Critical Behaviour in QCD

Helmut Satz

Universität Bielefeld, Germany
and
Instituto Superior Técnico, Lisboa, Portugal

QCD and Hadronic Physics **Beijing, June 2005**

Contents

1. The Physics of Complex Systems

- **Given constituents and dynamics of elementary systems, what is the behaviour of complex systems?**
- **What are the possible states of matter and how can they be specified?**
- **How do transitions from one state of matter to another occur?**

1.1 Critical Behaviour in Thermodynamics

basic feature of critical phenomena:

discontinuous or singular behaviour of physical observables

to summarize main features, recall Ising model

d**-dimensional lattice grid,** N^d **sites with spins** $s_i = \pm 1\ \forall i = 1, ..., N^d$**,**

uniform next neighbor interaction $-Js_i s_{i+1}$

properties of the system determined by partition function

$$Z(T, H=0, N) = \prod_{i=1}^{N^d} \sum_{s_i=\pm 1} \exp\{\, \beta J \sum_{i,j}^{nn} s_i s_j - \beta H \sum_i s_i\}$$

temperature $T = \beta^{-1}$, external field H; take $H = 0$

$Z(T, H=0; N)$ **has global symmetry (Z_2):**

$$s_i \longrightarrow -s_i \quad \forall\ i = 1, ..., N^d$$

leaves sum over all states $Z(T, H=0; N)$ invariant

for high temperatures, system agrees:
on the average, $\exists$ disorder, as many spins $\uparrow$ as $\downarrow$

but below a certain temperature:
$\exists$ order $\Rightarrow$ spontaneous symmetry breaking

on the average, more $\uparrow$ or more $\downarrow$

additional measure to specify state of system: order parameter

magnetization

$$m(T,N) = \frac{1}{Z(T,N)} \prod_{i=1}^{N^d} \sum_i \left[\frac{\sum_i s_i}{N^d}\right] \exp\{\,\beta J \sum_{i,j}^{nn} s_i s_j\}$$

under reflection $s_i \to -s_i \; \forall \; i = 1, ..., N^d$**:** $m(T,N) \to -m(T,N)$

order parameter is not invariant

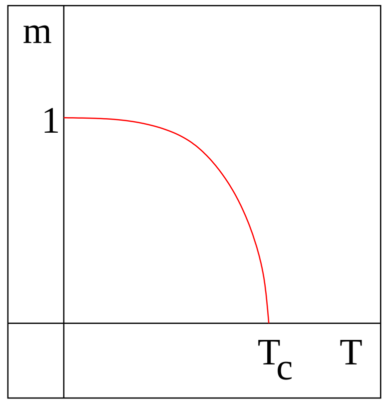
$$m(T) \begin{cases} \neq 0 & \text{for ordered state, broken symmetry} \\ = 0 & \text{for disordered state, symmetry} \end{cases}$$

thermodynamic limit $N \to \infty$**:**

$m(T, H=0)$ **is not analytic**

$$m(T) \sim \begin{cases} (T - T_c)^\beta > 0 & \forall \; T < T_c \\ 0 & \forall \; T > T_c \end{cases}$$

$\Rightarrow$ **critical exponent** β



also other observables (specific heat, susceptibility,...) diverge, leading to set of critical exponents $\alpha, \beta, \gamma, ...$

besides global also local observables diverge:

correlation function $\Gamma(r,t) \sim \langle s_i s_{i+r} \rangle \sim \exp -r/\xi$

correlation length diverges: $\xi \sim |T - T_c|^{-\nu}$

$t \neq 0$**: correlation length finite, dimensional scale, given spin does not see far-away other spins**

$t = 0$**: correlation length diverges, no scale, all spins are correlated, the system cannot be split into independent subsystems**

But: why is there singular behaviour?

transition $\sim$ **onset of spontaneous symmetry breaking: "either-or", nothing gradual or smooth; you cannot break symmetry "a little".**

infinite correlated system requires new physics:

$\Rightarrow$ **scaling and renormalization** (Kadanoff, Wilson)

rescale distances, temperature, external field

$$r \longrightarrow r' = br, \quad t \longrightarrow t' = b^{y_t} t, \quad h \longrightarrow h' = b^{y_h} h$$

all physics must remain the same

$\Rightarrow$ **all critical exponents given in terms of** y_t, y_h

$\Rightarrow$ **critical behaviour fully specified by** y_t, y_h

$\Rightarrow$ y_t, y_h **define universality class**

$\Rightarrow$ **Thermodynamic Critical Behaviour*** $\Leftarrow$

- **onset of spontaneous symmetry breaking**
- **singular behaviour of thermodynamic observables**
- **critical exponents, universality class**

---

* continous transitions

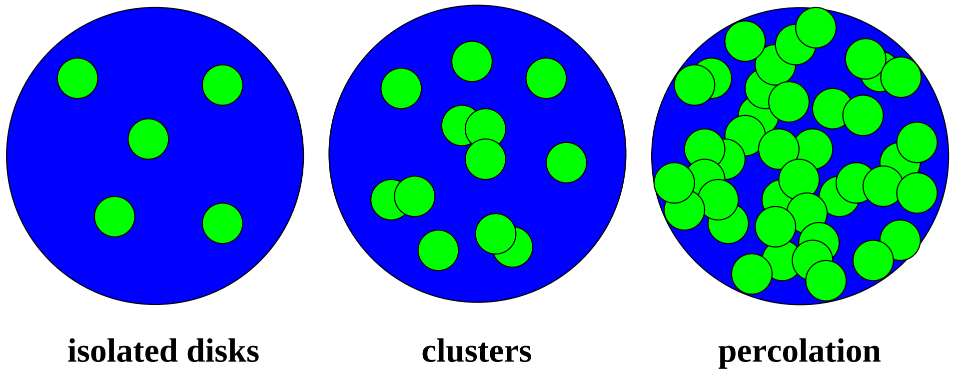
1.2 Cluster Formation and Percolation

thermal transitions, critical behaviour: dynamics ($\mathcal{H}$), symmetry

for constituents with intrinsic scale,
$\exists$ simpler, geometric form of critical behaviour:

$\Rightarrow$ formation of infinite connected clusters or networks

example: 2-d disk percolation (lilies on a pond)



distribute small disks of area $a = \pi r^2$ randomly on large area $A = \pi R^2$, $R \gg r$, with overlap allowed: when can an ant walk across?

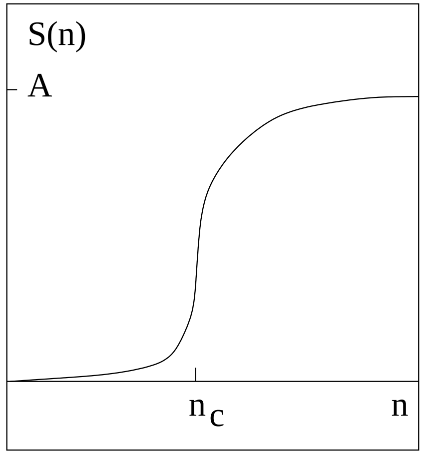
for N disks, disk density $n = N/F$

average cluster size $S(n)$ increases
with increasing density n

suddenly, for $n \to n_c$, $S(n)$ becomes
large enough to span the pond: $S \sim F$

for $N \to \infty$, $A \to \infty$:

$S(n_c)$ and $(dS(n)/dn)_{n=n_c}$ diverge: $\Rightarrow$ percolation



percolation is geometric critical behaviour

probability $P(n)$ that a given disk is in the infinite cluster

$$P(n) \begin{cases} = 0 & \forall\ n < n_c \\ \\ \sim (n - n_c)^{\beta} & \text{for } n \to n_c \text{ from above} \end{cases}$$

$\Rightarrow$ order parameter for percolation

... other observables (cluster size,...) also diverge

again singular behaviour $\rightarrow$ critical exponents, universality classes

instead of symmetry breaking: onset of connectivity

disconnected $\rightarrow$ connected system

$\Rightarrow$ Geometric Critical Behaviour $\Leftarrow$

- **onset of infinite cluster formation**
- **singular behaviour of geometric observables**
- **critical exponents, universality class**

Why singular behaviour?

onset of connection is "either-or", you cannot connect "a little".

2. Critical Behaviour in Statistical QCD

2.1 Phases of Strongly Interacting Matter

What happens to strongly interacting matter at high temperature and/or density?

- **quark liberation**

  hadronic matter: colorless constituents of hadronic dimension

  $$\Downarrow$$

  quark-gluon plasma: pointlike colored constituents

  $\Rightarrow$ **deconfinement: insulator-conductor transition in QCD**

- **quark mass shift**

  at $T = 0$, quarks 'dress' with gluons $\rightarrow$ constituent quarks

  bare quark mass $m_q \sim 0$ $\rightarrow$ constituent quark mass $M_q \sim 300$ MeV

  in hot medium, dressing 'melts' $M_q \rightarrow 0$

for $m_q = 0$, $\mathcal{L}_{QCD}$ has chiral symmetry

$M_q \neq 0 \rightarrow$ spontaneous chiral symmetry breaking

$M_q \rightarrow 0 \Rightarrow$ chiral symmetry restoration

- diquark matter

deconfined quarks $\sim$ attractive interaction

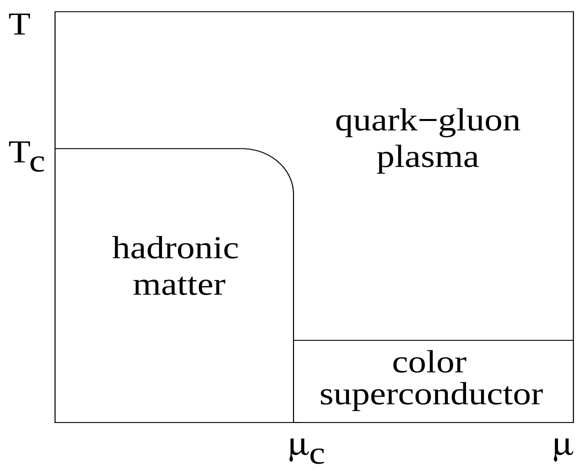
can form colored bosonic 'diquark' pairs (QCD's Cooper pairs)

form condensate $\Rightarrow$ color superconductor

- expected phase diagram of QCD:

baryochemical potential

$\mu \sim$ baryon density



2.3 Finite Temperature Lattice QCD

given QCD as dynamics input, calculate resulting thermodynamics, based on QCD partition function

$\Rightarrow$ lattice regularization

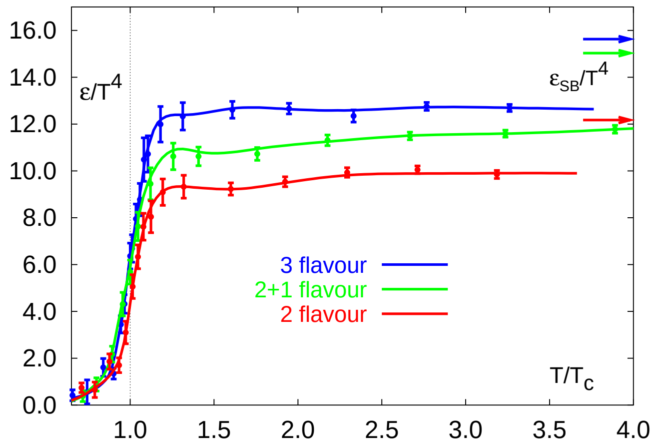
- energy density

  $\Rightarrow$ latent heat of deconfinement

  For $N_f = 2, 2+1$:

  $T_c \simeq$ 175 MeV

  $\epsilon(T_c) \simeq 0.5 - 1.0$ GeV/fm^3



explicit relation to deconfinement, chiral symmetry restoration?

$\Rightarrow$ order parameters

- **<u>deconfinement</u>** $\Rightarrow m_q \to \infty$

  Polyakov loop $L(T) \sim \exp\{-F_{Q\bar{Q}}/T\}$

  $F_{Q\bar{Q}}$: **free energy of** $Q\bar{Q}$ **pair for** $r \to \infty$

$$L(T) \begin{cases} = 0 & T < T_L \text{ confinement} \\ \\ \neq 0 & T > T_L \text{ deconfinement} \end{cases}$$

  variation defines <u>deconfinement temperature</u> T_L

- **<u>chiral symmetry restoration</u>** $\Rightarrow m_q \to 0$

  chiral condensate $\chi(T) \equiv \langle \bar{\psi}\psi \rangle \sim M_q$

  measures dynamically generated ('constituent') quark mass

$$\chi(T) \begin{cases} \neq 0 & T < T_\chi \text{ chiral symmetry broken} \\ \\ = 0 & T > T_\chi \text{ chiral symmetry restored} \end{cases}$$

  variation defines <u>chiral symmetry temperature</u> T_χ

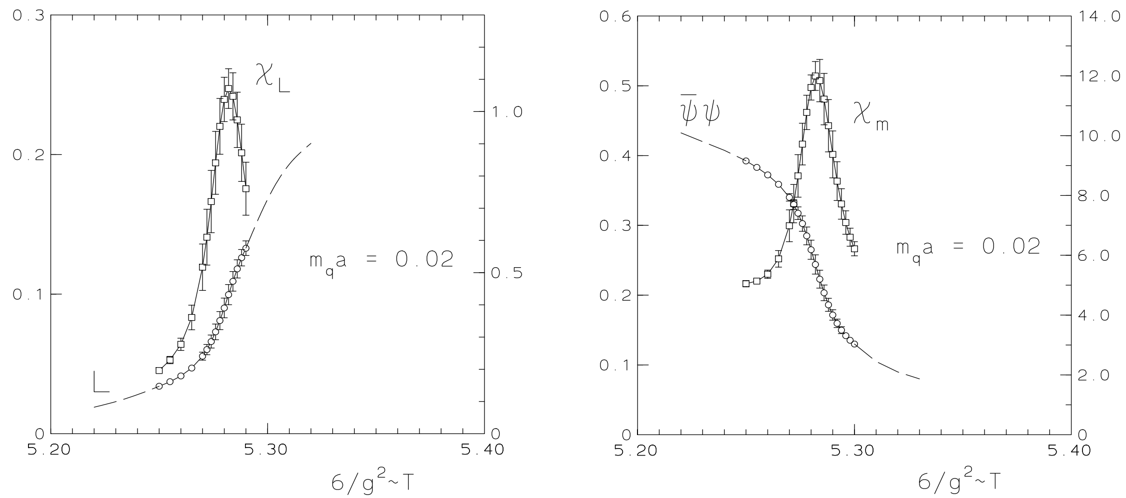
- **how are T_L and T_χ related?**

  pure $SU(N)$ gauge theory: $\sim$ spontaneous Z_N breaking at T_L

  full QCD, chiral limit: $\sim$ explicit Z_N breaking by $\chi(T) \to 0$ at T_χ

  chiral symmetry restoration $\Rightarrow$ deconfinement

  lattice results



Polyakov loop & chiral condensate vs. temperature

at $\mu = 0$, $\exists$ one transition hadronic matter $\to$ QGP

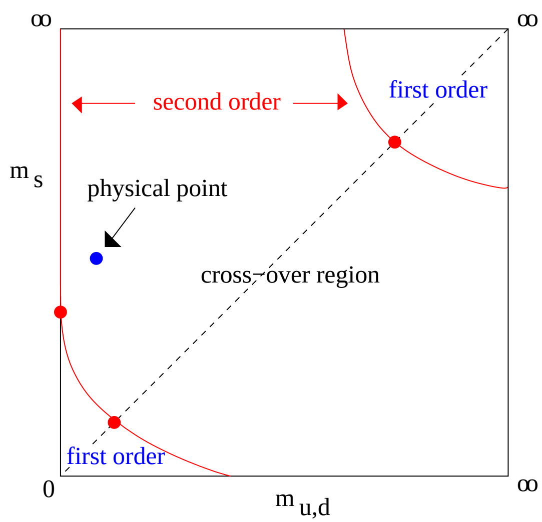
for $N_f = 2, m_q \to 0$ at $T_c = T_L = T_\chi \simeq 175$ MeV

- **nature of transition** at $\mu = 0$
  - for $m_q \to \infty$ (pure gauge theory) spontaneous Z_N breaking $\to$ **deconfinement transition**
  - for $m_q \to 0$, spontaneous chiral symmetry breaking $\to$ **chiral transition**
  - for finite quark masses, no spontaneous symmetry breaking or restoration, hence in general no singular behaviour
  - both $L(T)$ and $\chi(T)$ vary sharply for all m_q, define common transition point T_c
  - what kind of transition?

depends on N_f and m_q:

continuous, first order

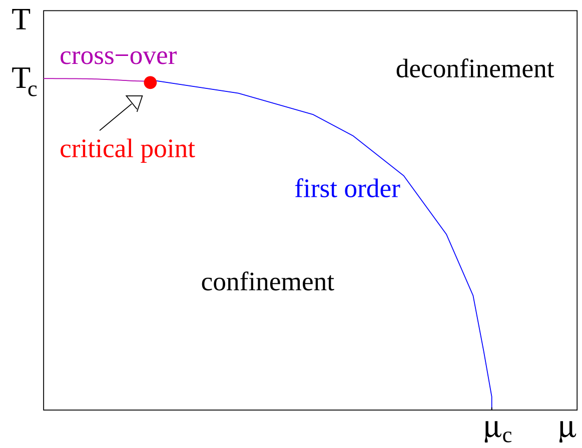
"rapid" cross-over



- non-zero net baryon density

  $(\mu \neq 0, N_b > N_{\bar{b}},\ N_f = 2+1)$

  computer algorithms break down: reweighting, analytic continuation, power series...; expect:



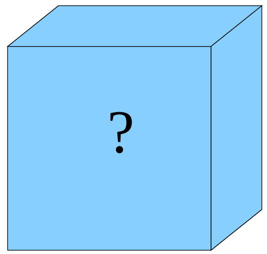
  critical point in $T-\mu$ plane depends on position of physical point in $m_s - m_{u,d}$ plane

- cross-over region (the real world): enigmatic
  - no thermal singularity, no thermal phase transition
  - so what does it mean: new state of matter?
  - observables change rapidly, define clear transition
  - what transition mechanism? percolation?

3. Probing Matter in Statistical QCD

given a box of strongly interacting matter in thermal equilibrium,
how can *theorists* determine its state through QCD calculations?

NB:
equilibrium thermodynamics, no collision dynamics, time dependence, equilibration, expansion, cooling, etc.



3.1 Interaction Range and Colour Screening

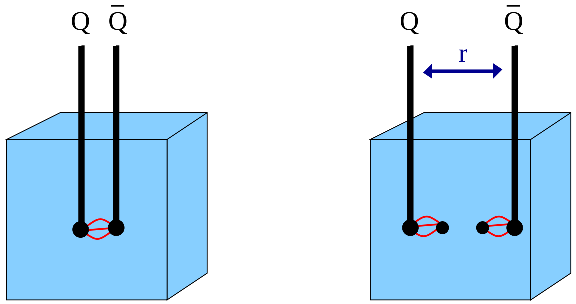
static quark/antiquark in medium: interaction vs. separation?

at $T = 0$, confining "string" potential

$$V(r) \sim \sigma\, r$$

string breaks for $V(r) \geq 2M_q$

$\Rightarrow$ two light-heavy mesons $(Q\bar{q}),\ (\bar{Q}q)$

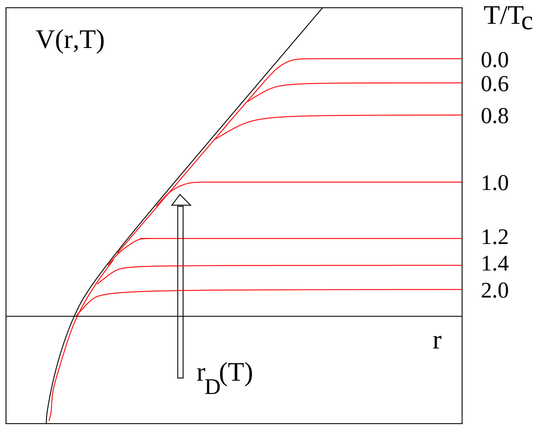


with increasing temperature,

potential strength and
range reduced (from LL^+ correlations)

string breaks earlier

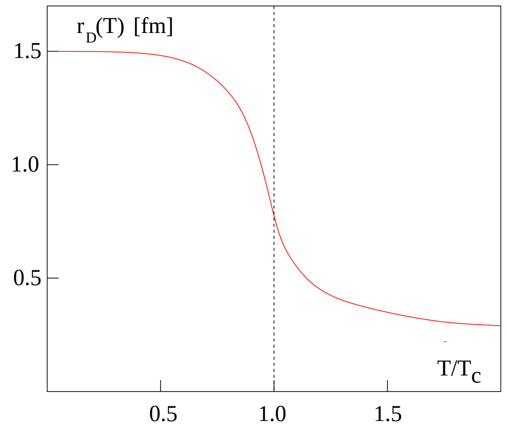
$\Rightarrow$ **colour screening**



(Bielefeld, $16^3 \times 4$**,** $N_f = 2$**,** $m_q/T = 0.4$**)**

screening radius $\sim$ **interaction range**
drop sharply as $T \to T_c$

string breaking point falls
from $r \simeq 1.5$ **fm to** $r \simeq 0.3$ **fm**
for $T/T_c = 0$ **to** $T/T_c = 2$



3.2 Light Hadron Spectroscopy

look at mass spectrum of virtual photons emitted from box

$$\gamma^* \longrightarrow e^+e^-$$

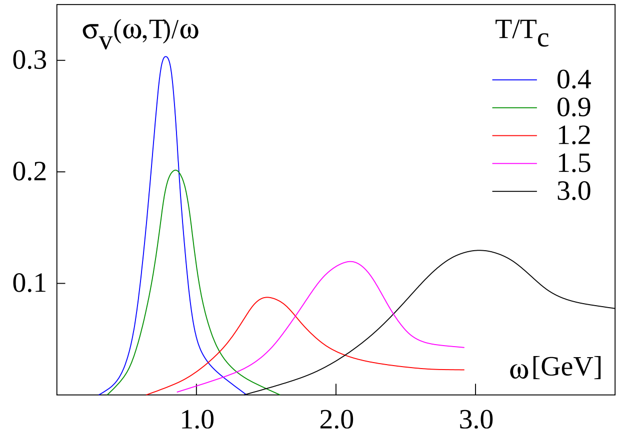
expect:

in hadronic phase $\rho \longrightarrow \gamma^* \longrightarrow e^+e^-$

so that $M(\gamma^*) \simeq M(\rho)$

in QGP phase $q\bar{q} \longrightarrow \gamma^* \longrightarrow e^+e^-$

so that $M(\gamma^*) \sim T$



lattice calculations:

(Bielefeld, quenched QCD, $64^3 \times 16$**)**

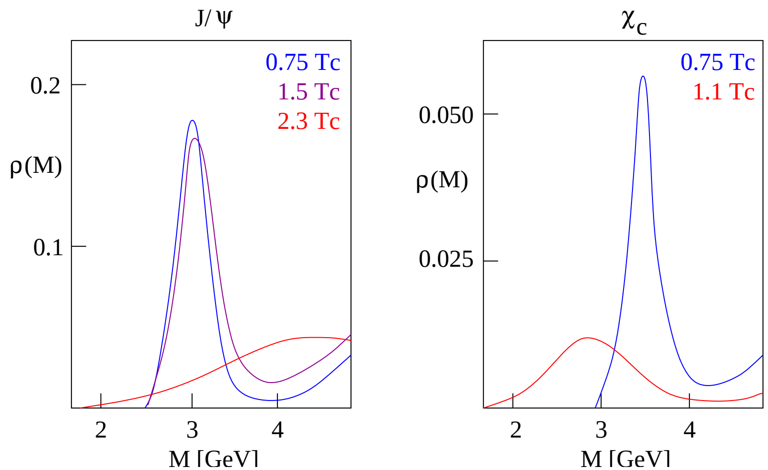
confined state: hadronic scale, peak at ρ **mass**
position $\sim$ **temperature-independent**

deconfined state: temperature scale, broad peak at position $\sim T$

3.2 Charmonium Spectroscopy

existence of heavy quark-antiquark bound states (J/ψ, χ_c, ψ',...) as indicator of nature and temperature of medium

lattice calculations for
spectral functions of $c\bar{c}$ systems
(**Bielefeld, quenched QCD,** $48^3 \times 10 - 24$)



J/ψ **persists up to** $2.3\ T_c > T \geq 1.5\ T_c$

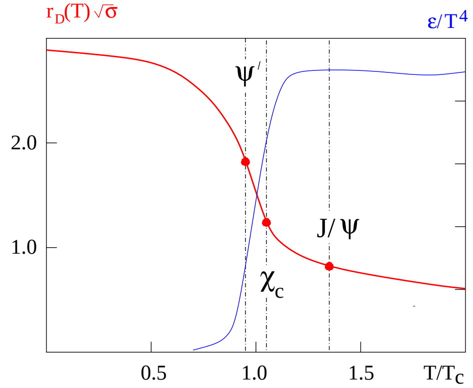
χ_c **is dissociated for** $T \geq 1.1\ T_c$

widths?

cross check:

compare to interaction range,
potential models (Schrödinger equ'n)

χ_c **and** ψ' **analyze deconfinement transition**

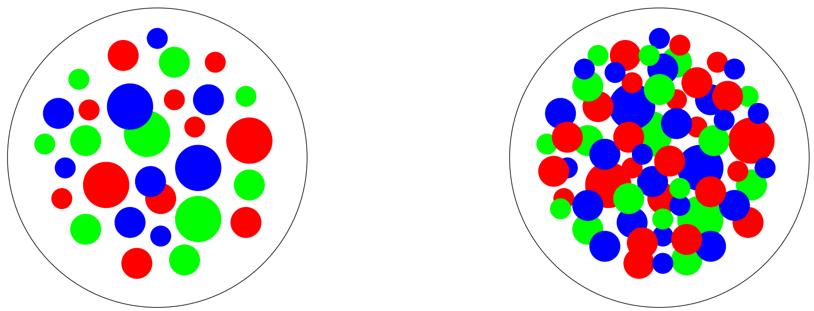


4. Parton Percolation and Saturation

Critical behaviour in initial state of hadronic/nuclear collisions

Pre-equilibrium aspects, no thermalisation ⇒ geometric features

Partons in transverse plane of incoming hadron or nucleus:



increasing $\sqrt{s}$ or A → higher density → clustering → ?

- **transverse size of hadron/nucleus $F = \pi R^2$, radius $R = A^{2/3} R_0$**
- **average transverse size of partons $\alpha_s \pi r^2$, $r \sim Q^{-1}$, intrinsic parton transverse momentum**

- **number of partons in hadron/nucleus** $N = A\ dN_p/dy$
- **deep inelastic scattering: as** $x \sim \sqrt{s}/Q$ **decreases, the density of partons increases,** $dN_p/dy \sim 1/x^{\delta}$ $\Rightarrow$ **"small x problem"**

What happens when combined partonic area $\gg F$ **?**

Percolation condition $$n = \frac{N}{F} = \frac{1.13}{\pi r^2}$$

becomes $$\frac{\alpha_s A}{R^2 Q^2} \frac{dN_p}{dy} = 1.13$$

determines onset of parton connectivity as function of $\sqrt{s},\ Q,\ A$

Beyond percolation point, partons for connected interacting network, cease to have individual existence $\Rightarrow$ **parton saturation**

Geometric transition from a system of individual partons to a new medium: parton network or colour glass condensate

Colour connectivity $\sim$ **prerequisite for subsequent thermalisation**

Critical Behaviour in QCD

In strong interaction thermodynamics

∃ a well-defined transition at which

- **deconfinement sets in, chiral symmetry is restored, latent heat increases energy density**
- **colour screening decreases interaction range**
- **dilepton spectra go from hadron decay to thermal annihilation**
- **charmonium dissociation analyses transition region**

In initial state of hadronic/nuclear collisions,

centerline∃ a well-defined geometric transition at which

- **initial state partons ⇒ connected parton network ⇒ saturation**