WHAT'S WRONG WITH THESE ELEMENTS OF REALITY?

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The subject of Einstein–Podolsky–Rosen correlations—those strong quantum correlations that seem to imply "spooky actions at a distance"—has just been given a new and beautiful twist. Daniel Greenberger, Michael Horne and Anton Zeilinger have found a clever and powerful extension of the two-particle EPR experiment to *gedanken* decays that produce more than two particles.[1] In the GHZ experiment the spookiness assumes an even more vivid form than it acquired in John Bell's celebrated analysis of the EPR experiment, given over 25 years ago.[2] The argument that follows is my attempt to simplify a refinement of the GHZ argument given by the philosophers Robert Clifton, Michael Redhead and Jeremy Butterfield.[3]

Consider *three* spin-$\frac{1}{2}$ particles, named 1, 2 and 3. They have originated in a spin-conserving *gedanken* decay and are now *gedanken* flying apart along three different straight lines in the horizontal plane. (It's not essential for the *gedanken* trajectories to be coplanar, but it makes it easier to describe the rest of the geometry.) I specify the spin state Ψ of the three particles in a time-honored manner, giving you a complete set of commuting Hermitian spin-space operators of which Ψ is an eigenstate.

Those operators are assembled out of the following pieces (measuring all spins in units of $\frac{1}{2}\hbar$): σ_z^i, the operator for the spin of particle i along its direction of motion; σ_x^i, the spin along the vertical direction; and σ_y^i, the spin along the horizontal direction orthogonal to the trajectory. (Any three orthogonal directions independently chosen for each particle would do. But we're going to be *gedanken* measuring x and y components of each particle's spin, so it's nice to think of the x and y directions as orthogonal to the direction of motion, since the components can then be straightforwardly measured by passage through a conventional Stern–Gerlach magnet.) The complete set of commuting Hermitian operators consists of

$$\sigma_x^1\sigma_y^2\sigma_y^3,\quad \sigma_y^1\sigma_x^2\sigma_y^3,\quad \sigma_y^1\sigma_y^2\sigma_x^3. \qquad (1)$$

Even though the x and y components of a given particle's spin anticommute—a fact of paramount importance in what follows—all three of the operators in (1) do indeed commute with one another, because the product of any two of them differs from the product in the reverse order by an even number of such anticommutations. Because they all commute, the three operators can be provided with simultaneous eigenstates. Since the square of each of the three is unity, the eigenvalues of each are $+1$ or -1, and the 2^3 possible choices are indeed just what we need to span the eight-dimensional space of three spins-$\frac{1}{2}$.

For simplicity of exposition let's focus our attention on the symmetric eigenstate in which each of the operators (1) has the eigenvalue $+1$. (Its state vector is $\Psi = (1/\sqrt{2})(|1,1,1\rangle - |-1,-1,-1\rangle)$, where 1 or -1 specifies spin up or down along the appropriate z axis, but you don't need to know this. I'm only telling you because discussions of EPR always write down an explicit form for the state vector and I wouldn't want you to think you were missing anything.) Because the spin vectors of distinct particles commute component by component, we can simultaneously measure the x component of one particle and the y components of the other two (using three Stern–Gerlach magnets in three remote regions of space). Since the three particles are in an eigenstate of all three operators (1) with eigenvalue unity, the product of the results of the three spin measurements has to be $+1$, regardless of which particle we single out for the x-spin measurement.

This affords an immediate application of the EPR reality criterion[4]: "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." The "element of physical reality" is that predictable value, and it ought exist whether or not we actually carry out the procedure necessary for its prediction, since that procedure in no way disturbs it. Because the product of the results of measuring one x component and two y components is unity in the state Ψ, we can predict with certainty the result of measuring the x component of the spin of any one of the three particles by measuring the y components of the two other, far away particles. For if both y components turn out to be the same then the x component, when measured, must yield the value $+1$; if the two y components turn out to be different, the subsequently measured x component will necessarily yield the value -1. In the absence of spooky actions at a distance or the metaphysical cunning of a Niels Bohr, the two far away y-component measurements cannot "disturb" the particle whose x component is subsequently to be measured. The EPR reality criterion therefore asserts the existence of elements of reality m_x^1, m_x^2 and m_x^3, each having the value $+1$ or -1, each waiting to be revealed by the appropriate pair of far away y-component measurements.

In much the same way, we can also predict the result of measuring the y component of the spin of any particle with certainty, by measuring one x component and one y component of the spins of the other two. There are thus elements of reality m_y^1, m_y^2 and m_y^3, with values $+1$ or -1, also waiting to be revealed by far away measurements. All six of the elements of reality m_x^i and m_y^i have to be there, because we can predict in advance what any one of the six values will be by measurements made

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so far away that they cannot disturb the particle that subsequently does indeed display the predicted value.

This conclusion is, of course, highly heretical, because σ_x^i does not commute with σ_y^i—in fact the two *anti*-commute—and therefore they cannot have simultaneous values. (The operators (1) are nicely chosen to hide this failure to commute, since the anticommutations always occur in pairs.) But heresy or not, since the result of either measurement can be predicted with probability 1 from the results of other measurements made arbitrarily

(1) whose eigenvalues define Ψ.

However:

Not only does (2) commute with each of the operators (1), but you can easily check that it is a simple explicit function of them, namely, *minus* the product of all three. The (crucial) minus sign arises because here, at last, in bringing the pairs of operators σ_y^i together to produce unity, one runs up against an *odd* number of anticommutations of σ_y^i's with σ_x^i's. Since Ψ is an eigenstate with eigenvalue $+1$ of each of the operators (1), it is therefore indeed an eigenstate of

other hand, the elements of reality require a class of outcomes to occur *all* of the time, while quantum mechanics *never* allows them to occur.

It is also appealing to see the failure of the EPR reality criterion emerge quite directly from the one crucial difference between the elements of reality (which, being ordinary numbers, necessarily commute) and the precisely corresponding quantum mechanical observables (which sometimes anticommute).

I was surprised to learn of this always-vs-ne