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Chiral effective action of QCD: precision tests, questions and electroweak extensions

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* SU(2) chiral expansion
  - $\pi\pi$ amplitude
* SU(3) chiral expansion
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  - π K amplitude
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Introduction

* Everyone's starting point

$$\mathcal{L}_{QCD} = \frac{-1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{f=1}^{6} \overline{q}_f [i \not{D} - m_f] q_f$$

$$\text{with} \quad \not{D} = (\partial_\lambda - i g_s G_\lambda)\gamma^\lambda$$

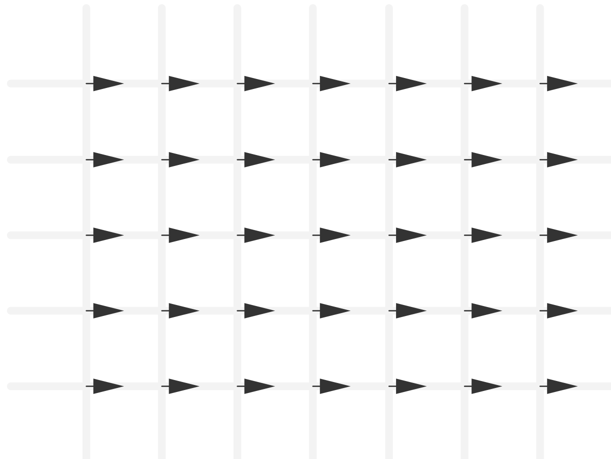
* One coupling constant g_s, one scale Λ_{QCD}

* Two remarkably different regimes (soft/hard)

* Very soft regime: chiral effective Lagrangian
  - Analytic approach ; precision physics ?
  - Light hadrons probe weak interaction: V_{ud}, V_{us}

Chiral symmetry and light quarks

* **Chiral limits:** set $N_F^0 (\geq 2)$ masses $= 0$
  – $\mathcal{L}_{QCD}$ has a global symmetry $SU_L(N_F^0) \times SU_R(N_F^0)$
* QCD vacuum breaks the symmetry to $SU_V(N_F^0)$ (like a ferromagnet)



* Order parameter $\langle \bar{u}_L u_R + \bar{u}_R u_L \rangle \neq 0$
* **Goldstone theorem:** $(N_F^0)^2 - 1$ massless pseudoscalar bosons
* Three quarks are light $u,\ d,\ s$, masses ($\mu_0 = 2$ GeV):

$$\frac{1}{2}(m_u + m_d)_{latt} = 4 \pm 1 \text{ MeV} \quad m_s|_{latt} = 100 \pm 25 \text{ MeV}$$
$$m_u/m_d|_{chpt} = 0.54 \pm 0.04$$

* **Two** relevant chiral limits: $N_F^0 = 2$, $N_F^0 = 3$

Chiral effective Lagrangian

∗ Dynamical field U, $UU^\dagger = 1$ (nonlinear sigma-model) $U = \exp(i\Phi/F)$

$$\mathcal{L}_2^{eff} = \frac{1}{4}F^2\langle D_\mu U D^\mu U^\dagger + 2B(\mathcal{M}^\dagger U + \mathcal{M}U^\dagger)\rangle$$

∗ Two coupling constants ($\langle \bar{u}u\rangle = BF^2$).

∗ Weakly interacting theory: expansion in powers of p, m_f

∗ Higher orders: Weinberg (1979)
- $O(p^4)$: L_1,...L_{10} (Gasser, Leutwyler (1985))
- $O(p^6)$: C_1,...,C_{90} (Bijnens, Colangelo, Ecker (1999))

∗ "Generalized" renormalizability, couplings encode dynamical informations on massive states.

SU(2) chiral expansion

$*$ $\pi\pi$ amplitude: three successive orders computed

$*$ $O(p^2)$ (Weinberg (1966))

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}, \quad a_0^2 = \frac{-M_\pi^2}{16\pi F_\pi^2}, \quad M_\pi^2 = 2\hat{m}B$$

$*$ $O(p^4)$ (Gasser-Leutwyler (1984)) depends on four coupling combinations $\bar{l}_i$ (chiral logs: $M_\pi^4 \log M_\pi^2$)

$*$ $O(p^6)$

– (Knecht et al. (1995)) using unitarity, analyticity, crossing and chiral counting: up to six parameter polynomial

– (Bijnens et al. (1996)) complete calculation from chiral Lagrangian

$\pi\pi$ amplitude: numerical results

* Theory (note: no isospin breaking)

order	a_0^0	$a_0^0 - a_0^2$
p^2	0.16	0.20
p^4	0.20 ± 0.01	0.24 ± 0.02
p^6 Bijnens et al (1998)	0.217	0.258
p^6 Colangelo et al (2000)	0.220 ± 0.005	0.264 ± 0.011

* Experiment

experiment	a_0^0	$a_0^0 - a_0^2$
Rosselet (1977)	0.27 ± 0.05	
E865 (2001)	$0.228 \pm 0.012 \pm 0.003$	
E865 (2003)	$0.216 \pm 0.013 \pm 0.003$	
DIRAC (2005) pionium		0.261 ± 0.033
NA48 (2005) $K \to 3\pi$		$0.281 \pm 0.007 \pm 0.014$

* SU(2) expansion works (main corrections from chiral logs)

$SU(3)$ chiral expansion

$*$ $N_F^0 = 3$. Obviously $m_s >> m_u,\ m_d$

$*$ Less obvious: OZI rule

- i.e. $s\bar{s} \not\to u\bar{u} + d\bar{d}$
- well satisfied for mesons (nonet structure, decay patterns)
- One exception: scalar mesons
- Explanation from QCD: $1/N_c$ expansion ?

$*$ Why is it interesting here ? It affects order parameters

OZI rule $\Rightarrow$

$$\langle \bar{u}u \rangle_{SU(2)} = \langle \bar{u}u \rangle_{SU(3)}$$
$$(F_\pi)_{SU(3)} = (F_\pi)_{SU(2)}$$

$*$ Is this true ?

Banks-Casher formula

* Relation with the density of small eigenvalues of the Dirac operator

$$\langle \bar{u}u \rangle_{SU(2)} = \frac{-\pi}{Z} \int d\mu[G] \rho_G(0) \, (\det i \not{D})^2 \det(i \not{D} + m_s) \mathrm{e}^{-S_{YM}(G)}$$

$$\text{with } \rho_G(\lambda) = \sum_n \delta(\lambda - \lambda_n) \qquad \not{D} \psi_n = \lambda_n \psi_n$$

* Since $\det i \not{D} = \Pi_n \lambda_n^2$ we expect

$$\langle \bar{u}u \rangle_{SU(3)} < \langle \bar{u}u \rangle_{SU(2)}$$

* Similar effect for $(F_\pi)_{SU(3)}$. CS breaking is weaker for $SU(3)$

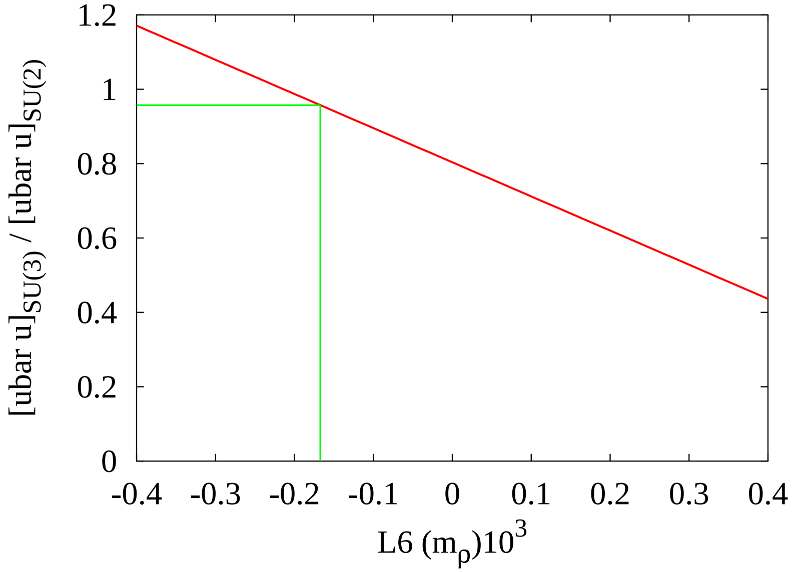
* $O(p^4)$ Lagrangian, OZI suppressed couplings

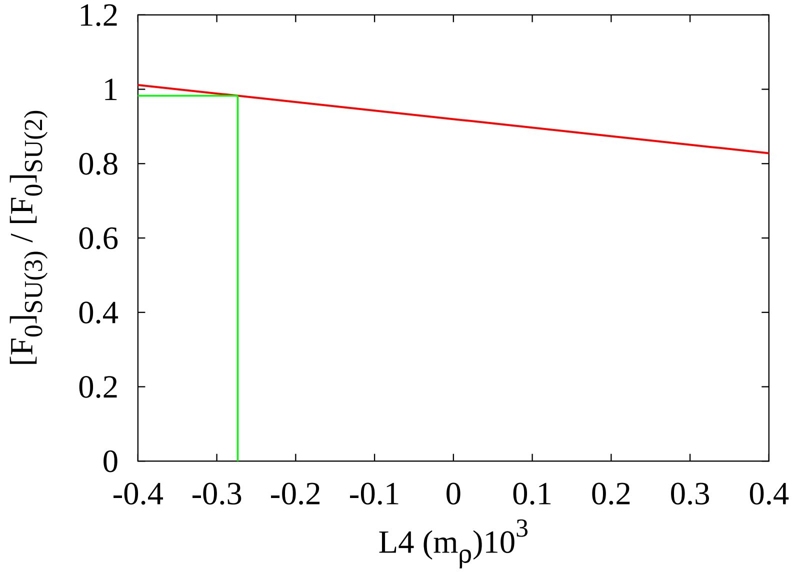
$$L_2 - 2L_1, \quad L_4, \quad L_6$$

* Relation with order parameters, $x_K = m_K^2 - \frac{1}{2}m_\pi^2$

$$\frac{\langle \bar{u}u \rangle_{SU(3)}}{\langle \bar{u}u \rangle_{SU(2)}} = 1 - \frac{x_K}{F^2}\left[32L_6^r - \frac{1}{144\pi^2}\left(11\log\frac{x_K}{\mu^2} + 2\log\frac{4}{3}\right)\right]$$

$$\frac{(F_\pi)_{SU(2)}}{(F_\pi)_{SU(3)}} = 1 - \frac{x_K}{F^2}\left[8L_4^r - \frac{1}{32\pi^2}\log\frac{x_K}{\mu^2}\right] \qquad (1)$$





Determination of OZI suppressed couplings

$*$ Model independent determination:

$L_2 - 2L_1$: $Kl_4, \pi K$

L_4 : πK,

L_6 : no (Kaplan-Manohar invariance)

$*$ Model dependent determination: L_4, L_6 from scalar form factors

$$\langle \pi^i(p)|\bar{u}u(0) + \bar{d}d(0)|\pi^j(q)\rangle = \delta^{ij} F_u^\pi(t). \quad t = (p-q)^2$$

$$\langle \pi^i(p)|\bar{s}s|\pi^j(q)\rangle = \delta^{ij} F_s^\pi(t)$$

consider also

$$\Pi_6(p^2) = i \int d^4x e^{ipx} \langle 0|\bar{s}s(x)[\bar{u}u(0) + \bar{d}d(0)]|0\rangle_{conn.}$$

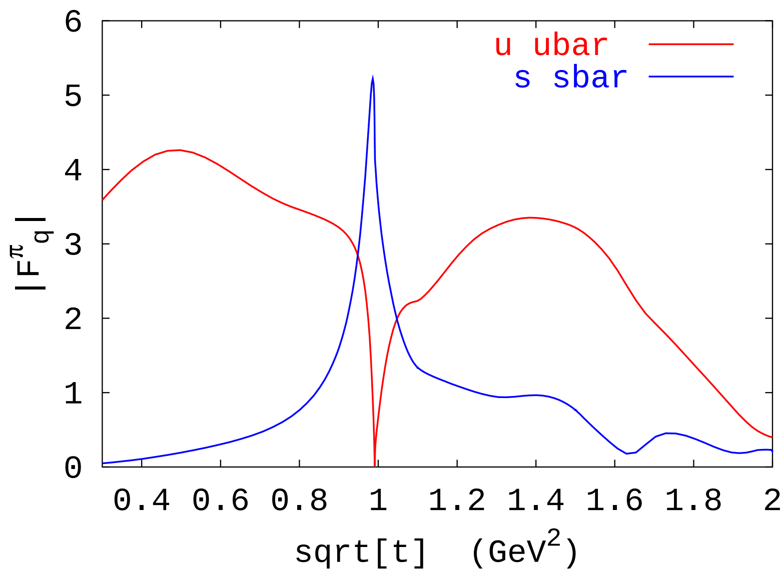
("OZImeter")

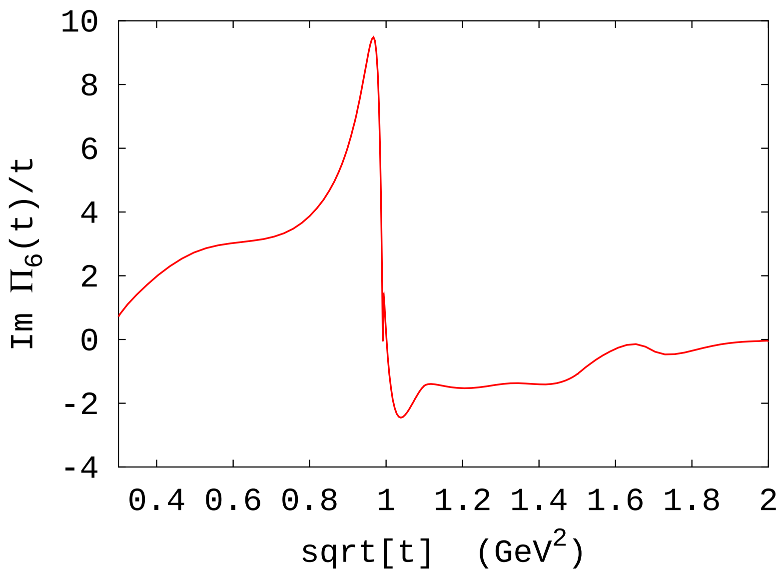
Construction of scalar form-factors

∗ Method: Donoghue, Gasser, Leutwyler (1990):

– Analyticity, Muskhelishvili-Omnès equations based on 2-channel $\pi\pi$ and $K\bar{K}$ unitarity

– $F_q^\pi(0)$ given from CHPT $\longrightarrow$ $F_q^\pi(t)$ $t \leq 1$ GeV





∗ Relation with scalar meson physics

Results from scalar form-factors

* relation with couplings

$$L_4 = \frac{1}{8}\sqrt{\frac{2}{3}}\dot{F}_s^\pi(0) + log's + O(p^6)$$
$$L_6 = \frac{1}{64}\Pi_6(0) + log's + O(p^6)$$

* Chiral computations to p^6: Π_6 (BM (2000)), form-factors (Bijnens, Dhonte (2003))

	$10^3\, L_4^r(m_\rho)$	$10^3\, L_6^r(m_\rho)$	method
GL (85)	-0.3 ± 0.3	-0.2 ± 0.5	OZI
BM (00)	$+0.3 \pm 0.2$	$+0.3 \pm 0.3$	$F_s^\pi,\ \Pi_6$
BD (03)	$+0.4 \pm 0.2$	$+0.1 \pm 0.3$	$F_s^\pi,\ F_u^\pi,\ F_s^K,\ F_u^K$

πK amplitude

* Experimental informations:

* Medium energy: production experiments

  - $K^{\pm}p \rightarrow K^{\pm}\pi^{+}n$, Estabrooks et al (1978), Aston et al. (1988) obtain $\pi K \rightarrow \pi K$ PW amplitudes $l = 0...5$ in range $0.8\ GeV < E < 2.5\ GeV$

  - $\pi N \rightarrow K\overline{K}N$, Cohen et al. (1980), Etkin et al. (1982) obtain $\pi\pi \rightarrow K\overline{K}$ in range $2m_K < E < 2.5$ GeV

* Low energy:

  - $D^{+} \rightarrow \pi^{+}K^{-}\mu\nu$ FOCUS (2002) $l = 0\ \pi K\ E = 895$ MeV, $\tau \rightarrow \pi K\nu$, $l = 1\ (\pi K)_{atom}$ in the future.

* Low-energy amplitude can be constructed from analyticity.

πK amplitude at low energy

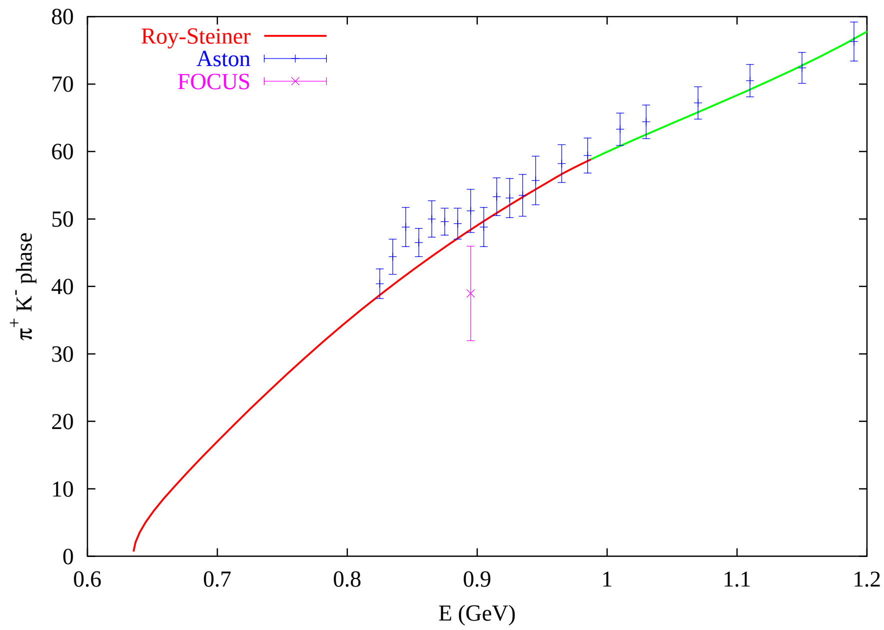
$*$ Roy-Steiner equations : $\pi K \to \pi K$ $f_0^{1/2},\ f_0^{3/2},\ f_1^{1/2},\ f_1^{3/2}$

$\pi\pi \to K\overline{K}$ $g_0,\ g_1$

$*$ Two (subtraction) parameters: $a_0^{1/2}, a_0^{3/2}$

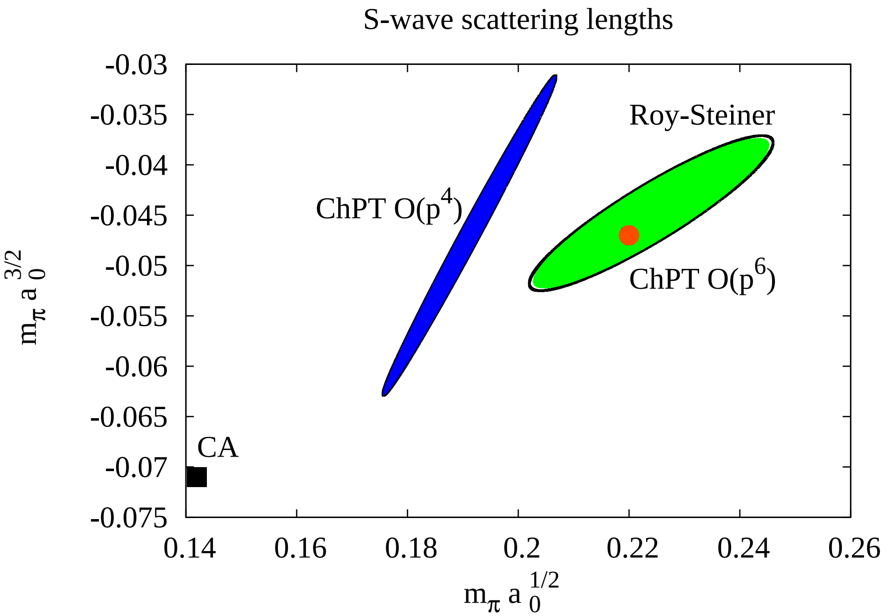
$*$ Boundary value problem approach (Pennington-Protopopescu (1974)) unique solution with proper boundary conditions.

$*$ New solutions: Descotes-Genon et al. (2004)



Comparison with SU(3) expansion at p^4

* Chiral calculation Bernard, Kaiser, Meissner (1991)



* Expand the amplitudes around $s - u = 0$, $t = 0$ (unphysical point)

L_2	L_3	$L_2 - 2L_1$	L_4	$\times 10^3$
1.3 ± 0.1	-4.5 ± 0.1	-0.8 ± 0.2	0.5 ± 0.4	πK
1.5 ± 0.2	-3.2 ± 0.8	$+0.6 \pm 0.5$	$-$	Kl_4

Comparison with SU(3) expansion at p^6

$*$ Calculation by Bijnens, Dhonte (2004)

$*$ Strategy: simple resonance model for C_i while L_i's fitted in previous work (masses, πl_2, $K l_2$, $K l_4$ etc...)

$*$ I=0 scattering lengths: good

$*$ Comparison with sub-threshold expansion parameters not so good

sub. coeff.	c_{10}^{+}	10 c_{20}^{+}	c_{01}^{+}	c_{10}^{-}
p^4	0.6	0.2	1.7	0.2
p^6	0.9	0.03	3.8	0.1
dispersive	0.9 ± 0.1	0.2 ± 0.1	2.1 ± 0.1	0.31 ± 0.01

$*$ More refined estimates of C_i's needed

Electroweak extensions

∗ Aim: Accurate evaluation of EM corrections and isospin breaking e.g. to determine $m_u - m_d$ (Dashen's theorem) or V_{ud}, V_{us} etc...

∗ Extension to dynamical photons Urech (1995)
∗ Extension to dynamical leptons Knecht et al. (2000)

∗ Classification method: $Q \longrightarrow$ 2 spurions $q_L(x)$, $q_R(x)$, chiral counting $Q(e) \sim O(p)$

– $O(e^2)$ One coupling C
$\mathcal{L}_{e^2} = C\langle q_R U q_L U^\dagger \rangle$

– $O(e^2p^2)$ $K_1,\ldots,K_{13}$
$X_1,\ldots,X_6$ with leptons (charged current)

∗ This is the effective theory of the full standard model

Chiral sum rules

∗ Das et al. (1967) [$\pi^+ - \pi^0$ mass difference]

$$2C = i\int d^4x < 0|T\left(V^3_\mu(x)V^3_\nu(0) - A^3_\mu(x)A^3_\nu(0)\right)|0 > D^{\mu\nu}_\gamma(x)$$

∗ BM (1997) trick: use spurions as sources

$\langle V^3V^3 - A^3A^3\rangle$	C, K_{13}
$\langle V^{ud}V^{du} - V^{us}V^{su}\rangle$	$K_9 + K_{10}$
$\langle A^{ud}A^{du} - A^{us}A^{su}\rangle$	$K_9 - K_{10}$
$\frac{d}{dm_s}\langle V^3V^3\rangle$	K_7
$\frac{d}{dm_s}\langle A^3A^3\rangle$	K_8
$\langle V^1A^2P^3\rangle$	$K_{11},\ K_{12}$
$\langle A^aA^bA^cA^d\rangle$, $\langle A^aA^bV^cV^d\rangle$ $\langle V^aV^bA^cA^d\rangle$ $\langle V^aV^bV^cV^d\rangle$, $\langle A^aV^bA^cV^d\rangle$	$K_1, ..., K_6$

∗ Practical evaluations, euclidian space + simple models

∗ Question of convergence

Role of QED counterterms

* Terms quadratic in e in QED Lagrangian:

$$Z_2^{\overline{MS}} = \frac{\mu_0^{-2\epsilon}}{16\pi^2}(-\xi)\left\{\frac{1}{\epsilon} - \gamma + \log 4\pi\right\}$$

$$Z_s^{\overline{MS}} = \frac{\mu_0^{-2\epsilon}}{16\pi^2}(-\xi - 3)\left\{\frac{1}{\epsilon} - \gamma + \log 4\pi\right\}$$

* Counterterms in terms of spurions:

$$\mathcal{L}_{QED}^{CT} = \frac{i}{2} Z_2^{\overline{MS}} e^2\, \overline{\psi}_L\, [q_L, D^\mu q_L]\gamma_\mu \psi_L + (L \leftrightarrow R)$$
$$+ \frac{1}{2} Z_2^{\overline{MS}} e^2\, \overline{\psi}_R \left(q_R^2(s + ip) + (s - ip) q_L^2\right) \psi_L + h.c.$$
$$- Z_s^{\overline{MS}} e^2\, \overline{\psi}_R q_R (s + ip) q_L \psi_L + h.c.$$

* Generating functional $O(e^2)$ **well defined** $q_L(x)$,$q_R(x)$

Consequences of the counterterms

$*$ Contributions to 4 $K_i's$

$$K_9^{CT} = -\frac{1}{8} Z_2 \qquad K_{10}^{CT} = \frac{1}{8} Z_s$$
$$K_{11}^{CT} = -\frac{1}{8} Z_s \qquad K_{12}^{CT} = -\frac{1}{4} Z_2$$

$\Rightarrow$ Short distance constraints

$*$ Observables involve

$K_{10} + K_{11}$ (finite)
$4K_{12} - X_6$ (finite)
$K_9 + K_{10} \longrightarrow$ quarks masses

Definition of a purely strong quark mass

* Gasser, Rusetsky, Scimemi (2003): physical mass $m_f(\mu_0)$ runs with QCD +QED, proposed a definition of "strong" mass ((i.e. only QCD running) $\overline{m}_f(\lambda, \mu_0)$ such that

$$\overline{m}_f(\lambda, \lambda) = m_f(\lambda)$$

* Alternative (more convenient ?) definition from chiral Lagrangian

$$\overline{m}_f(\mu, \mu_0) = m_f(\mu_0)[1 + 4e^2 Q_f^2 (K_9 + K_{10})]$$

* Determination of $K_9 + K_{10}$

$$\left(K_9^r(\mu) + K_{10}^r(\mu) - \frac{1}{16\pi^2} \chi\, log's \right) (m_s - m) B_0 =$$

$$\frac{1}{F_0^2} \int d^d x < 0 | V_\mu^{ud}(x) V_\nu^{du}(0) - V_\mu^{us}(x) V_\nu^{su}(0) | 0 > D_\gamma^{\mu\nu}(x)$$

$$+ (Z_s - Z_2)(m_s - m) B_0 + O(m_s^2)$$

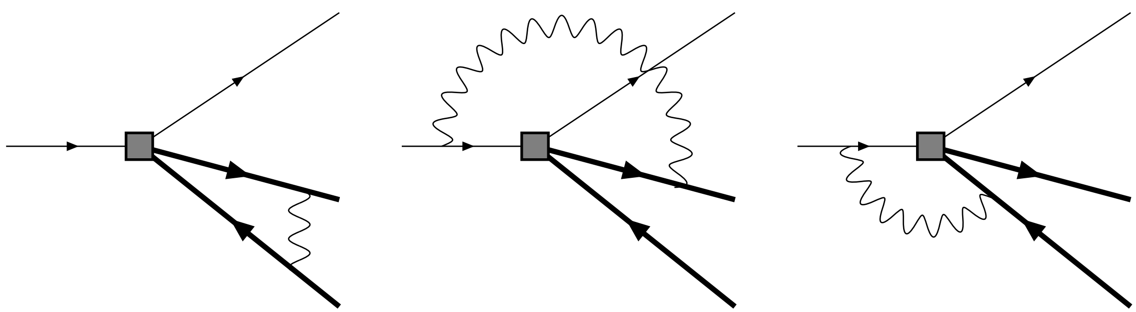
* Possible evaluation using τ decay input.

X_i couplings sum rules

[Descotes-Genon, BM (2005)] * There are two effective theories involved

$$\mathcal{L}_{Fermi} = \frac{-4G_F V_{ud}}{\sqrt{2}} \{\bar{l}_L \gamma^\lambda \nu_L \times \bar{d}\gamma_\lambda u_L + h.c.\} \text{ and } \mathcal{L}_{Chiral}$$

* Perform matching in two steps at order one loop.
* Complete SM calculation by Braaten and Li (1990) at order one loop of generic process $l(p) \rightarrow \bar{u}(q) + d(q') + \nu(p')$
* Calculation from Fermi theory:



* This determines the counterterms (trick: use Pauli-Villars regularization)

X_i **results**

* Exact relations found

$$X_3 = 4X_2 - \frac{3}{2}\frac{1}{16\pi^2}$$
$$X_5 = -2X_2$$

* Sum rules

$< 0\|V^3V^3 - A^3A^3\|\pi >$	X_1
$< 0\|V^3V^3 - A^3A^3\|0 >$	X_2
$< 0\|V^3V^3 + A^3A^3\|\pi >$	$X_6 - 4K_{12}$

* One explicit expression $X_6 - 4K_{12} =$

$$\frac{1}{32\pi^2}\int_0^{M_Z^2} dx[\Gamma_{VV}(-x) + \Gamma_{AA}(-x)] + \frac{1}{16\pi^2}\left[-6\log\frac{M_Z}{\mu} + \frac{5}{2}\right]$$

* Relation with Sirlin's universal factor

$$S_{EW} = 1 - \frac{1}{2}e^2(X_6 - 4K_{12})$$

Some applications

* Marciano-Sirlin formula for F_π

$$\sqrt{2}F_\pi = 130.7\left(\frac{0.9750}{V_{ud}}\right) \pm 0.1 + 0.15\, C_1 \ \mathrm{MeV}$$

* Explicit chiral expression: $C_1 =$

$$-4\pi^2\left[\tfrac{8}{3}(K_1^r+K_2^r)+\tfrac{20}{9}(K_5^r+K_6^r)-\tfrac{4}{3}X_1+4(-X_2^r+X_3^r)-(\tilde{X}_6^r-4K_{12}^r)\right]_{\mu=M_\rho^2}$$
$$+\ \frac{C}{4F^4}\left(3+2\log\frac{M_\pi^2}{M_\rho^2}+\log\frac{M_K^2}{M_\rho^2}\right)-\frac{1}{2}$$

* Result: $F_\pi = 92.20 \pm 0.20$ MeV

$*$ Compatibility of K_{l3}^+ and K_{l3}^0 decays

$K^+ \to \pi^0 l^+ \nu$ [E865, ISTRA]
$K^0 \to \pi^+ l^- \nu$ [NA48, KTEV, KLOE]

E865 $V_{us} = 0.2272 \pm 0.0036$
KTEV: $V_{us} = 0.2252 \pm 0.0021$ problem with CKM unitarity ?

$*$ Compatibility test:

$$\left.\frac{f_+^{K^+}(0)}{f_+^{K^0}(0)}\right|_{ChPT} = 1.022 \pm 0.003 - 16\pi\alpha X_1 \quad \left.\frac{f_+^{K^+}(0)}{f_+^{K^0}(0)}\right|_{exp} = 1.038 \pm 0.007$$

$*$ Sum rule result: $16\pi\alpha X_1 \simeq 0.001$

Some conclusions

* Restricted to meson sector. Many 2-loop computations performed.

* SU(2): fair predictivity and convergence

* SU(3):
  - indication for some weakening of CS breaking
  - πK amplitude important, low-energy data welcome
  - work in progress on modelling couplings
  - information on scalar currents correlators, perhaps from lattice QCD

* Electromagnetic corrections: possibly under good control