# Mixed QCD and Weak Corrections to $t\bar{t}$ Production at Hadron Colliders*

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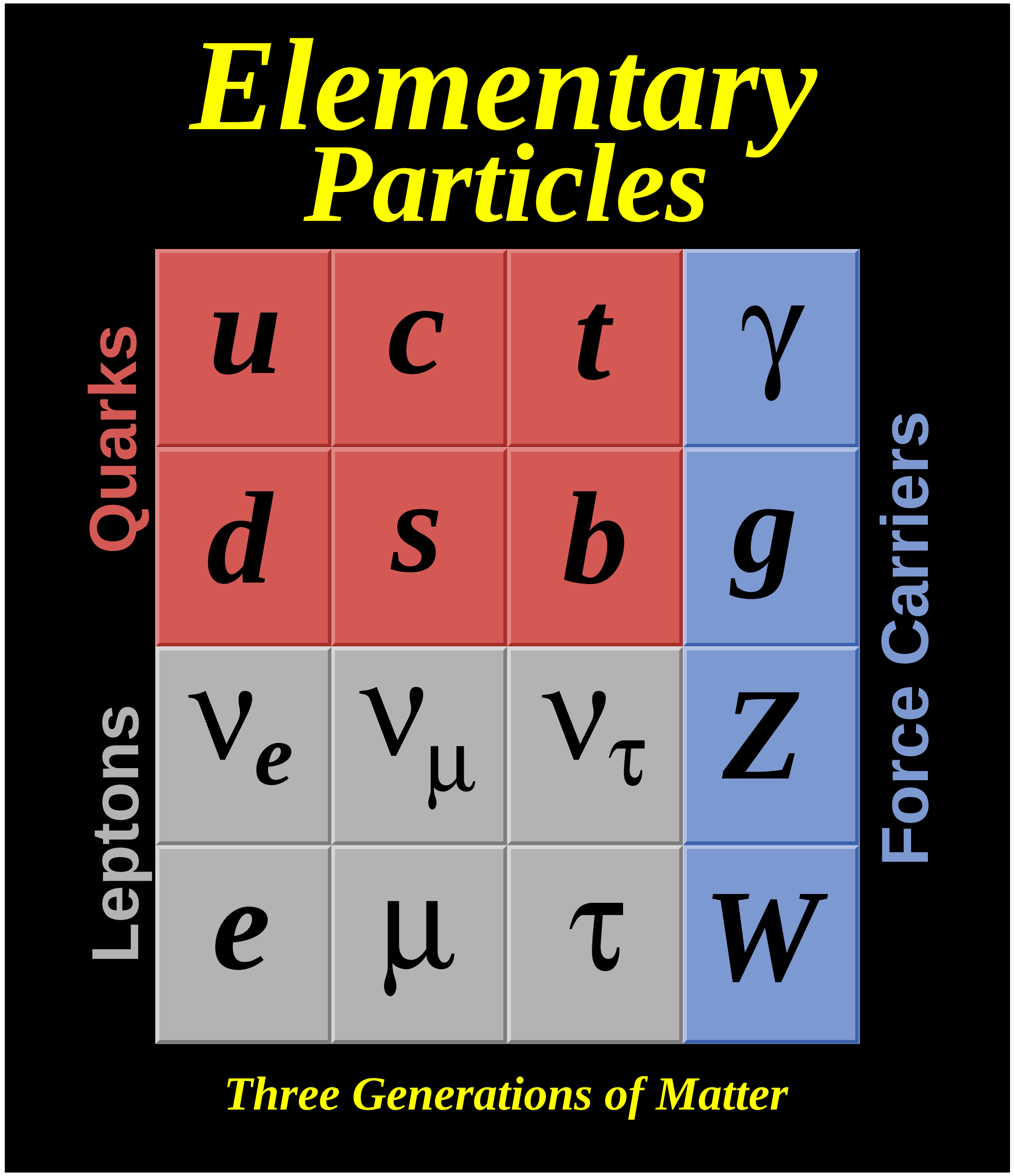
*Finished together with W. Bernreuther and M. Fuecker

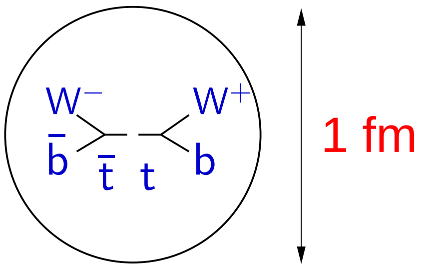
# Introduction



# Introduction(cont'd)

- Top quark: $\mathbf{m_t} \sim \mathbf{180}$ GeV as heavy as gold atom $\longrightarrow$ yet pointlike particle!?
- **Extremely instable**:

  **Lifetime** $\sim 4 \times 10^{-25}$s $\ll$ characteristic **Hadronization time** $\sim 28 \times 10^{-25}$s




$\Longrightarrow$ Their interactions are governed by short distance dynamics!

$\Longrightarrow$ Top-Quark properties not polluted by hadronization!

$\Longrightarrow$ Theoretical predictions related to top quark are reliable!!

- Within SM, Top quark Spin $\xrightarrow{V-A \;\; int.}$ Decay Product

$\Longrightarrow$ Spin effects will be important tools to study the interaction of top quarks in their production and decay once large data samples are available

# Introduction(cont'd)

## Production of top quarks at (future) accelerators

| Collider | Tevatron Run 1 | Tevatron Run 2 | LHC | LC |
|---|---|---|---|---|
| Type<br>Runtime | $p\bar{p}$<br>1992-1996 | $p\bar{p}$<br>2001-2008(?) | pp<br>2007-? | $e^+e^-$<br>2015(?)-? |
| $E_{CM}$ (TeV)<br>$\sigma(t\bar{t})$ (pb)<br>$\sigma$(single t) (pb) | 1.80<br>$\sim 5$<br>$\sim 1$ | 1.96<br>$\sim 7$<br>$\sim 1.5$ | 14.0<br>$\sim 800$<br>$\sim 300$ | $< 2m_t \sim 1.0$<br>$\sim 0.8$<br>$\sim 0$ |

Tevatron (Run II): $\sim 10^4/\mathrm{y}$ $t\bar{t}$ Pairs

Large Hadron Collider: $10^{7-8}/\mathrm{y}$ $t\bar{t}$ Pairs

Linear Collider: $10^5/\mathrm{y}$ $t\bar{t}$ Pairs

$\Longrightarrow$ Top Quark: an ideal 'laboratory' to test SM and search for 'new' physics

**TASK**: *$t\bar{t}$ production and decay must be predicted as precisely as possible within SM*

esp. Weak interaction contributions to $t\bar{t}$ production should be taken into account!!

# Introduction(cont'd)

**Aim**:

To Calculate the Mixed QCD and Weak radiative corrections to $t\bar{t}$ Production(keeping the full spin information of top quarks) at Hadron Colliders

**Present Status**:

For the process:

$$pp/p\bar{p} \to t\bar{t}X \to a_1 a_2 + X,$$

- the full differential distribution: $d\sigma \sim A + B$
  - $B$ induced by top quark spin effects
  - AT LO QCD, including exp. cuts, the contributions from B cannot be ignored
  - in present event generator, only $A$ taken into account

# Introduction(cont'd)

- Angular distribution(NLO QCD avilable):

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_1\ \cos\theta_2}=\frac{1}{4}\left\{1+B_1\ \cos\theta_1+B_2\ \cos\theta_2-C\ \ \cos\theta_1\ \cos\theta_2\right\}$$

$\theta_1=\sphericalangle(\hat{\mathbf{a}}_1,\hat{\mathbf{a}})$, $\theta_2=\sphericalangle(\hat{\mathbf{a}}_2,\hat{\mathbf{b}})$, $\hat{\mathbf{a}},\hat{\mathbf{b}}$: interpreted as **Spin-Quantum Axis**

- $C$ reflects spin-spin correlations between $t$ and $\bar{t}$
  - contr. from initial $q\bar{q}$ and $gg$ induced by pure QCD effects have different sign

  $\Longrightarrow C$ can be used as a tool to determine PDF

- $B_1$ and $B_2$ reflects top quark spin polarization
  - for pure QCD effects, only component normal to scattering plane
  - Weak int. leads to the component parrallel to scattering plane

# Introduction(cont'd)

- Mixed QCD and Weak radiative corrections
  - Parity-even and parity-odd $\alpha\alpha_s^2$ vertex corr. considered in

    Beenakker, Denner, Holik, & Mertig, Nucl. Phys. B **411** (1994) 343

    Kao, Ladinsky, Yuan, J. Mod. Phys. A **12** (1997) 1341

    Kao, Wackeroth, Phys. Rev. D **61** (2000) 055009
  - The box contributions to $q\bar{q} \to t\bar{t}$ not taken into account

**Our Present Work**

- focus on investigating mixed QCD and Weak radiative corrections to $q\bar{q} \to t\bar{t}$

  Leading order: $\alpha\alpha_s^2$

# Theoretical Framework

Consider

$$p\bar{p}, pp \to t\bar{t}X \to \begin{cases} 2\ell + n \geq 2 \text{ jets} + P_T^{miss} \\ \ell + n \geq 4 \text{ jets} + P_T^{miss} \\ n \geq 6 \text{ jets} \end{cases}$$

**Present Task:** Within SM, Determine Mixed QCD and Weak corrections

$$q\bar{q} \xrightarrow{t\bar{t}} b + \bar{b} + 4\ f(+\text{gluons}),$$

Proper treatment of unstable $t, \bar{t}, W^+, W^-$ needed

**Narrow resonances:** $\Gamma_{t,W} \ll m_{t,W}$

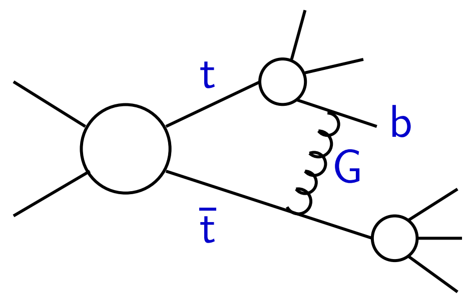
Consistent way to compute perturbative S matrix:

- expand complete amplitude around complex poles of unstable particle propagators
- keep only the leading pole terms

# Theoretical Framework(cont'd)

In the leading pole approximation, there are two types of radiative corrections:
factorizable non-factorizable

Example for Non-factorizable Corr.




In factorizable radiative corrections on-shell approximation may be applied:

$$\lim_{\Gamma/m \longrightarrow 0} \left| \frac{1}{k^2 - m^2 + im\Gamma} \right|^2 \longrightarrow \frac{\pi}{m\Gamma} \delta(k^2 - m^2)$$

associated error is of order $\alpha_s \Gamma / m$

# Theoretical Framework(cont'd)

At parton level

$$d\sigma_{fact.} = d\ \mathcal{L}ips\ [\mathrm{Tr}_{t,\bar{t}\ spins}(R\ \rho\ \bar{\rho})]$$

$R$ is the production spin density matrix of $t\bar{t}$ system

$\rho(\bar{\rho})$ is decay density matrix of $t(\bar{t})$

eg., for polarized semileptonic $t$ decay: $\rho = \frac{1}{2}\{1\!\!1 + \kappa_+ \hat{\sigma} \cdot \hat{\mathbf{q}}_+\}$ and $\bar{\rho} = \frac{1}{2}\{1\!\!1 - \kappa_- \hat{\sigma} \cdot \hat{\mathbf{q}}_-\}$

$\kappa_+(\kappa_-)$ signifies the top-spin analyzing power of the charged lepton

**Definition**

$$R_{\alpha\beta,\alpha'\beta'} = \widehat{\sum} \langle t(k_t, \alpha)\bar{t}(k_{\bar{t}}, \beta)X|\mathcal{T}|q\bar{q}\rangle \times \langle t(k_t, \alpha')\bar{t}(k_{\bar{t}}, \beta')X|\mathcal{T}|q\bar{q}\rangle^*$$

# Theoretical Framework(cont'd)

## Decomposition of $R$

$$R = A\, 1\!\!1 \otimes 1\!\!1 + \mathbf{B}^{+}\sigma \otimes 1\!\!1 + 1\!\!1 \otimes \sigma \mathbf{B}^{-} + C_{ij}\, \sigma^{i} \otimes \sigma^{j}$$

- A determines the $t\bar{t}$ production cross section
- $C_{ij}$ encodes full information on spin-spin-correlations of $t$ and $\bar{t}$
- $B^{+}(B^{-})$ describes polarization of $t(\bar{t})$

$\Longrightarrow$ Pure QCD, only a component normal to the scattering plane

$\Longrightarrow$ mixed QCD and Weak corr., components parrellel to the scattering plane

# Preliminary Results

For the Processes

$$q(p_1)+\bar{q}(p_2)\longrightarrow t(k_1,s_t)+\bar{t}(k_2,s_{\bar{t}}),$$

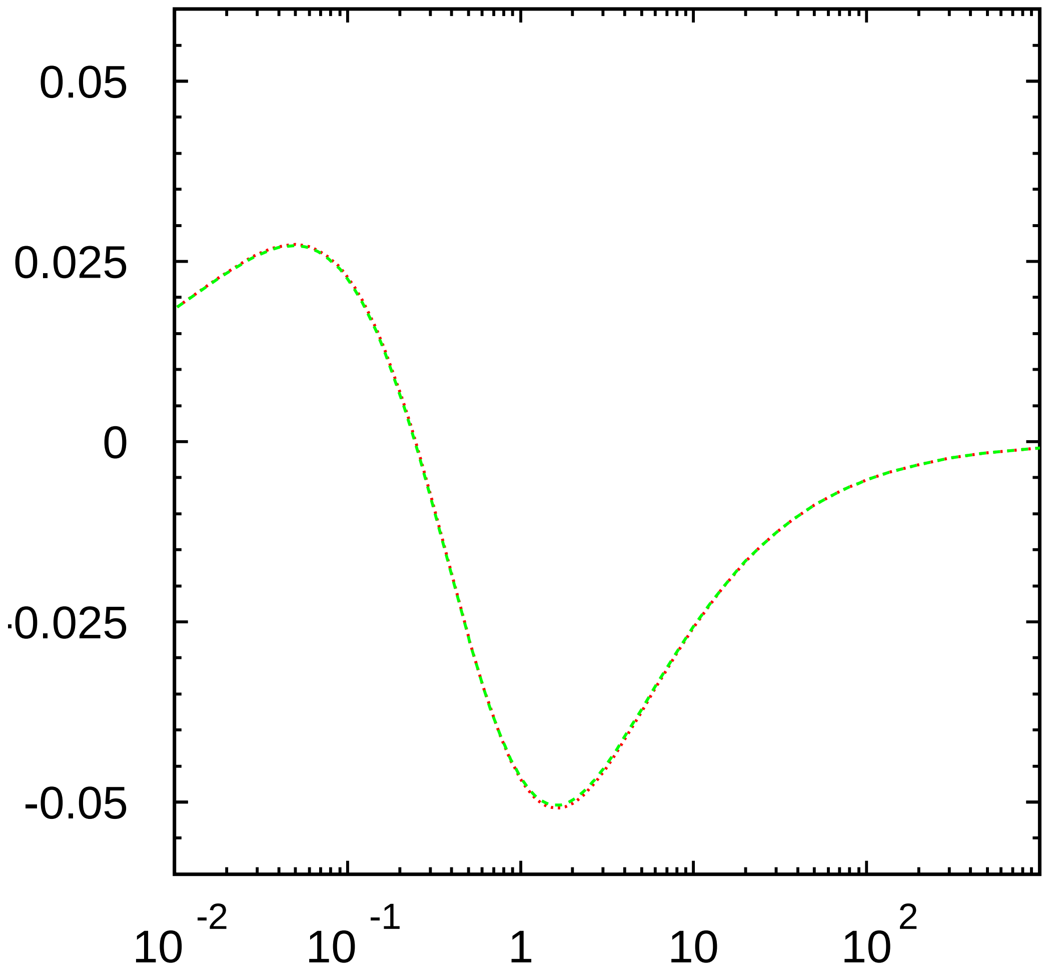
$$q(p_1)+\bar{q}(p_2)\longrightarrow t(k_1,s_t)+\bar{t}(k_2,s_{\bar{t}})+g(k_3),$$

the respective contibutions to the cross section are of the form

$\Longrightarrow \quad \alpha^2\,|\mathcal{M}_2|^2+\alpha\,\alpha_s^2\,\delta\mathcal{M}_2+\alpha\,\alpha_s^2\,\delta\mathcal{M}_3$

- $\mathcal{M}_2 \longrightarrow$ $\gamma$ and $Z$ exchange diagrams
- $\delta\mathcal{M}_2 \longrightarrow$ the Mixed QCD and Weak corrections(virtual)
  UV div. in the vertex corr. are removed by on-shell scheme defining the quark wave function renormalization and the top quark mass
- $\delta\mathcal{M}_3 \longrightarrow$ the Mixed QCD and Weak corrections(real gluon rad.)
  two methods can be used to seperate the IR divergence:
  1. Phase space slicing method
  2. subtraction method

# Preliminary Results(cont'd)




*Results obtained by phase space slicing and subtraction method, respectively*

# Preliminary Results(cont'd)

$$\sigma_{q\bar{q}}(\hat{s}, m_t) = \frac{4\pi\alpha}{m_t^2}\left[\alpha\ f_{q\bar{q}}^{(0)}(\rho) + \alpha_s^2 f_{q\bar{q}}^{(1)}(\rho)\right]$$

$$\sigma_{q\bar{q}}\langle 2\mathbf{s}_t \cdot \mathbf{a}\rangle_{q\bar{q}} = \frac{4\pi\alpha}{m_t^2}\left[h_{q\bar{q}}^{(0),a}(\rho) + \alpha_s^2 h_{q\bar{q}}^{(1),a}(\rho)\right]$$

$$\sigma_{q\bar{q}}\langle 4\mathbf{s}_t \cdot \mathbf{s}_{\bar{t}}\rangle_{q\bar{q}} = \frac{4\pi\alpha}{m_t^2}\left[e_{q\bar{q}}^{(0)}(\rho) + \alpha_s^2 e_{q\bar{q}}^{(1)}(\rho)\right]$$

$$\sigma_{q\bar{q}}\langle 4(\mathbf{s}_t \cdot \mathbf{a})(\mathbf{s}_{\bar{t}} \cdot \mathbf{b})\rangle_{q\bar{q}} = \frac{4\pi\alpha}{m_t^2}\left[g_{q\bar{q}}^{(0),a}(\rho) + \alpha_s^2 g_{q\bar{q}}^{(1),a}(\rho)\right]$$

$$4(\mathbf{s}_t \cdot \hat{\mathbf{a}})(\mathbf{s}_{\bar{t}} \cdot \hat{\mathbf{b}}\rangle = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}$$

$$\langle 2\mathbf{s}_t \cdot \hat{\mathbf{a}}\rangle = \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}, \qquad \rho = \frac{\hat{s}}{4m_t^2} - 1$$

# Preliminary Results(cont'd)

$\mathbf{a}$ and $\mathbf{b}$ are spin axixes chosen in $t\bar{t}$ CMS

For pure QCD-induced process, Good choice for spin-spin correlations are as follows:

at Tevatron,

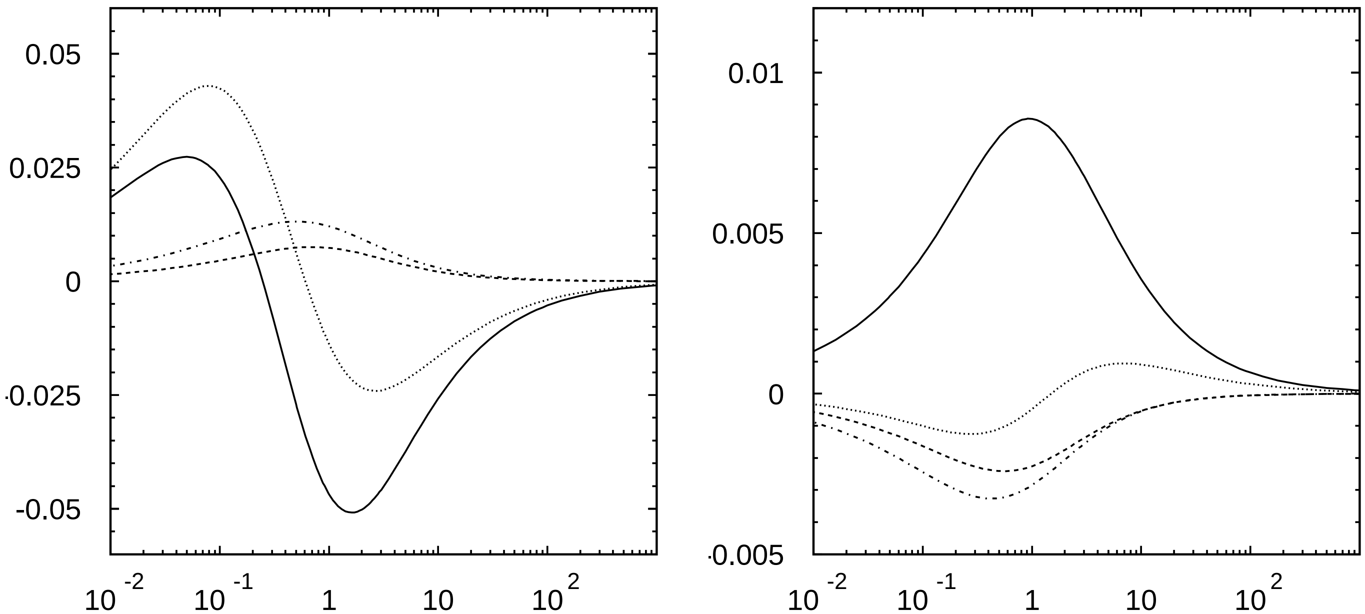
$$\hat{\mathbf{a}} = \hat{\mathbf{b}} = \mathbf{d} = \frac{-\mathbf{p} + (1-\gamma)(\mathbf{p}\cdot\mathbf{k})}{\sqrt{1-(\mathbf{p}\cdot\mathbf{k})^2(1-\gamma^2)}}, \quad \gamma = E_t/m_t$$

$$\hat{\mathbf{a}} = \hat{\mathbf{b}} = \hat{\mathbf{p}},$$

at LHC,

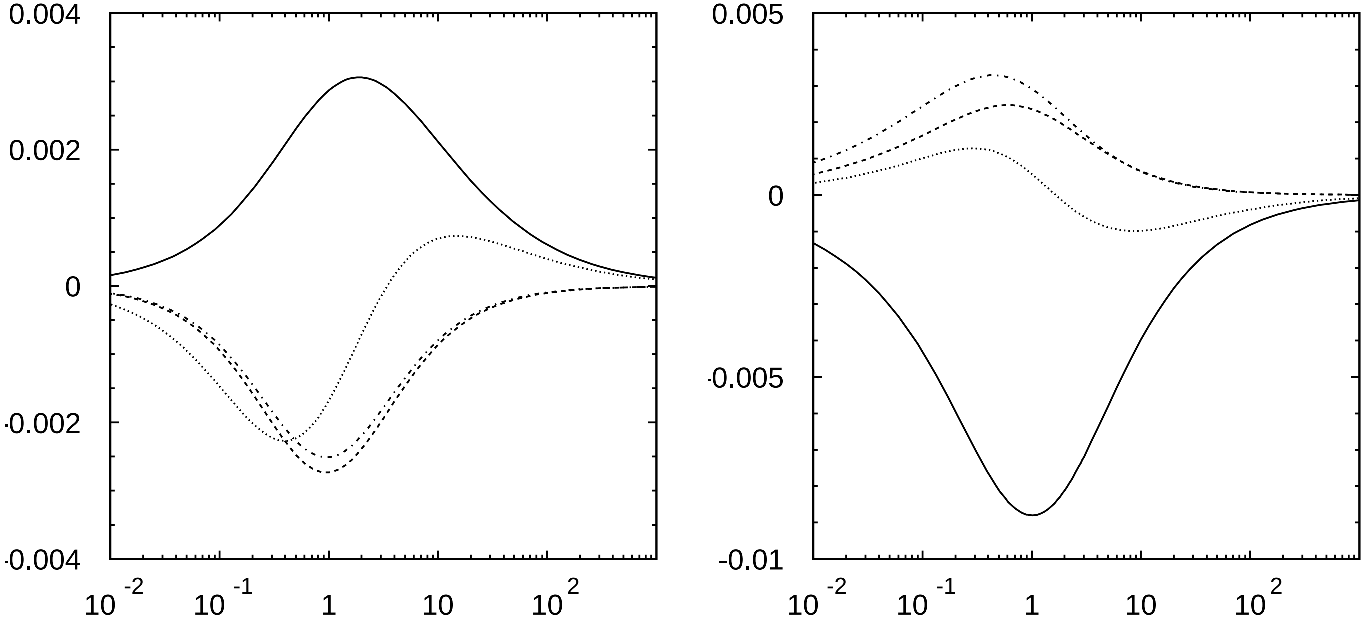
$$\hat{\mathbf{a}} = -\hat{\mathbf{b}} = \hat{\mathbf{k}}$$

# Preliminary Results(cont'd)



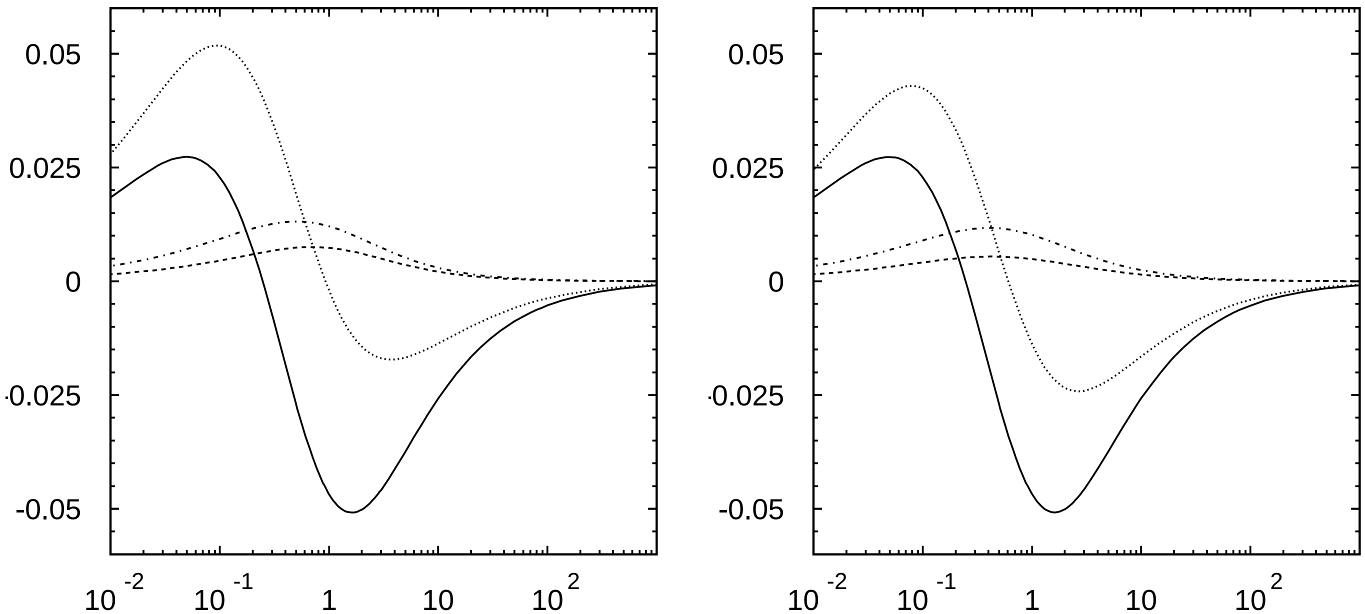

*Left: Scaling functions $f^{(0)}(\rho)_{q\bar{q}}$ (dashed for $q = d$-type, dash-dotted for $q = u$-type); $f^{(1)}(\rho)_{q\bar{q}}$ (solid for $q = d$-type, dotted for $q = u$-type); Right: Scaling functions $h^{(0),\mathbf{P}}(\rho)_{q\bar{q}}$ (dashed for $q = d$-type, dash-dotted for $q = u$-type); $h^{(1),\mathbf{P}}(\rho)_{q\bar{q}}$ (solid for $q = d$-type, dotted for $q = u$-type).*

# Preliminary Results(cont'd)



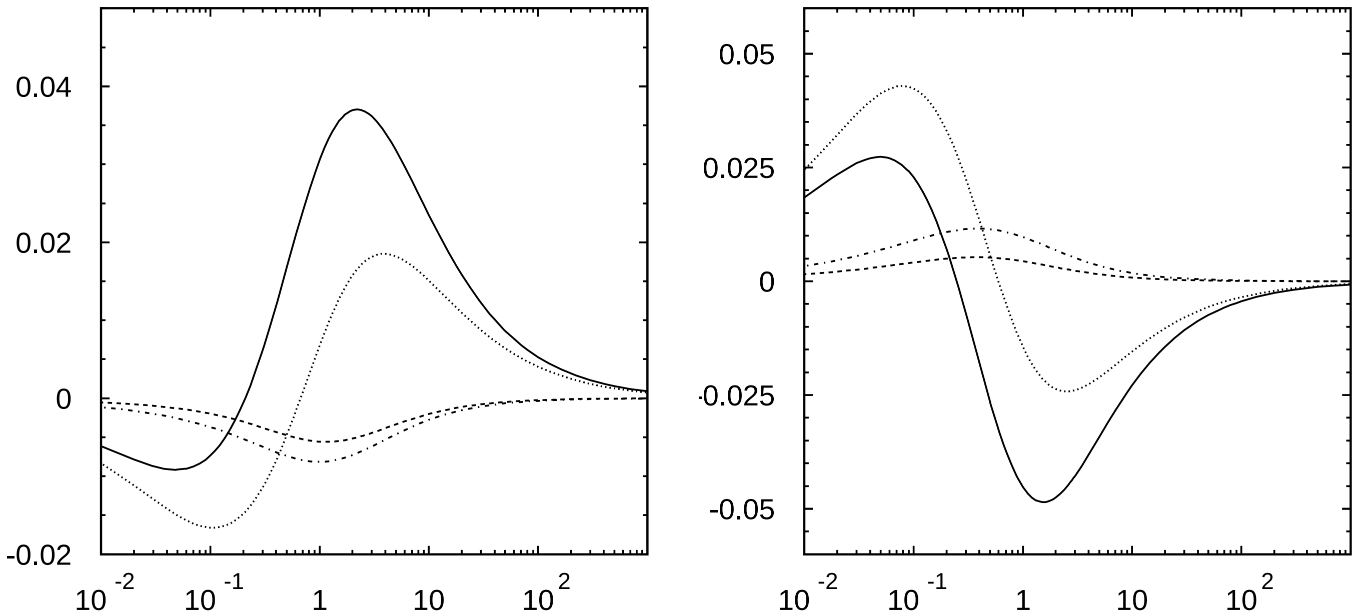

*Left: Scaling functions* $h^{(0),\mathbf{k}}(\rho)_{q\bar{q}}$ *(dashed for* $q = d$*-type, dash-dotted for* $q = u$*-type);* $h^{(1),\mathbf{k}}(\rho)_{q\bar{q}}$ *(solid for* $q = d$*-type, dotted for* $q = u$*-type); Right: Scaling functions* $h^{(0),\mathbf{d}}(\rho)_{q\bar{q}}$ *(dashed for* $q = d$*-type, dash-dotted for* $q = u$*-type);* $h^{(1),\mathbf{d}}(\rho)_{q\bar{q}}$ *(solid for* $q = d$*-type, dotted for* $q = u$*-type).*

# Preliminary Results(cont'd)




*Left: Scaling functions* $e^{(0)}(\rho)_{q\bar{q}}$ *(dashed for* $q = d$*-type, dash-dotted for* $q = u$*-type);* $e^{(1)}(\rho)_{q\bar{q}}$ *(solid for* $q = d$*-type, dotted for* $q = u$*-type); Right: Scaling functions* $g^{(0),\mathbf{d}}(\rho)_{q\bar{q}}$ *(dashed for* $q = d$*-type, dash-dotted for* $q = u$*-type);* $g^{(1),\mathbf{d}}(\rho)_{q\bar{q}}$ *(solid for* $q = d$*-type, dotted for* $q = u$*-type).*

# Preliminary Results(cont'd)




*Left: Scaling functions $g^{(0),\mathbf{k}}(\rho)_{q\bar{q}}$ (dashed for $q = d$-type, dash-dotted for $q = u$-type); $g^{(1),\mathbf{k}}(\rho)_{q\bar{q}}$ (solid for $q = d$-type, dotted for $q = u$-type); Right: Scaling functions $g^{(0),\mathbf{p}}(\rho)_{q\bar{q}}$ (dashed for $q = d$-type, dash-dotted for $q = u$-type); $g^{(1),\mathbf{p}}(\rho)_{q\bar{q}}$ (solid for $q = d$-type, dotted for $q = u$-type).*

# Summary and Outlook

- We calculated the mixed QCD and Weak corrections to $q\bar{q} \to t\bar{t}$, keeping full dependence on $t$ and $\bar{t}$ spins

- The detailed predictions for spin-induced angular correlations and asymmetries within SM will be available
- The full differential distribution keeping full information on top quark spins will be available

- Such predictions are necessary for the a detailed analysis of large $t\bar{t}$ data samples at the LHC.

**Top-Quark Physics will play an important role**

**in particle physics!**

*Thanks a lot for your attention!*