2G^2}\right)+\alpha_8\left(\frac{g^4}{8}-\frac{g^2{g'}^2}{4}+\frac{{g'}^4}{4}-\frac{{g'}^6}{4G^2}\right) \\
& +\alpha_2\left(\frac{5g^2{g'}^2}{2}-\frac{3{g'}^4}{4}\right)+\alpha_3\frac{5g^2{g'}^2}{2}+\alpha_9\left(\frac{-g^4}{2}+\frac{3g^2{g'}^2}{4}\right) \\
& -\alpha_4\left(\frac{15g^2{g'}^2}{4}+\frac{15{g'}^4}{8}\right)-\alpha_5\left(\frac{3g^2{g'}^2}{2}+\frac{3{g'}^4}{4}\right)
\end{aligned}
$$

$$-\alpha_6\left(\frac{3g^4}{4}+\frac{33G^4}{8}\right)-\alpha_7\left(3g^4+3G^4\right)-\alpha_{10}\left(\frac{9G^4}{2}\right). \qquad (56)$$

3.b Some features

- The anomalous couplings α_i always combined with gauge couplings in $\alpha_i g^2$ form.
- Both β_g and $\beta_{g'}$ are affected by anomalous couplings.

$$\begin{aligned}\beta_g &= g^2\Big\{-\frac{29}{4}-\frac{\beta}{6}+\alpha_1 g'^2-\frac{13\alpha_8 g^2}{6} \\ &\quad -\frac{5\alpha_2 g'^2}{6}+\alpha_3(\frac{28g^2}{3}+\frac{g'^2}{2})+\frac{13\alpha_9 g^2}{6}\Big\}, \qquad (57)\end{aligned}$$

$$\beta_{g'} = g'^2\left(\frac{1}{12} - \frac{\beta}{3} + 2\alpha_1 g^2 + \alpha_2 g^2 - \frac{5\alpha_3 g^2}{3}\right) \tag{58}$$

– Although both α_1, α_8 and β belong to the quadratic anomalous, the β function of β parameter receives the radiative corrections from quartic anomalous couplings while those of α_1 and α_8 do not.

$$\beta_{\alpha_1} = \frac{1}{12} + 2\alpha_1 g^2 + \alpha_8 g^2 + \frac{5\alpha_2 g^2}{2} - \frac{5\alpha_3 g^2}{6} + \frac{\alpha_9 g^2}{2}, \tag{59}$$

$$\beta_{\alpha_8} = \frac{\beta}{2} - \alpha_1 g'^2 + \frac{37\alpha_8 g^2}{6} + \frac{5\alpha_2 g'^2}{6} - \frac{\alpha_3 g'^2}{2} + \frac{17\alpha_9 g^2}{6}, \tag{60}$$

$$\beta_\beta = \frac{-3g'^2}{8} - \frac{15\beta g^2}{4} - \frac{3\beta g'^2}{4} + \alpha_1\left(\frac{9g^2 g'^2}{4} + \frac{g'^4}{2} - \frac{g'^6}{2G^2}\right) + \alpha_8\left(\frac{g^4}{8} - \frac{g^2 g'^2}{4} + \frac{g'^4}{4} - \frac{g'^6}{4G^2}\right)$$

$$+\alpha_2\left(\frac{5g^2g'^2}{2}-\frac{3g'^4}{4}\right)+\alpha_3\frac{5g^2g'^2}{2}+\alpha_9\left(\frac{-g^4}{2}+\frac{3g^2g'^2}{4}\right)$$
$$-\alpha_4\left(\frac{15g^2g'^2}{4}+\frac{15g'^4}{8}\right)-\alpha_5\left(\frac{3g^2g'^2}{2}+\frac{3g'^4}{4}\right)$$
$$-\alpha_6\left(\frac{3g^4}{4}+\frac{33G^4}{8}\right)-\alpha_7\left(3g^4+3G^4\right)-\alpha_{10}\left(\frac{9G^4}{2}\right) \quad (61)$$

– We notice that α_2 and α_3 can affect the running of α_1.

$$\beta_{\alpha_1} = \frac{1}{12}+2\alpha_1 g^2+\alpha_8 g^2+\frac{5\alpha_2 g^2}{2}-\frac{5\alpha_3 g^2}{6}+\frac{\alpha_9 g^2}{2} \quad (62)$$

– We can formulate the linear part of the β function into matrix form,

$$\frac{d\alpha_i}{dt} = \frac{1}{16\pi^2}\delta_{\mathbf{ij}}c_j+\frac{1}{16\pi^2}X_{ij}\alpha_j \quad (63)$$

3.c Checking points

c.1 Gauge couplings' β functions without anomalous couplings.

c.2 The constant parts in β functions should agree with previous results.

c.3 The explicit $U_{em}(1)$ symmetry is carefully examined in each step.

c.4 The custodinal symmetry in the limit $g' = 0$ is carefully examined in each step.

c.5 The whole computational procedure is checked with the nonlinearly realized Higgs SM and the renormalizability is confirmed.

c.6 Calculate in unitary gauge and find differences.

2.1 Quadratic and Triple AC's bounds from Experiments

1.a Quadratic AC's bounds The relations between the quadratic AC and the oblique precision test parameter

$$S = -16\pi\alpha_1 \,, \tag{64}$$

$$T = \frac{\rho - 1}{\alpha_{em}} = -\frac{2\beta}{\alpha_{em}} \,, \tag{65}$$

$$U = -16\alpha_8 \,, \tag{66}$$

T. Appelquist and G. H. Wu, Phys. Rev. D **48**, 3235 (1993) [arXiv:hep-ph/9304240].

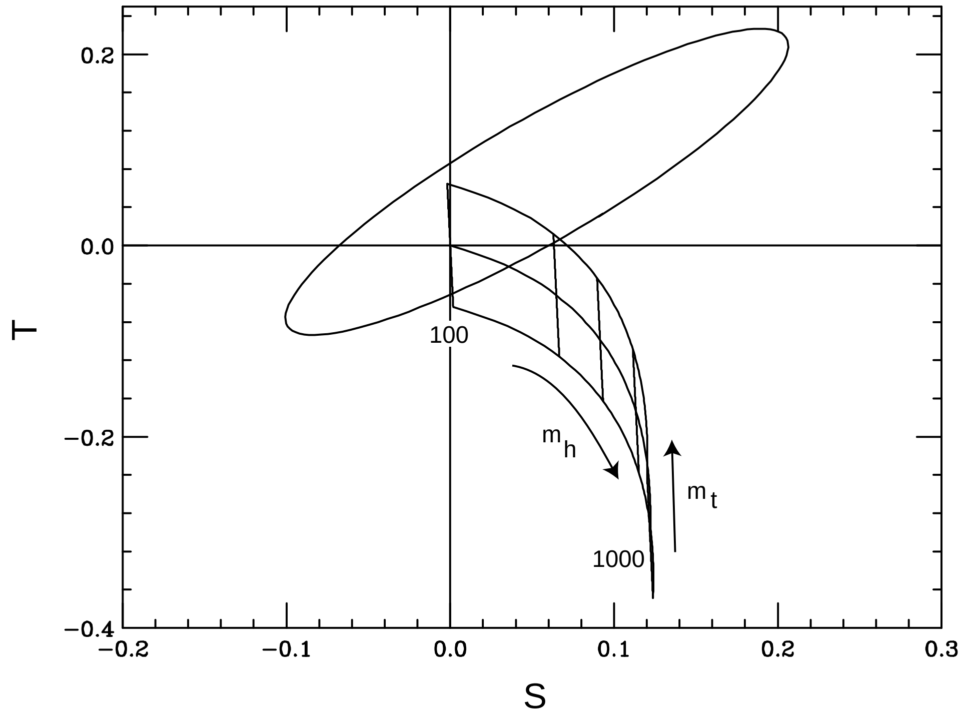
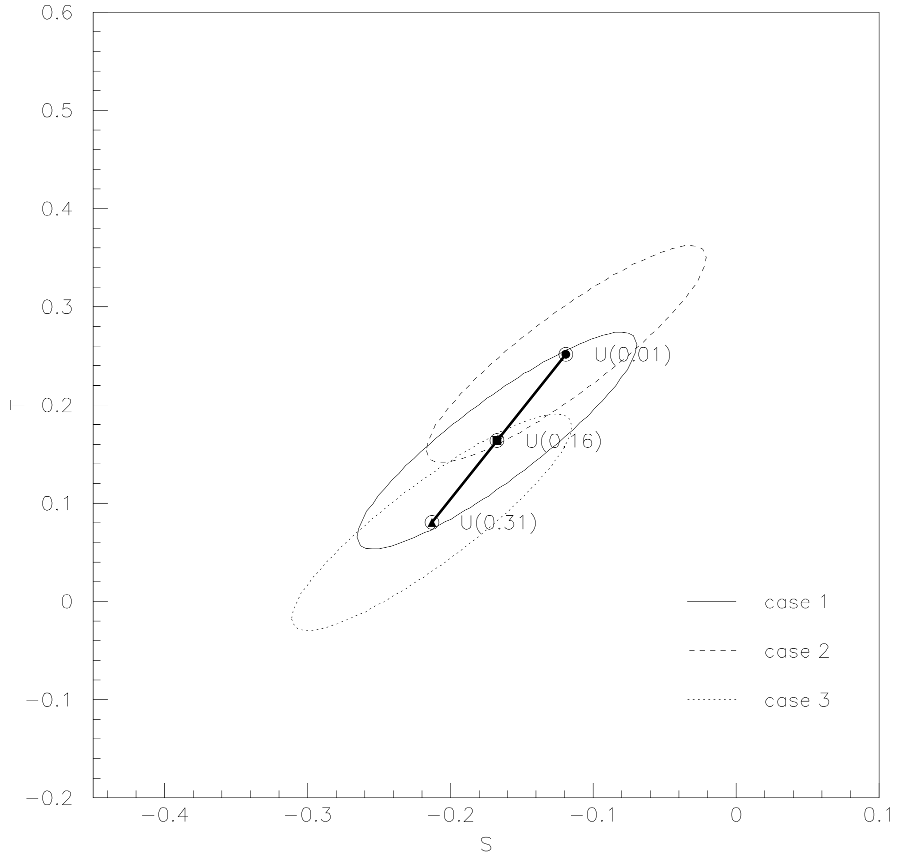


Fig. 2: M. E. Peskin and J. D. Wells, Phys. Rev. D **64**, 093003 (2001) [arXiv:hep-ph/0101342].

$$
\begin{array}{rl} S = & (-0.17 \pm 0.11) \\ T = & (0.16 \pm 0.14) \\ U = & (0.17 \pm 0.15) \end{array} \quad \rho_{corr.} = \begin{pmatrix} 1 & 0.9 & -0.42 \\ 0.9 & 1 & -0.60 \\ -0.42 & -0.60 & 1 \end{pmatrix} . \quad (67)
$$

The fitting with $m_h = 1TeV$.



0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
T
−0.4
−0.3
−0.2
−0.1
0
0.1
S
U(0.01)
U(0.16)
U(0.31)
case 1
case 2
case 3

1.b Triple AC's bounds

Relations between experimental observables to triple ACs:

$$\delta k_\gamma = -(\alpha_1 + \alpha_8 + \alpha_2 + \alpha_3 + \alpha_9) g^2 \,, \tag{68}$$

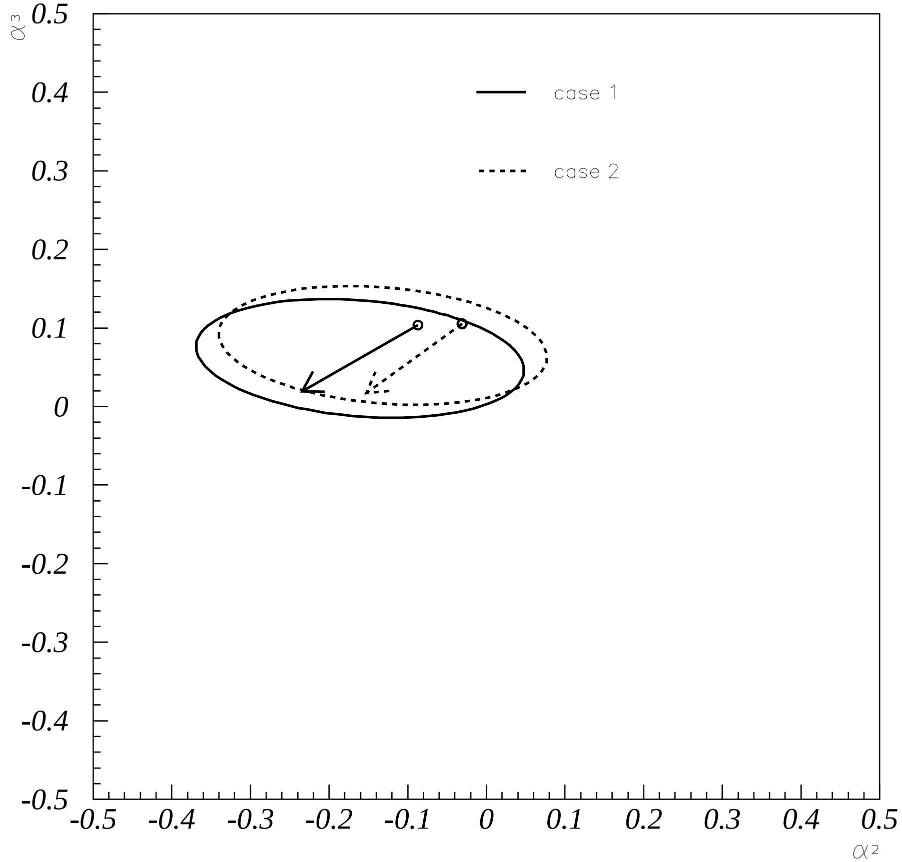
$$\delta k_z = -(\alpha_8 + \alpha_3 + \alpha_9) g^2 + (\alpha_1 + \alpha_2) g'^2 \,, \tag{69}$$

$$\delta g_Z^1 = -\alpha_3 G^2 \,. \tag{70}$$

Data taken from P. Achard *et al.* [L3 Collaboration], Phys. Lett. B **586**, 151 (2004) [arXiv:hep-ex/0402036].

$$\begin{aligned} \delta k_z &= -0.076 \pm 0.081 \\ \delta k_\gamma &= 0.013 \pm 0.092 \\ \delta g_z &= -0.034 \pm 0.048 \ . \end{aligned} \tag{71}$$

$$\begin{array}{l} \alpha_2 = \ (-0.16 \pm 0.22) \\ \alpha_3 = \ (0.06 \pm 0.09) \\ \alpha_9 = \ (0.07 \pm 0.18) \end{array} \quad \rho_{corr.} = \begin{pmatrix} 1 & 0.0 & -0.34 \\ 0.0 & 1 & -0.49 \\ -0.34 & -0.49 & 1 \end{pmatrix}. \qquad (72)$$


case 1
case 2
α^3
α^2
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

2.2 $S-T$ at $\mu=\Lambda$

2.a The Power Counting: β functions and Γ^2

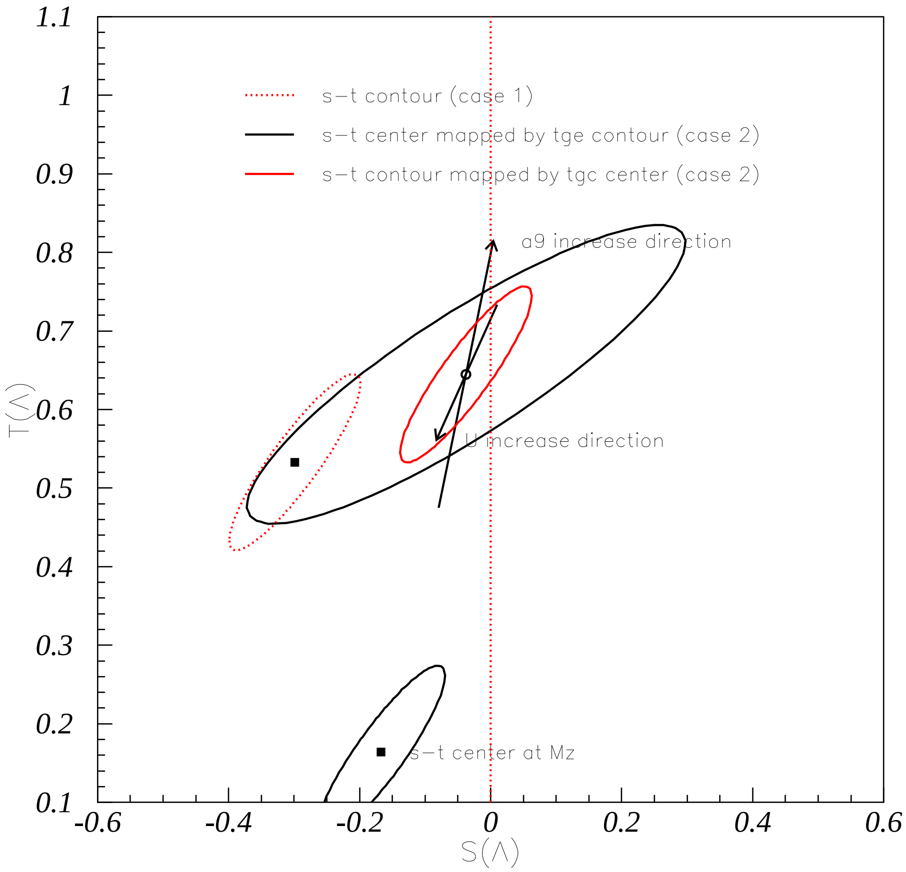
$$\beta = \frac{1}{16\pi^2}c^{1-loop} + \frac{1}{16\pi^2}f(g^2\alpha_i) + (\frac{1}{16\pi^2})^2 c^{2-loop} \qquad (73)$$

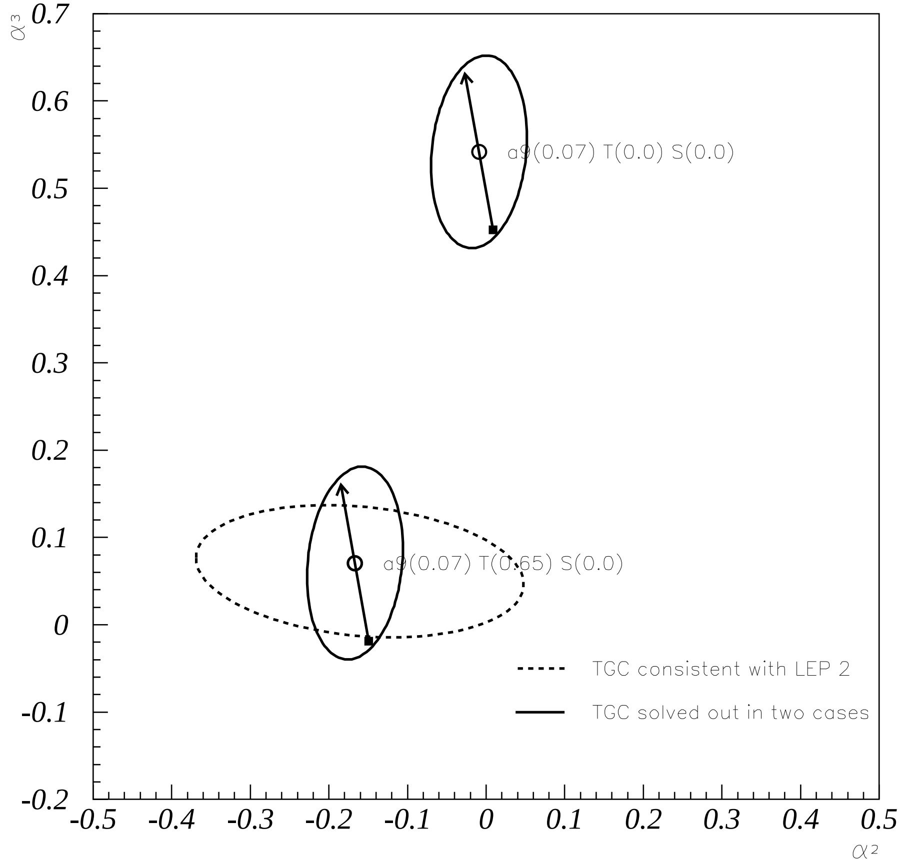
The improved power counting rules

H. Georgi, Phys. Lett. B **298** (1993) 187 [arXiv:hep-ph/9207278]. H. J. He, Y. P. Kuang and C. P. Yuan, Phys. Lett. B **382**, 149 (1996) [arXiv:hep-ph/9604309]. M. Harada and K. Yamawaki, Phys. Repts. 381 (2003) 1.

2.b Why $S-T$ at $\mu=\Lambda$?

The effective Lagrangian is matched with the fundamental dynamics at $\mu=\Lambda$ ($\Lambda = 1TeV$).


s−t contour (case 1)
s−t center mapped by tge contour (case 2)
s−t contour mapped by tgc center (case 2)
a9 increase direction
U increase direction
s−t center at Mz
T(Λ)
S(Λ)
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


a9(0.07) T(0.0) S(0.0)
a9(0.07) T(0.65) S(0.0)
TGC consistent with LEP 2
TGC solved out in two cases
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

2.c What've we learned?

a LEP 2 triple gauge couplings measurement have greatly constrained the parameter space of EWCL

b Uncertainty from the triple gauge couplings measurement can affect the value of $S(\Lambda)$ significantly; the chance for the triple guage couplings inducing a negative $S(m_z)$ still exists.

c $T(\Lambda)$ is always positive and big (0.6 or so)

d Future Giga Z and future triple gauge couplings measurements will further shrunk the parameter space of EWCL and help to pinpoint EWSB mechanism.

2.3 Summary

3.a What we have done

- D.1 We have obtained the improved one-loop renormalization group equations of the gauged nonlinear EW with the Feynman-'t Hooft gauge.
- D.2 We have demonstrated how to use the RGEs to phenomenology study.

3.b What's new

- W.1 The impact of TGC measurement to the EW global fit.
- W.2 The existence of a new solution to the negative $S(m_z)$ question in the EWCL.

3.c One Puzzle

– How to intepret the differences between the calculations in the unitary gauge and the renormalizable gauge?

Thank you!