The Kaon B-parameter in quenched QCD

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Topics:

* $*$ Defintion of B_K & wanted precision
* $*$ B_K and Wilson type quarks
* $*$ Non-perturbative renormalisation
* $*$ A few simulation details & (preliminary) results
* $*$ Conclusions

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Work done by a subset of the ALPHA Collaboration

- Petros Dimopoulos (University of Cyprus)
- Jochen Heitger (Universität Münster, Germany)
- Filippo Palombi (DESY Hamburg, Germany)
- Carlos Pena (CERN, Switzerland)
- Stefan Sint (Unversidad Autónoma de Madrid, Spain)
- Anastassios Vladikas (INFN Rome "Tor Vergata", Italy)

The numerical simulations were (mostly) carried out on APE-1000 machines at DESY Zeuthen, Germany

Definition of B_K

The B_K parameter is defined in QCD with dynamical u, d, s quarks:

$$\langle \bar{K}^0 | O^{\Delta S=2}_{(\mathrm{V-A})(\mathrm{V-A})} | K^0 \rangle = \tfrac{8}{3} F_K^2 m_K^2 B_K$$

The local operator

$$O^{\Delta S=2}_{(\mathrm{V-A})(\mathrm{V-A})} = \sum_{\mu} \left[\bar{s} \gamma_\mu (1-\gamma_5) d \right]^2$$

is the effective local interaction induced by integrating out the massive gauge bosons and t, b, c quarks in the Standard Model.

- The approximation of $K^0 - \bar{K}^0$ mixing by a matrix element of a local operator is estimated to be good to about 5 percent.

$\Rightarrow$ defines the precision to be reached in lattice simulations

- only the parity-even part contributes to B_K

$$O_{(\mathrm{V-A})(\mathrm{V-A})} = \underbrace{O_{\mathrm{VV+AA}}}_{\mathrm{parity-even}} - \underbrace{O_{\mathrm{VA+AV}}}_{\mathrm{parity-odd}}$$

B_K & Wilson type quarks

Lattice QCD with Wilson quarks:

+ computationally cheap

+ no mixing of flavour and spin degrees of freedom (as opposed to staggered quarks)

− but axial symmetries are explicitly broken:

  – mixing of operators with opposite chirality
  – leading cutoff effects of O(a)
  – quenched approximation: zero modes at finite quark masses (exceptional configurations) $\Rightarrow m_{PS} > m_K$ in practice

N.B. mixing problem most relevant for B_K:

$$
\begin{aligned}
[O_{\mathrm{VV+AA}}]_{\mathrm{R}} &= Z_{\mathrm{VV+AA}}\Big\{O_{\mathrm{VV+AA}} + \sum_{i=1}^{4} z_i\, O_i^{d=6}\Big\} \\
[O_{\mathrm{VA+AV}}]_{\mathrm{R}} &= Z_{\mathrm{VA+AV}} O_{\mathrm{VA+AV}}
\end{aligned}
$$

⇒ parity-odd component renormalizes multiplicatively!

Wilson quarks with chirally rotated mass terms

Question: Can we avoid the mixing problem by using the multiplicatively renormalized operator $O_{\mathrm{VA+AV}}$ to compute B_K?
Answer: YES, by introducing the quark mass terms in a non-standard way (Frezzotti, Grassi, S., Weisz '01):

- consider continuum theory for a light quark doublet ψ and the s-quark:

$$\mathcal{L}_f = \overline{\psi}\left(\not{D} + m + i\mu_q\gamma_5\tau^3\right)\psi + \bar{s}\left(\not{D} + m_{\mathrm{s}}\right)s.$$

- perform a chiral rotation of the doublet fields

$$\psi' = \exp\left(i\alpha\gamma_5\frac{\tau^3}{2}\right)\psi, \qquad \overline{\psi}' = \overline{\psi}\exp\left(i\alpha\gamma_5\frac{\tau^3}{2}\right), \qquad \text{with} \quad \tan\alpha = \mu_{\mathrm{q}}/m$$

$$\Rightarrow \quad \mathcal{L}_f' = \overline{\psi}'\left(\not{D} + m'\right)\psi' + \bar{s}\left(\not{D} + m_{\mathrm{s}}\right)s, \qquad m' = \sqrt{m^2 + \mu_q^2}$$

- the field rotation redefines the symmetries and maps composite fields:

$$\begin{aligned} O'_{\mathrm{VV+AA}} &= \cos(\alpha)O_{\mathrm{VV+AA}} - i\sin(\alpha)O_{\mathrm{VA+AV}} \\ &= -iO_{\mathrm{VA+AV}} \quad (\alpha = \pi/2) \end{aligned}$$

Maximally twisted light doublet ($\pi/2$ szenario)

- The operators are mapped to each other at "maximal twist" $\alpha = \pi/2 \Leftrightarrow m = 0$

$\Rightarrow$ use Wilson quarks with a maximally twisted doublet and a standard s-quark

$$S_{\mathrm{f}} = a^4 \sum_x \{\overline{\psi}(x)(D_W + m_0 + i\mu_{\mathrm{q}}\gamma_5\tau^3)\psi(x) + \bar{s}(x)(D_W + m_{0,s})s(x)\}$$

- additional benefit: the twisted mass term eliminates unphysical zero modes

$\Rightarrow$ simulations can get close to physical situation: mass degenerate u, d-quarks and a heavier s-quark

- HOWEVER:
  - benchmark results in lattice QCD usually performed for $m_s = m_d$
    $\Rightarrow$ zero mode problem is back for the s-quark
  - quenched approximation: quenched chiral log for $m_s \neq m_d$

Mass degenerate (d, s) doublet ($\pi/4$ scenario)

Exchange the roles of s and u quarks, i.e. $\psi = (s, d)^T$:

- after the chiral rotation one then finds

$$\begin{aligned} O'_{\mathrm{VV+AA}} &= \cos(2\alpha) O_{\mathrm{VV+AA}} - i \sin(2\alpha) O_{\mathrm{VA+AV}} \\ &= -i O_{\mathrm{VA+AV}} \quad (\alpha = \pi/4) \end{aligned}$$

$\Rightarrow$ again the operators are mapped to each other provided $\alpha = \pi/4 \Leftrightarrow m = \mu_{\mathrm{q}}$

- can push down simultaneously d and s quark masses

- no zero modes (exceptional configurations): the u quark does not participate in correlation functions

- we know all finite renormalization constants and O(a) improvement coefficients which are necessary for the parameter tuning.

$\Rightarrow$ need to determine the multiplicative renormalization constant

Non-perturbative renormalization of $O_{\mathrm{VA+AV}}$

We use a finite volume scheme based on the Schrödinger functional which

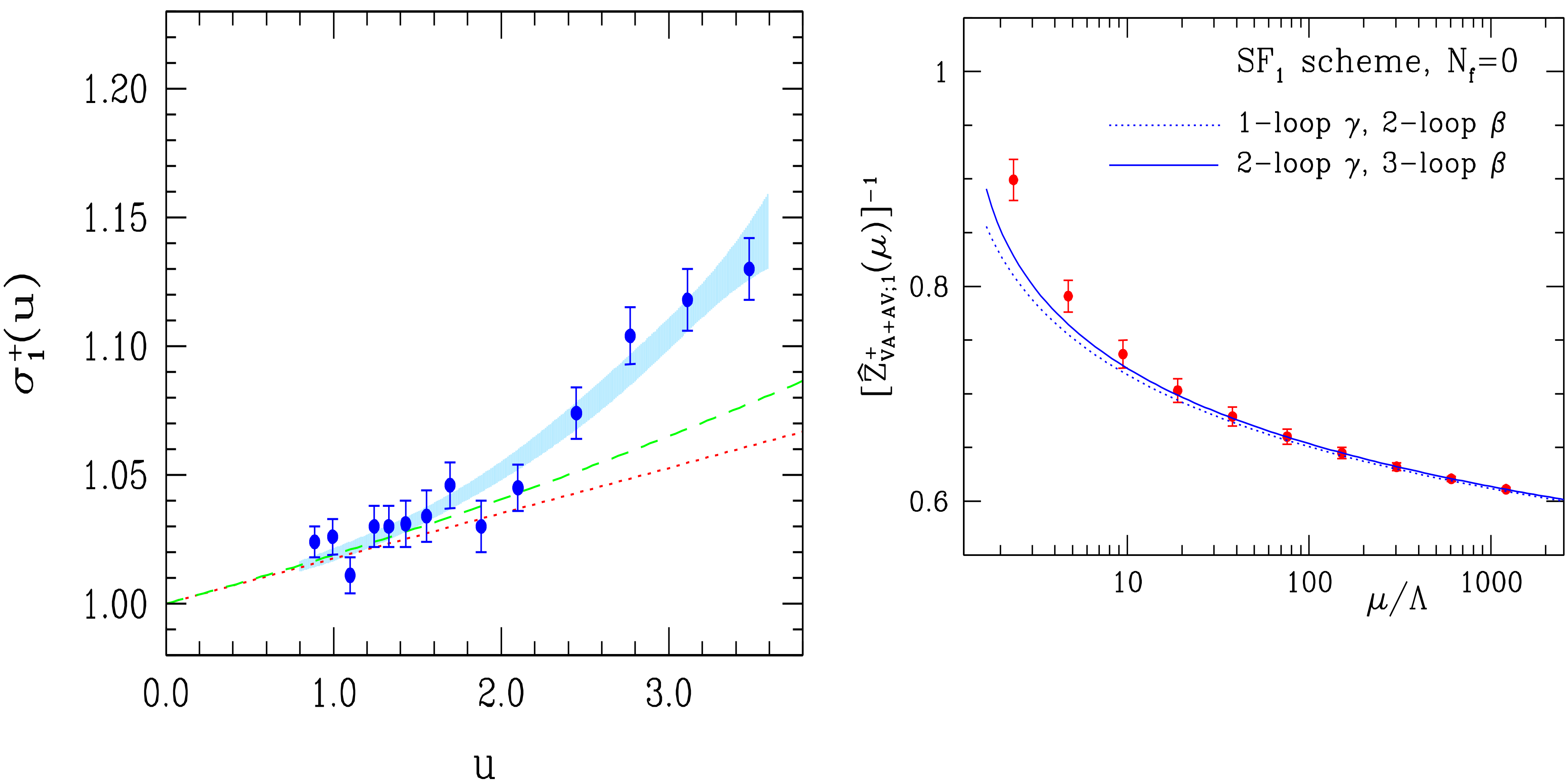
- is gauge invariant
- is quark mass independent: renormalization conditions are imposed in the chiral limit (S. Weinberg '73)
- allows for simulations at zero quark mass, due to SF boundary conditions (Dirichlet conditions in time direction)
- has the renormalization scale set by the space-time volume, $\mu = 1/L$

⇒ recursive procedure allows to connect scales which differ by orders of magnitude

⇒ renormalization fully controlled at the non-perturbative level

refs.: Guagnelli, Heitger, Palombi, Pena, S., Vladikas (Alpha collaboration), `hep-lat/0505002` & `hep-lat/0505003`

Renormalization group evolution of B_K (SF scheme)

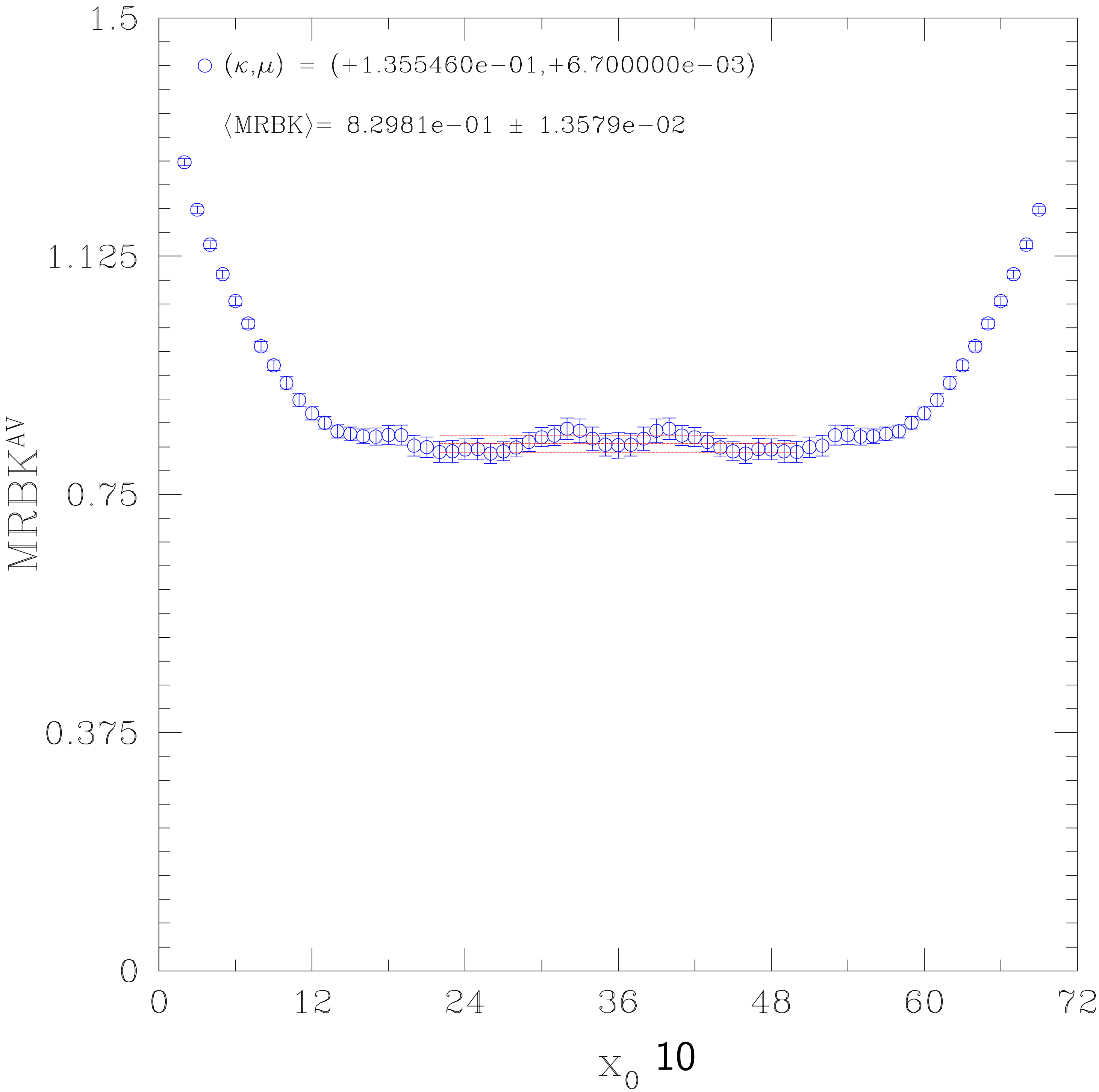


A few simulation details

Use O(a) improved Wilson quarks with twisted mass term, standard Wilson plaquette action:

- 4-5 β-values $\in [6.0 - 6.45]$, corresponding to lattice spacings $a = 0.05 - 0.1$ fm (scale from $r_0 = 0.5$ fm)
- lattice volumes range from $16^3 \times 48$ to $32^3 \times 72$
- quark masses tuned to achieve $\alpha = \pi/2$ or $\alpha = \pi/4$ and pseudoscalar masses above or around m_K
- finite volume effects below statistical errors (checked at $\beta = 6.0$); At $\beta = 6.45$ the 32^3-lattice is too small $\Rightarrow$ small chiral extrapolation required even in $\pi/4$ case

Extract B_K directly from suitable ratios of correlation functions:



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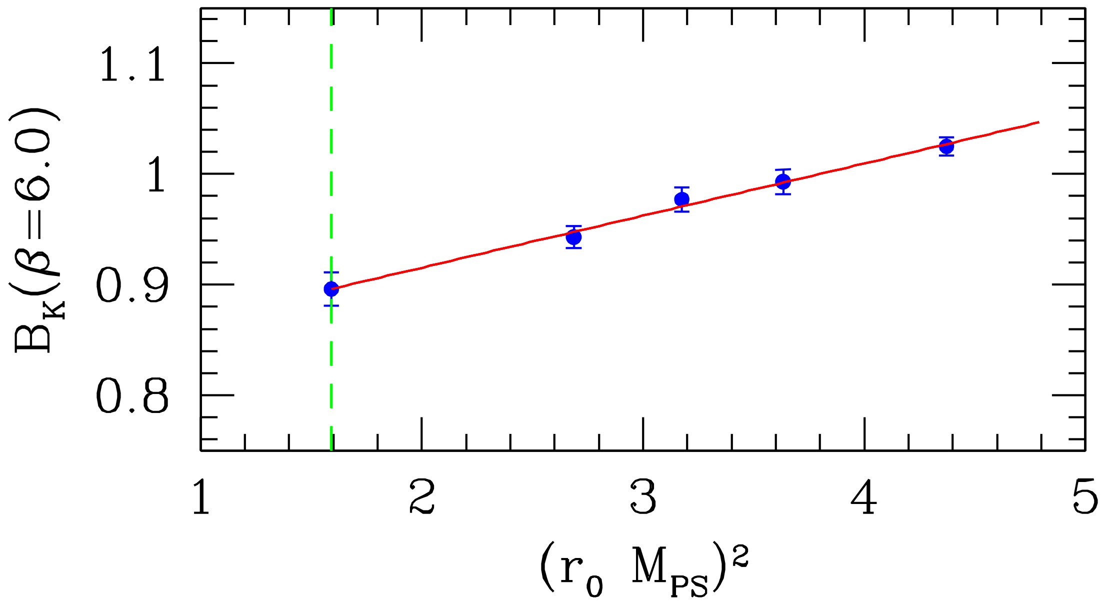
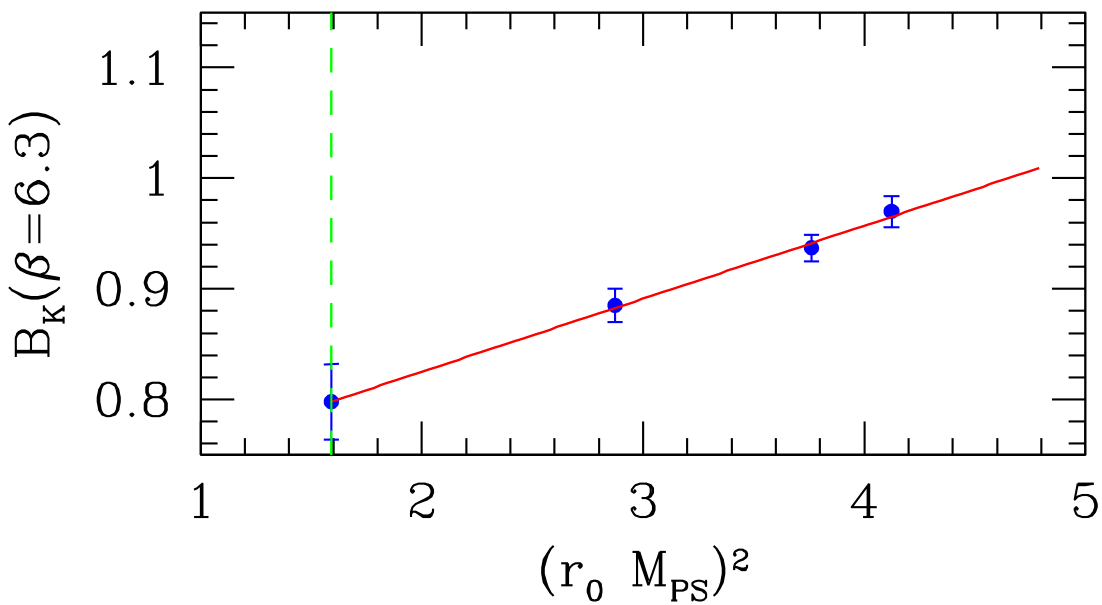
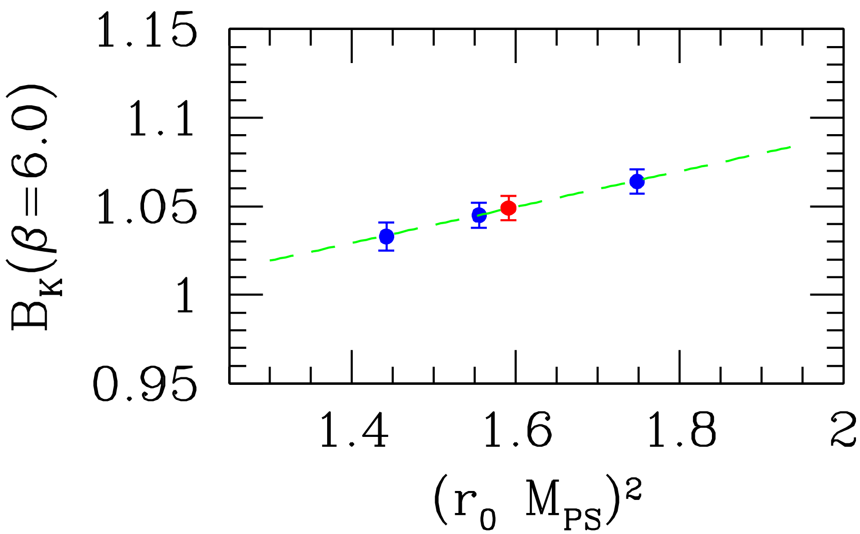
- $\pi/2$ case:

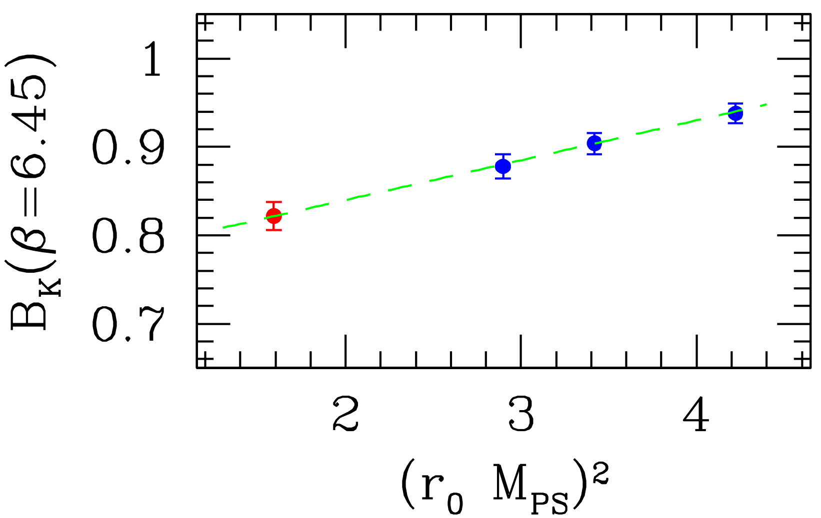




- $\pi/4$ case:

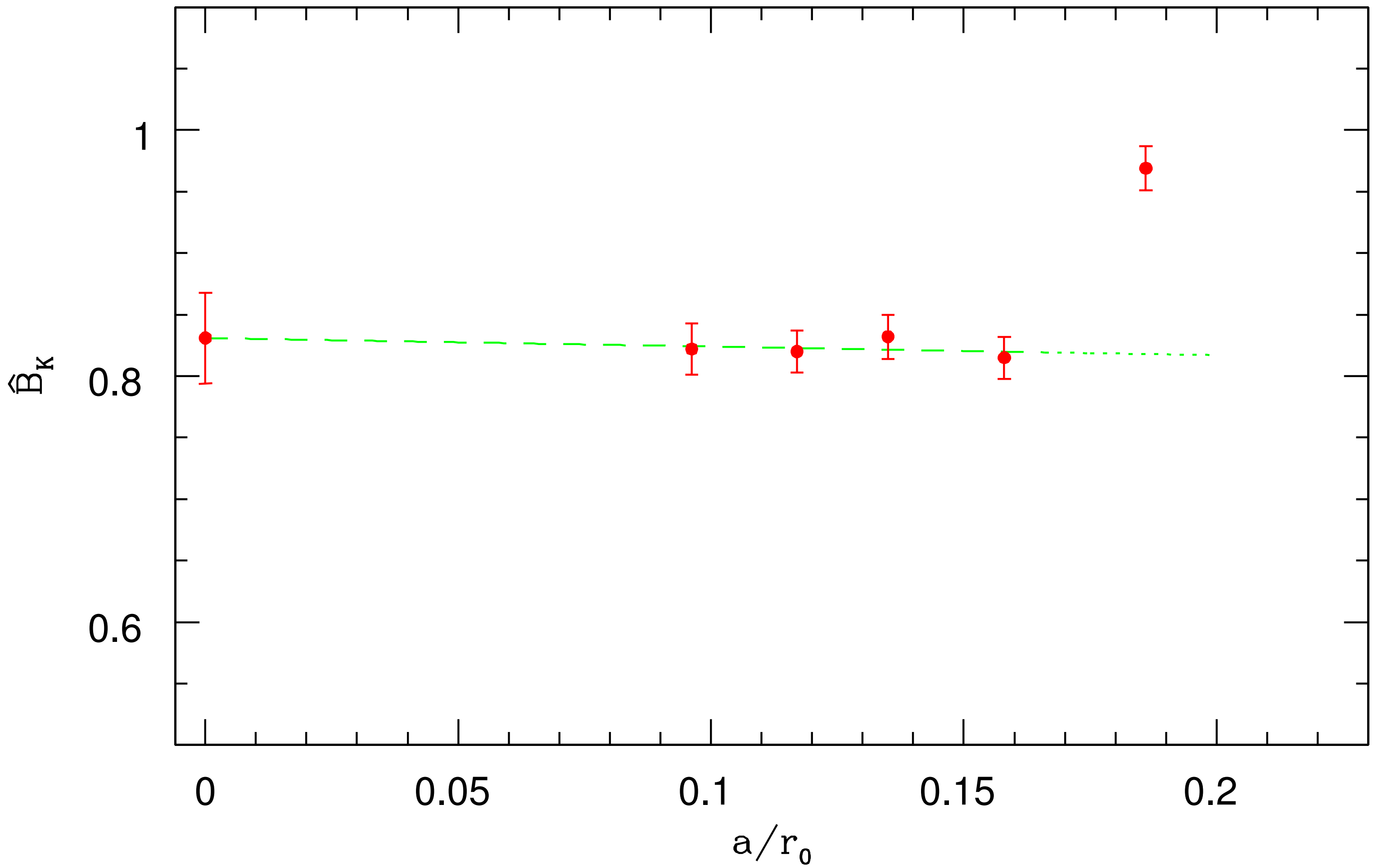


1.15
1.1
1.05
1
0.95
1.4
1.6
1.8
2
$B_K(\beta=6.0)$
$(r_0\ M_{PS})^2$

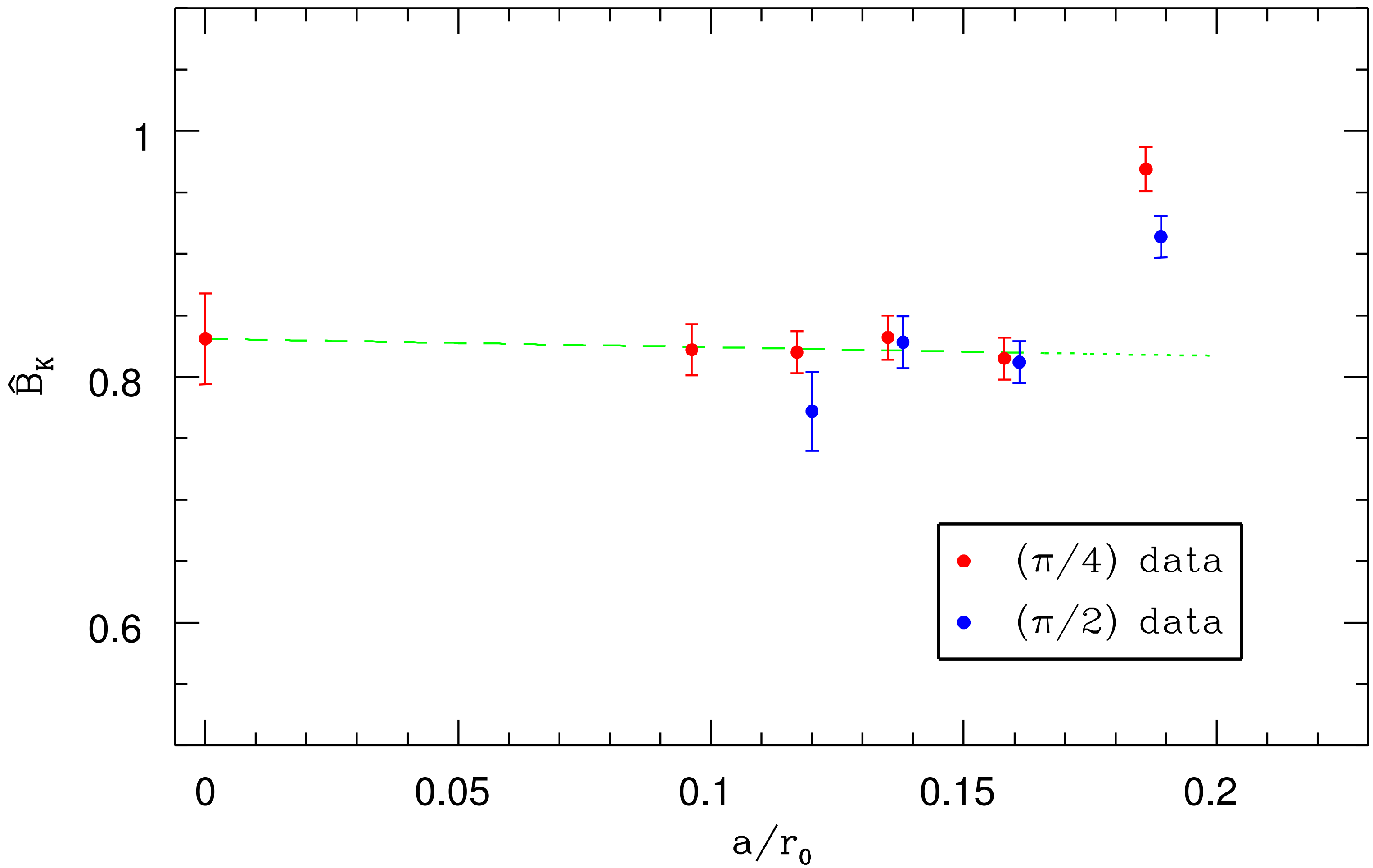


1
0.9
0.8
0.7
2
3
4
$B_K(\beta=6.45)$
$(r_0\ M_{PS})^2$

Continuum extrapolation of B_K ($\pi/4$ data):



(Preliminary) result: $\hat{B}_K = 0.834(37) \quad \Leftrightarrow \quad B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.604(27)$

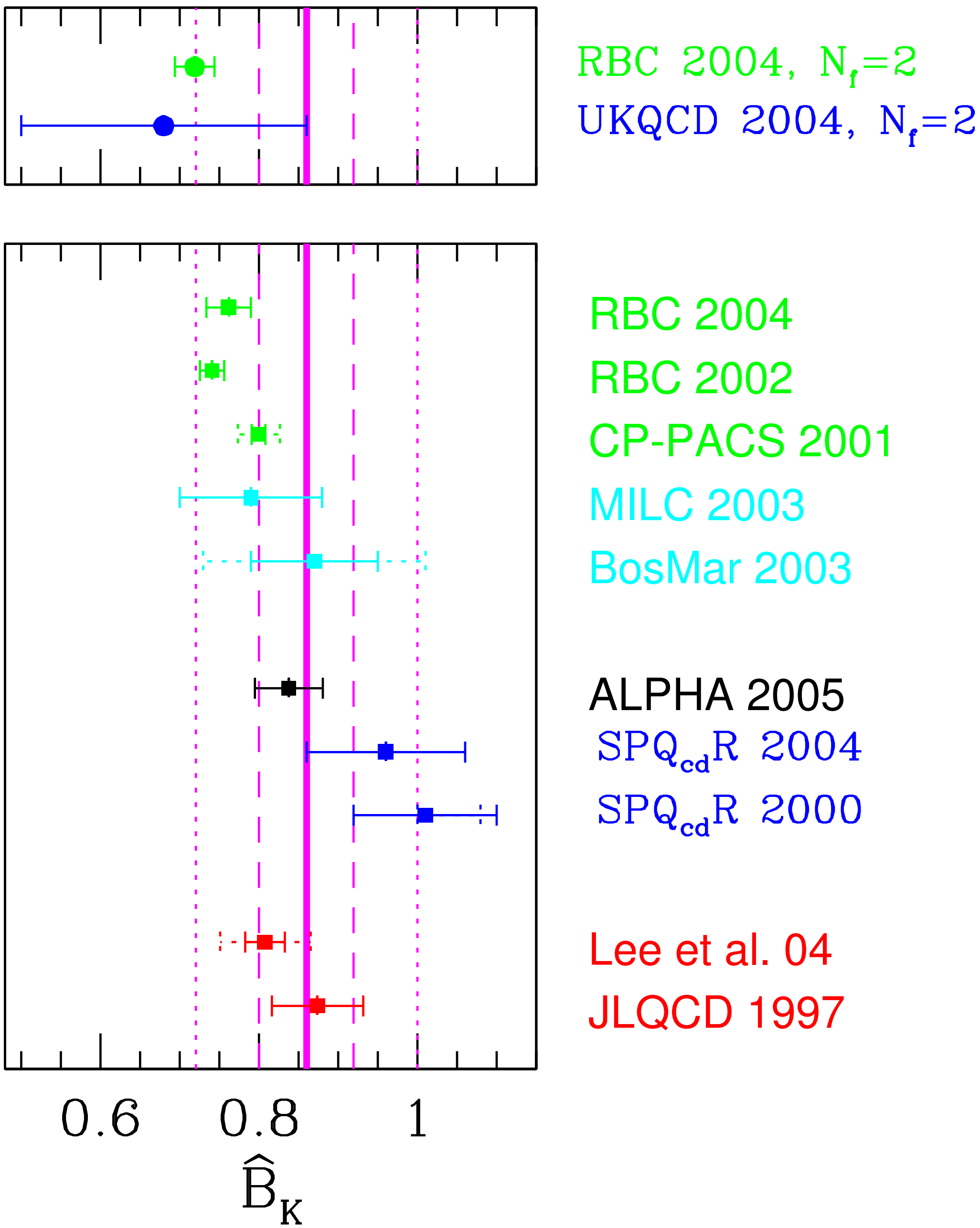


Conclusions

- We have performed a benchmark calculation of B_K in quenched model for QCD with controlled systematic errors:
  - finite size effects
  - excited states contributions (more complicated in tmQCD)
  - chiral interpolations/(small) extrapolations
  - non-perturbative renormalization
  - continuum extrapolation using 2 alternative formulations
  - SU(3) breaking: no dependence up to $(M_s - M_d)/(M_s + M_d) \approx 0.5$
- the (preliminary) total error meets the requirement (< 5 percent)
- compatible with results using other lattice regularisations (usually less controlled)
- Further progress requires dynamical quarks;

  good prospects for Wilson type quarks from algorithmic improvements

  (fall of the "Berlin wall")



(compiled by H. Wittig)