General Relativity and Nonlinear Representation of Supersymmetriy for Unity of Nature

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OUTLINE

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1. Motivation

@ MSSM with the low TeV mass scale for SUSY breaking looks excluded by LHC data.

@ The origin of D-term?, R-parity?, $\cdots$

@ Distinction of Boson and Fermion
$\Longleftrightarrow$ $\mathbf{h \neq 0}$, i.e. SUSY breaking around Planck scale space-time physics!?

@ The space-time symmetry nature of SUSY:

- Supergravity(SUGRA)
$\Longleftrightarrow$ Geometry of superspace (**Mathematical:** $[x^\mu, \theta_\alpha]$, sPoicaré)

While,

- General Relativity(GR)
$\Longleftrightarrow$ Geometry of Riemann space (**Physical**:$[x^\mu]$, GL(4,R))

$\Longrightarrow$ **New SUSY paradigm on particular physical space-time.**

A brief review of NLSUSY:

- Take flat space-time specified by x^a and ψ_α.
- Consider one form $\omega^a = dx^a - \frac{\kappa^2}{2i}(\bar\psi\gamma^a d\psi - d\bar\psi\gamma^a\psi)$,
- κ is an arbitrary constant with the dimension l^{+2}.
- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar\zeta\gamma^a\psi - \bar\psi\gamma^a\zeta)$ and $\delta\psi = \zeta$

with a global spinor parameter ζ.

- An invariant acction($\sim$ invariant volume) is obtained:

$S = -\frac{1}{2\kappa^2}\int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA}$,

L_{VA} is N=1 Volkov-Akulov model of NLSUSY given by

$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right]$,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,

$t^a{}_b = -i\kappa^2(\bar\psi\gamma^a\partial_b\psi - \partial_b\bar\psi\gamma^a\psi)$,

which is invariant under N=1 NLSUSY transformation:

$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar\zeta\gamma^a\psi - \bar\psi\gamma^a\zeta)\partial_a\psi$. $\longleftrightarrow$ NG fermion for SB SUSY

- ψ is NG fermion (the coset space coordinate) of $\frac{superPoincare}{Poincare}$.
- ψ is quantized canonically in compatible with SUSY algebra.

2. Nonlinear Supersymmetric General Relativity (NLSUSY GR)

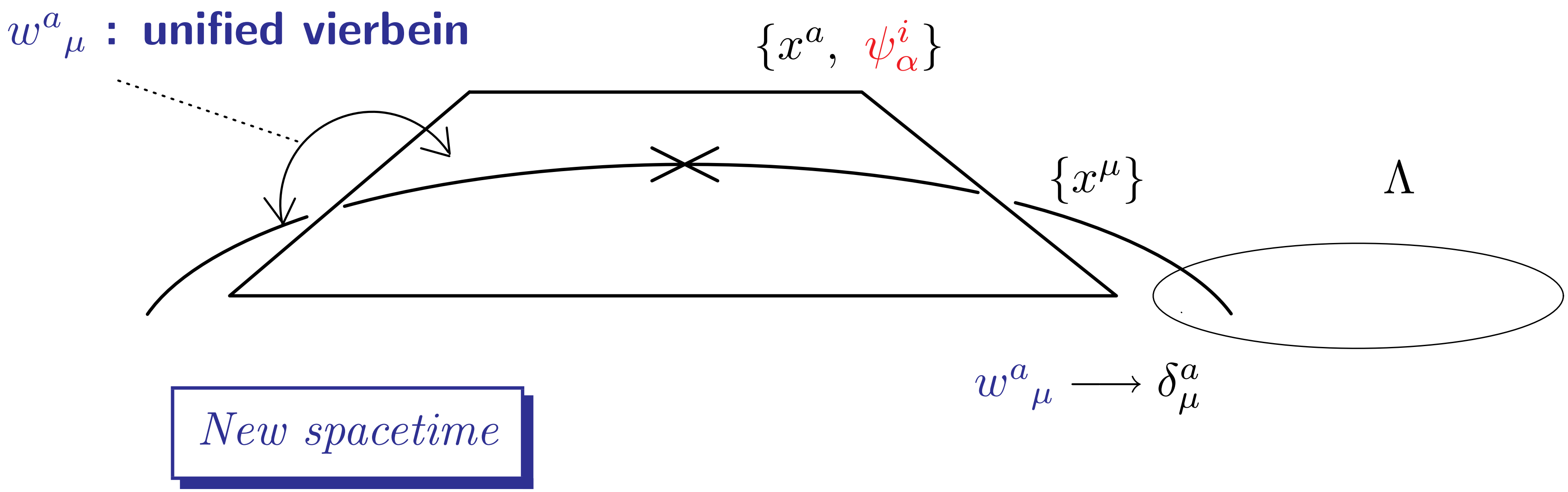
2.1 New Space-time as Ultimate Shape of Nature

New (unstable) Riemann space-time inspired by nonlinear(NL) SUSY :

The tangent space of new space-time is specified by
SL(2,C) Grassmann coordinates ψ_α for NLSUSY
and
the ordinary SO(1,3) Minkowski coordinates x^a,

i.e the coordinates of the the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning subsequently to the NLSUSY NG fermion ψ_α (called $superon$ hereafter) and x^a are attached at every curved space-time point.

- **Ultimate shape of nature** $\Longleftrightarrow$ **(empy) unstable space-time:**



(Locally homomorphic non-compact groups SO(1,3) and SL(2,C) for space-time symmetry are analogous to compact groups SO(3) and SU(2) for gauge symmetry of 't Hooft-Polyakov monopole, though SL(2,C) is realized nonlinearly.)

- Note that $SO(1,3) \cong SL(2,C)$ is crucial for NLSUSY GR scenario.

4 dimensional space-time is singled out.

2.2. Nonlinear Supersymmetric General Relativity (NLSUSY GR)

We extend geometrical arguments of Einstein general relativity(EGR) to **new (unstable) space-time** : $[x^\mu; x^a, \psi]$

- Unified vierbein of new space-time:

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi),$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a,$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar\psi^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar\psi^I \gamma^a \psi^I), (I = 1, 2, .., N)$$

(Note: The first and the second indices of t represent those of γ-matrix and the covariant derivative, respectively. The order of indices are important.)

- N-extended NLSUSY GR action of EH-type in new (empty) space-time:

(Note: Mach principle is encoded geometrically by NLSUSY.)

N-extended NLSUSY GR action:

$$L_{NLSUSYGR}(w) = -\frac{c^4}{16\pi G}|w|(\Omega(w) + \Lambda), \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \tag{2}$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, .., N) \tag{3}$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: the unified vierbein of new space-time,
- $e^a{}_\mu(x)$: the ordinary vierbein for the local SO(1,3) of EGR,
- $t^a{}_\mu(\psi(x))$: the mimic vierbein for the local SL(2,C) subsequently turning to the stress-energy-momentum of NG fermion $\psi(x)^I$(called *superons*),
- $\Omega(w)$: the unified Ricci scalar curvature of new space-time in terms of $w^a{}_\mu$,
- $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$, $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) w^{\nu a}(x)$: unified metric tensors of new space-time.
- G : the Newton gravitational constant.
- Λ : ($small$) cosmological constant in new space-time indicating the NLSUSY structure of new space-time.

- The arbitrary constatnt of NLSUSY κ^2 with the dimension $(length)^4$ is related to cosmological quantities:

$$\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$$

by NLSUSY GR scenario.

- Remarkably $\Lambda > 0$ in the action gives subsequently

(i) the correct sign to the kinetic term of $\psi(x)$

(ii) the competence of Λ for the negative dark energy density of space-time. ($\rightarrow$ Sec.4).

- NLSUSY GR scenario predicts 4 dimensional space-time.

i.e. $SO(1, D-1) \cong SL(d, C)$,

$$\frac{D(D-1)}{2} = 2(d^2 - 1)$$

holds for only $\underline{D = 4, d = 2}$.

2.3 Symmetries of NLSUSY GR (N-extended SUSY action)

• NLSUSY GR action is invariant under the following space-time symmetries which are isomorphic to sP:

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4,\text{R})] \otimes [\text{local Lorentz}] \otimes [\text{local spinor translation}] \quad (4)$$

and

the following internal symmetries for N-extended NLSUSY GR (with N-superons ψ^I $(I = 1, 2, ..N)$) :

$$[\text{global SO(N)}] \otimes [\text{local U(1)}^{\text{N}}] \otimes [\text{chiral}]. \quad (5)$$

For Example:

- Invariance under the new NLSUSY transformation;

$$\delta_{\zeta^I}\psi = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_\rho\psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_{[\mu}e^a{}_{\rho]}, \tag{6}$$

Because (6) induce GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{7}$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and $\xi^\rho = -i\kappa\bar{\zeta}^I\gamma^\rho\psi^I)$.

Commutators of two new NLSUSY transformations (6) on ψ^I and $e^a{}_\mu$ close to GL(4,R),

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu\partial_\mu\psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \tag{8}$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2 - \xi_1^\rho\xi_2^\sigma e_a{}^\mu\partial_{[\rho}e^a{}_{\sigma]}$. *Q.E.D.*

- New NLSUSY transformation (6) is the square-root of GL(4,R);

$$[\delta_1, \delta_2] = \delta_{GL(4,R)}, \quad i.e. \quad \delta \sim \sqrt{\delta_{GL(4,R)}}.$$

c.f. SUGRA

$$[\delta_1, \delta_2] = \delta_P \underline{+\delta_L + \delta_g}$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- Invariance under the local Lorentz transformation;

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I)\gamma_5\gamma_d\psi^I(\partial_\mu \epsilon_{bc}) \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

Because (9) induce the familiar local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{10}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra, for example, on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu \beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$. $Q.E.D.$

3. Vacuum Structure of NLSUSY GR

3.1 Phase Transition of New Space-Time: Big Decay

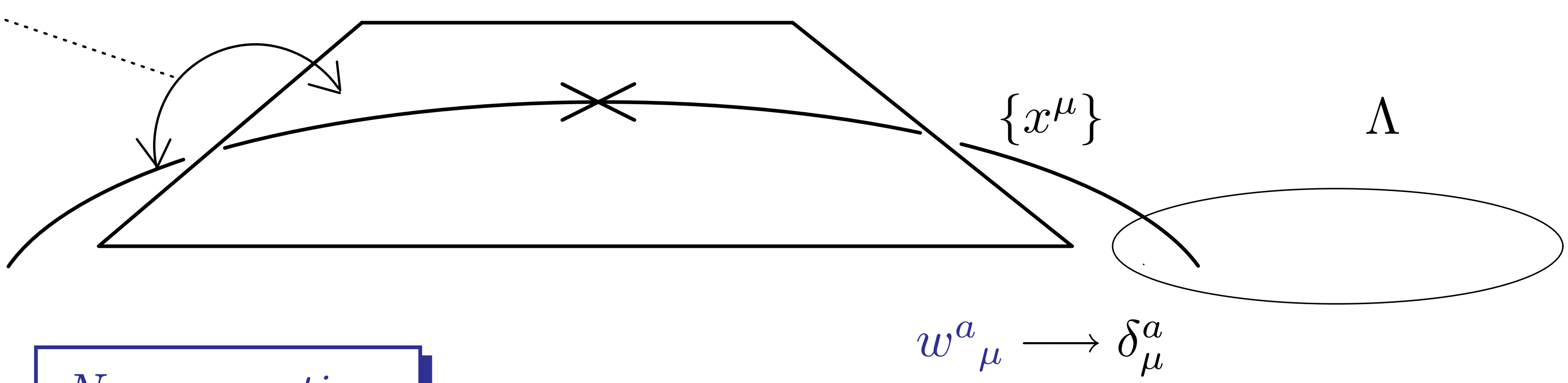
New space-time described by $L_{NLSUSYGR}(w)$ is unstable
due to the NLSUSY nature of tangent space-time and
breaks down spontaneously to **ordinary Riemann space-time($EH\ action$)**
and massless NG fermion($superons$) *Superon-Graviton Model(SGM)*
(inflating by Pauli principle):

$$L_{NLSUSYGR}(w) = L_{SGM}(e,\psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + |w_{VA}(\psi)|\Lambda + \tilde{T}(e,\psi)\}. \quad (12)$$

- $R(e)$: the ordinary Ricci scalar curvature of EH action
- Λ : the cosmological term; $V_{P.E} = \Lambda > 0$
- $\tilde{T}(e,\psi)$: the gravitational interaction of superon.
- $|w_{VA}(\psi)| = \det w^a{}_b = \det(\delta^a{}_b + t^a{}_b(\psi))$, $t^a{}_b(\psi) = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \partial_b\bar{\psi}\gamma^a\psi)$

Note that $L_{SGM}(e,\psi)$ produces N-extended NLSUSY action with $\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$
in Riemann-flat($e^a{}_\mu(x) \to \delta^a{}_\mu$) space-time.

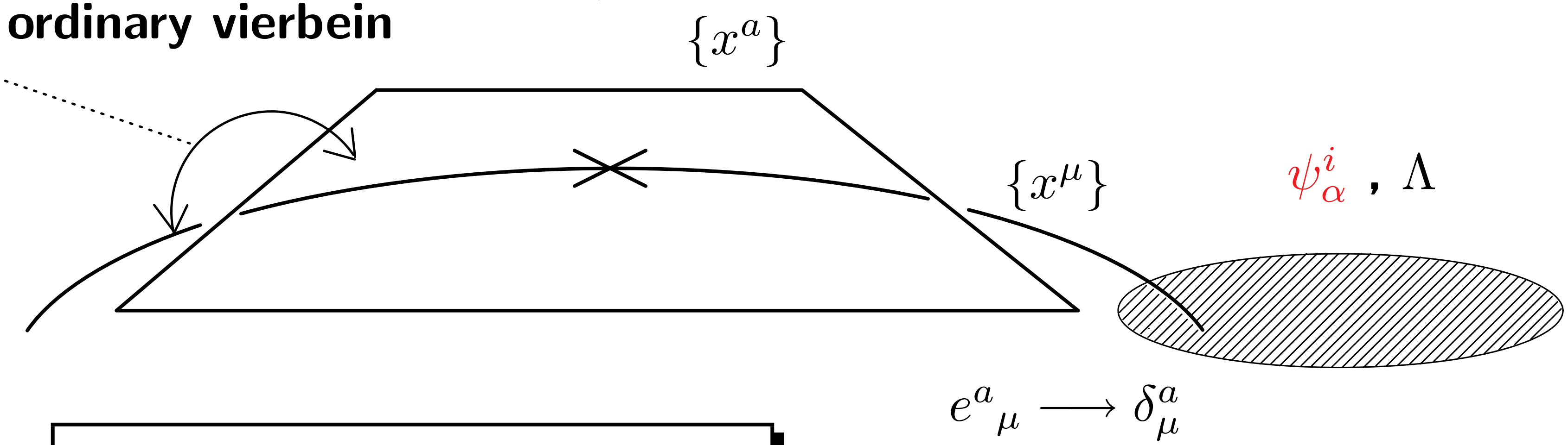
$w^a{}_\mu$: **unified vierbein**



New spacetime

$\Downarrow$ **Big Decay**

$e^a{}_\mu$: **ordinary vierbein**



Riemann spacetime $\oplus$ **matter**

Ignition of Big Bang towards the true vacuum

3.2 Vacuum structure of $L_{SGM}(e,\psi)$

Representation of Space-time symmetry $\Longleftrightarrow$ particle spectrum

Despite the high nonlinearity of $L_{SGM}(e,\psi)$
SUSY dictates the vacuum particle configuration of SGM,
which contains massless states for the SM(MSSM).

By **respecting SUSY algebra throughout** we show in the local flat frame:

- N-LSUSY theory emerges in the true vacuum of N-NLSUSY theory as (massless) composites of NG fermions.

$\Longleftrightarrow$ NL/L SUSY relations $\longleftrightarrow$ **BCS/LG**

- We have found the NL/L SUSY relation in flat space-time for $N=1$(toy model), 2(SUSY QED), 3(SUSY QCD).

- These phenomena are the phase transition of NLSUSY $L_{SGM}(e,\psi)$ towards the true vacuum with $V_{P.E.}=0$ achieved by forming massless composite states of SM(MSSM).

3.3 NL/L Relation for N=2 SUSY :

We demonstrate NL/L relation for N=2 SUSY as Low Energy Theory of N=2 SGM.

- $N = 1$ SUSY is not a realistic case in SGM scenario.

The realistic gauge field with $J^P = 1^-$ can be constructed in $N \geq 2$ SUSY.

i.e. $\bar{\psi}_M \gamma^a \psi_M \equiv 0$, for $N = 1$

$\Longrightarrow$ Realistic minimal model in SGM scenario emerges from $N \geq 2$ SUSY.

- N=2 SGM reduces to $N = 2$ NLSUSY in flat $(e^a{}_\mu(x) \rightarrow \delta^a{}_\mu)$ space-time :

$L_{N=2SGM}(e, \psi) \longrightarrow L_{N=2NLSUSY}(\psi)$ $\leftrightarrow$ cosmological constant of NLSUSY GR.

$N = 2$ NL/L SUSY relation (two dimensional space-time for simplicity):

N=2, d=2 NLSUSY model:

$$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \tag{13}$$

where,
$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,
$t^a{}_b = -i\kappa^2(\bar{\psi}_j \gamma^a \partial_b \psi^j - \partial_b \bar{\psi}_j \gamma^a \psi^j)$, $(j = 1, 2)$,

which is invariant under N=2 NLSUSY transformation,

$\delta_\zeta \psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar{\zeta}_k \gamma^a \psi^k - \bar{\psi}_k \gamma^a \zeta^k)\partial_a \psi^j, (j = 1, 2)$.

N=2, d=2 LSUSY Theory (SUSY QED):

- Helicity states of N=2 vector supermultiplet:

$$\begin{pmatrix} +1 \\ +\frac{1}{2}, \ +\frac{1}{2} \\ 0 \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell vector supermultiplet: $(v^a, \lambda^i, A, \phi, D$;i=1,2). in WZ. ($A$ and ϕ are two singlets, 0^+ and 0^-, scalar fields.)

- Helicity states of N=2 scalar supermultiplet:

$$\begin{pmatrix} +\frac{1}{2} \\ 0, \ 0 \\ -\frac{1}{2} \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets:
$(\chi,\ B^i,\ \nu,\ F^i;\ i = 1, 2)$.

• **The most genaral $N=2, d=2$ SUSYQED action ($m=0$ case)** :

$$L_{N=2SUSYQED} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf}, \tag{14}$$

$$\begin{aligned}
L_{V0} &= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D, \\
L'_{\Phi 0} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}(F^i)^2, \\
L_e &= e\Big\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5 \nu \\
&\quad + B^i(\bar{\lambda}^i \chi - \epsilon^{ij}\bar{\lambda}^j \nu) - \frac{1}{2}(B^i)^2 D \Big\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2, \\
L_{Vf} &= f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}.
\end{aligned} \tag{15}$$

• Note that
$J=0$ states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.

$L_{\mathrm{N=2SUSYQED}}$ is invariant under $N=2$ LSUSY transformation:

- For the vector off-shell supermultiplet:

$$
\begin{aligned}
&\delta_\zeta v^a = -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
&\delta_\zeta \lambda^i = (D - i\partial\!\!\!/ A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\partial\!\!\!/\phi\zeta^j, \\
&\delta_\zeta A = \bar{\zeta}^i\lambda^i, \\
&\delta_\zeta \phi = -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
&\delta_\zeta D = -i\bar{\zeta}^i\partial\!\!\!/\lambda^i.
\end{aligned}
\tag{16}
$$

$$
[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{17}
$$

where $\zeta^i, i = 1, 2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\, v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the two scalar off-shell supermultiplets:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial\!\!\!/ B^i)\zeta^i - e\epsilon^{ij} V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij} \bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial\!\!\!/ B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial\!\!\!/ \chi - i\epsilon^{ij}\bar{\zeta}^j \partial\!\!\!/ \nu \\
&\quad -e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i) B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\chi = \Xi^a \partial_a \chi - e\theta\nu, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]B^i = \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\nu = \Xi^a \partial_a \nu + e\theta\chi, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]F^i = \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{18}$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ and the U(1) gauge parameter θ.

$N=2$ NL/L SUSY relation:

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} = L_{\mathrm{N=2NLSUSY}} + [\text{surface terms}], \qquad (19)$$

achieved by the followings:

(i) Construct SUSY invariant relations which express component fields of LSUSY supermultiplet as the composites of superons ψ_j of NLSUSY.

(ii) Show that
performing NLSUSY transformations of constituent superons ψ^j
in SUSY invariant relations
reproduces familiar LSUSY transformations among the composite LSUSY supermultiplet.

(iii) Substituting SUSY invariant relations into $L_{\mathrm{N=2LSUSYQED}}$,
the NL/L SUSY relation is established.

- SUSY invariant relationsns for the vector off-shell supermultiplet:

$$
\begin{aligned}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|,\\
\lambda^i &= \xi\psi^i|w|,\\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|,\\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|,\\
D &= \frac{\xi}{\kappa}|w|. \qquad (20)
\end{aligned}
$$

- D-term originates from NLSUSY.

- Note that the global SU(2) emerges for N=2 SGM.

• SUSY invariant relations for scalar off-shell supermultiplets:

$$\chi = \xi^i \left[\psi^i |w| + \frac{i}{2}\kappa^2 \partial_a \{\gamma^a \psi^i \bar{\psi}^j \psi^j |w|\}\right]$$

$$B^i = -\kappa \left(\frac{1}{2}\xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j\right) |w|,$$

$$\nu = \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2}\kappa^2 \partial_a \{\gamma^a \psi^j \bar{\psi}^k \psi^k |w|\}\right],$$

$$F^i = \frac{1}{\kappa}\xi^i \left\{|w| + \frac{1}{8}\kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|)\right\} - i\kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|)$$

$$-\underline{\frac{1}{4} e\kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|}. \qquad (21)$$

The quartic fermion self-interaction term in F^i is the origin of the local $U(1)$ gauge symmetry of LSUSY.

• SUSY invariant relations produce under NLSUSY
a new off-shell commutator algebra which closes on only a translation:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{22}$$

where $\delta_P(v)$ is a translation with a parameter $v^a = i\bar{\zeta}_1\gamma^a\zeta_2$.

• Note that the commutator does not induce the U(1) gauge transformation,
which is different from the ordinary LSUSY.

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P + \delta_G, \tag{23}$$

• Substituting SUSY invariant relations to $L_{N=2LSUSYQED}$ gives **NL/L SUSY relation**:

$$L_{N=2LSUSYQED} = f(\xi, \xi^i) L_{\mathrm{N=2NLSUSY}} + [\mathrm{suface\ terms}], \tag{24}$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \tag{25}$$

$\Rightarrow$ composite eigenstates of global space-time ($N = 2$ sPoincaré) symmetry !?

• NL/L SUSY relation describes the reletion between the cosmology and the low energy particle physics in SGM scenario. ($\rightarrow$ Sec. 4).

• SGM scenario predicts the magnitude of the bare gauge coupling constant.

$$e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c} \tag{26}$$

• The direct linearization(NL/L relation) of highly nonlinear SGM action (12) in curved space remains to be carried out.

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
as massless composites of NG fermion
from
the NLSUSY cosmological constant of SGM.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

♣ Systematics of NL/L SUSY relation and $N=2$ SUSY QED

SUSY invariant relations: in the superfield formulation.

Linearization of NLSUSY in the $d=2$ superfield formulation

- General superfields are given for the $N=2$ vector supermultiplet by

$$\begin{aligned}\mathcal{V}(x,\theta^i) \ =\ & C(x)+\bar\theta^i\Lambda^i(x)+\frac{1}{2}\bar\theta^i\theta^j M^{ij}(x)-\frac{1}{2}\bar\theta^i\theta^i M^{jj}(x)+\frac{1}{4}\epsilon^{ij}\bar\theta^i\gamma_5\theta^j\phi(x) \\ & -\frac{i}{4}\epsilon^{ij}\bar\theta^i\gamma_a\theta^j v^a(x)-\frac{1}{2}\bar\theta^i\theta^i\bar\theta^j\lambda^j(x)-\frac{1}{8}\bar\theta^i\theta^i\bar\theta^j\theta^j D(x), \qquad (27)\end{aligned}$$

and for the $N=2$ scalar supermultiplet by

$$\begin{aligned}\Phi^i(x,\theta^i) \ =\ & B^i(x)+\bar\theta^i\chi(x)-\epsilon^{ij}\bar\theta^j\nu(x)-\frac{1}{2}\bar\theta^j\theta^j F^i(x)+\bar\theta^i\theta^j F^j(x)-i\bar\theta^i\partial\!\!\!/ B^j(x)\theta^j \\ & +\frac{i}{2}\bar\theta^j\theta^j(\bar\theta^i\partial\!\!\!/\chi(x)-\epsilon^{ik}\bar\theta^k\partial\!\!\!/\nu(x))+\frac{1}{8}\bar\theta^j\theta^j\bar\theta^k\theta^k\partial_a\partial^a B^i(x). \qquad (28)\end{aligned}$$

- Take the following ψ^i-dependent specific supertranslations with $-\kappa\psi(x)$,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{29}$$

and denote the resulting superfields on (x'^a, θ'^i) and their θ-epansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \tag{30}$$

- Hybrid global SUSY transformations $\delta^h = \delta^L(x.\theta) + \delta^{NL}(\psi)$ on (x'^a, θ'^i) give:

$$\delta^h\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \delta^h\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{31}$$

- Therefore, the following conditions, i.e. SUSY invariant constraints

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \xi^I_{\mathcal{V}}(\mathrm{constant}) \quad \tilde{\varphi}^I_{\Phi}(x) = \xi^I_{\Phi}(\mathrm{constant}), \tag{32}$$

are invariant (conserved quantities) under hybrid supertrasformations,
which provide SUSY invariant relations.

• Putting in general constants as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \tag{33}$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{34}$$

where mass dimensions of constants (or constant spinors) in $d = 2$ are defined by (-1, $\frac{1}{2}$, 0, 0, 0, $-\frac{1}{2}$) for (ξ_c, ξ_Λ^i, ξ_M^{ij}, ξ_ϕ, ξ_v^a, ξ_λ^i), (0, $-\frac{1}{2}$, $-\frac{1}{2}$) for (ξ_B^i, ξ_χ, ξ_ν) and 0 for ξ^i for convenience.

• we obtain straightforwardly the following SUSY invariant relations $\varphi_{\mathcal{V}}^I = \varphi_{\mathcal{V}}^I(\psi)$ for the vector supermultiplet

$$\begin{aligned} C \ &= \ \xi_c + \kappa\bar{\psi}^i\xi_\Lambda^i + \frac{1}{2}\kappa^2(\xi_M^{ij}\bar{\psi}^i\psi^j - \xi_M^{ii}\bar{\psi}^j\psi^j) + \frac{1}{4}\xi_\phi\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - \frac{i}{4}\xi_v^a\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j \\ &\quad -\frac{1}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\xi_\lambda^j - \frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\ \Lambda^i \ &= \ \xi_\Lambda^i + \kappa(\xi_M^{ij}\psi^j - \xi_M^{jj}\psi^i) + \frac{1}{2}\xi_\phi\kappa\epsilon^{ij}\gamma_5\psi^j - \frac{i}{2}\xi_v^a\kappa\epsilon^{ij}\gamma_a\psi^j \end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{2}\xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2}\kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi_\lambda^j) \\
& -\frac{1}{2}\xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i\kappa \partial\!\!\!/ C(\psi)\psi^i, \\
M^{ij} &= \xi_M^{ij} + \kappa \bar{\psi}^{(i} \xi_\lambda^{j)} + \frac{1}{2}\xi\kappa \bar{\psi}^i \psi^j + i\kappa \epsilon^{(i|k|} \epsilon^{j)l} \bar{\psi}^k \partial\!\!\!/ \Lambda^l(\psi) - \frac{1}{2}\kappa^2 \epsilon^{ik}\epsilon^{jl} \bar{\psi}^k \psi^l \partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \xi_\lambda^j - \frac{1}{2}\xi\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \partial\!\!\!/ \Lambda^j(\psi) + \frac{1}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 C(\psi), \\
v^a &= \xi_v^a - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \xi_\lambda^j - \frac{i}{2}\xi\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j - \kappa \epsilon^{ij} \bar{\psi}^i \partial\!\!\!/ \gamma^a \Lambda^j(\psi) + \frac{i}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j \partial^2 C(\psi) \\
& -i\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^b \psi^j \partial^a \partial_b C(\psi), \\
\lambda^i &= \xi_\lambda^i + \xi \psi^i - i\kappa \partial\!\!\!/ M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa \epsilon^{ab} \epsilon^{ij} \gamma_a \psi^j \partial_b \phi(\psi) \\
& -\frac{1}{2}\kappa \epsilon^{ij} \left\{ \psi^j \partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab} \gamma_5 \psi^j F_{ab}(\psi) \right\} \\
& -\frac{1}{2}\kappa^2 \{ \partial^2 \Lambda^i(\psi) \bar{\psi}^j \psi^j - \partial^2 \Lambda^j(\psi) \bar{\psi}^i \psi^j - \gamma_5 \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma_5 \psi^j
\end{aligned}
$$

$$-\gamma_a\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j + 2\partial\!\!\!/\partial_a\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j\} - \frac{i}{2}\kappa^3\partial\!\!\!/\partial^2 C(\psi)\psi^i\bar{\psi}^j\psi^j,$$

$$\begin{aligned}
D &= \frac{\xi}{\kappa} - i\kappa\bar{\psi}^i\partial\!\!\!/\lambda^i(\psi) \\
&+\frac{1}{2}\kappa^2\left\{\bar{\psi}^i\psi^j\partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2\phi(\psi)\right. \\
&\left.+\frac{i}{2}\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial^2 v^a(\psi) - i\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial_a\partial_b v^b(\psi)\right\} \\
&-\frac{i}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\partial\!\!\!/\partial^2\Lambda^j(\psi) + \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j\partial^4 C(\psi),
\end{aligned} \tag{35}$$

and the following SUSY invariant relations for the vector multiplet $\varphi^I_\Phi = \varphi^I_\Phi(\psi)$:

$$\begin{aligned}
B^i &= \xi^i_B + \kappa(\bar{\psi}^i\xi_\chi - \epsilon^{ij}\bar{\psi}^j\xi_\nu) - \frac{1}{2}\kappa^2\{\bar{\psi}^j\psi^j F^i(\psi) - 2\bar{\psi}^i\psi^j F^j(\psi) + 2i\bar{\psi}^i\partial\!\!\!/B^j(\psi)\psi^j\} \\
&-i\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) - \epsilon^{ik}\bar{\psi}^k\partial\!\!\!/\nu(\psi)\} + \frac{3}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 B^i(\psi), \\
\chi &= \xi_\chi + \kappa\{\psi^i F^i(\psi) - i\partial\!\!\!/B^i(\psi)\psi^i\}
\end{aligned}$$

$$
\begin{aligned}
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\chi(\psi)\bar\psi^i\psi^i - \epsilon^{ij}\{\psi^i\bar\psi^j\partial\!\!\!/\nu(\psi) - \gamma^a\psi^i\bar\psi^j\partial_a\nu(\psi)\}] \\
& +\frac{1}{2}\kappa^3\psi^i\bar\psi^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\partial\!\!\!/F^i(\psi)\psi^i\bar\psi^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar\psi^i\psi^i\bar\psi^j\psi^j, \\
\nu \;=\;& \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\partial\!\!\!/B^i(\psi)\psi^j\} \\
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\nu(\psi)\bar\psi^i\psi^i + \epsilon^{ij}\{\psi^i\bar\psi^j\partial\!\!\!/\chi(\psi) - \gamma^a\psi^i\bar\psi^j\partial_a\chi(\psi)\}] \\
& +\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar\psi^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial\!\!\!/F^i(\psi)\psi^j\bar\psi^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar\psi^i\psi^i\bar\psi^j\psi^j, \\
F^i \;=\;& \frac{\xi^i}{\kappa} - i\kappa\{\bar\psi^i\partial\!\!\!/\chi(\psi) + \epsilon^{ij}\bar\psi^j\partial\!\!\!/\nu(\psi)\} \\
& -\frac{1}{2}\kappa^2\bar\psi^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar\psi^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar\psi^i\partial\!\!\!/F^j(\psi)\psi^j \\
& +\frac{1}{2}\kappa^3\bar\psi^j\psi^j\{\bar\psi^i\partial^2\chi(\psi) + \epsilon^{ik}\bar\psi^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar\psi^j\psi^j\bar\psi^k\psi^k\partial^2 F^i(\psi). \qquad (36)
\end{aligned}
$$

• Choosing the following simple SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \tilde{D} = \frac{\xi}{\kappa}, \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{37}$$

give previous simple SUSY invariant relations.

Actions in the $d=2$, $N=2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \to (x'^a, \theta'^i)$, we can confirm systematically that LSUSY actions reduce to NLSUSY representation.

(a) The kinetic (free) action with the Fayet-Iliopoulos (FI) D term for the $N=2$ vector supermultiplet $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathcal{V}\mathrm{free}} = \int d^2x \left\{ \int d^2\theta^i \, \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \, \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i=0}$$
$$= \xi^2 S_{N=2\mathrm{NLSUSY}}, \qquad (38)$$

where

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \qquad (39)$$

(Note) The FI D term gives the correct sign of the NLSUSY action.

(b) Yukawa interaction terms for $\mathcal{V}$ vanish,

i.e.

$$
\begin{aligned}
S_{\mathcal{V}f} &= \frac{1}{8}\int d^2x\ f\left[\int d^2\theta^i\ \mathcal{W}^{jk}(\mathcal{W}^{jl}\mathcal{W}^{kl}+\mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\right. \\
&\qquad \left. +\int d\bar{\theta}^i d\theta^j\ 2\{\mathcal{W}^{ij}(\mathcal{W}^{kl}\mathcal{W}^{kl}+\mathcal{W}_5^{kl}\mathcal{W}_5^{kl})+\mathcal{W}^{ik}(\mathcal{W}^{jl}\mathcal{W}^{kl}+\mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\}\right]_{\theta^i=0} \\
&= 0,
\end{aligned}
\tag{40}
$$

by means of cancellations among four NG-fermion self-interaction terms.

[Note]

• General mass terms for $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$ vanish as well. $\rightarrow$ Chirality is encoded in the false vacuum.

(c) The *most general* gauge invariant action for Φ^i coupled with $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$
\begin{aligned}
S_{\mathrm{gauge}} &= -\frac{1}{16}\int d^2x \int d^4\theta^i e^{-4e\mathcal{V}}(\Phi^j)^2 \\
&= -(\xi^i)^2 S_{N=2\mathrm{NLSUSY}}.
\end{aligned}
\tag{41}
$$

• Here $U(1)$ gauge interaction terms with the gauge coupling constant e produce

four NG-fermion self-interaction terms as

$$
S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{ \frac{1}{4} e\kappa\xi(\xi^i)^2 \bar{\psi}^j\psi^j\bar{\psi}^k\psi^k \right\}, \tag{42}
$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding four NG-fermion self-interaction terms as (21):

$$
F^i(\psi) \longrightarrow F^i(\psi) - \frac{1}{4} e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w_{VA}|. \tag{43}
$$

Therefore,

• under SUSY invariant relations,
the $N = 2$ NLSUSY action $S_{N=2\mathrm{NLSUSY}}$ is related to $N = 2$ SUSY QED action:

$$f(\xi, \xi^i) S_{N=2\mathrm{NLSUSY}} = S_{N=2\mathrm{SUSYQED}} \equiv S_{\mathcal{V}\mathrm{free}} + S_{\mathcal{V}f} + S_{\mathrm{gauge}} \tag{44}$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

$\Longrightarrow$ NL/L SUSY relation gives the relation between
the cosmology and the low energy particle physics in NLSUSY GR (in Sec. 4).

• **SGM scenario predicts the magnitude of the bare gauge coupling constant.**

More general SUSY invariant constraints, i.e. NLSUSY vevs of 0^+ auxiliary fields:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \tag{45}$$

produce

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \; e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \tag{46}$$

where e is the bare gauge coupling constant.

• This mechanism is natural and favorable for SGM scenario as a theory for everything.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

4. Cosmology and Low Energy Physics in NLSUSY GR

The variation of SGM action $L_{N=2SGM}(e,\psi)$ with respect to $e^a{}_\mu$ yields the equation of motion for $e^a{}_\mu$ in Riemann space-time:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e,\psi) - g_{\mu\nu}\frac{c^4\Lambda}{16\pi G}\}, \tag{47}$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

• Note that $\frac{c^4\Lambda}{16\pi G}$ can be interpreted as the negative energy density of space-time, i.e. the dark energy density ρ_D.
(The negative sign in r.h.s is unique.)

4.1. Low Energy Particle Physics of NLSUSY GR :

We have seen in the preceding section that
$N=2$ SGM is essentially N=2 NLSUSY action in Riemann-flat (tangent) space-time.

- The low energy theorem for NLSUSY gives the following superon(massless NG fermion matter)-vacuum coupling

$$< \psi^j{}_\alpha(x)|J^{k\mu}{}_\beta|0 >= i\sqrt{\frac{c^4\Lambda}{16\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{jk} + \cdots, \tag{48}$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{16\pi G}}\gamma^\mu\psi^k + \cdots$ is the conserved supercurrent.

$\sqrt{\frac{c^4\Lambda}{16\pi G}} \equiv \sqrt{\rho_D}$ is the coupling constant (g_{sv}) of superon with the vacuum.

For extracting the low energy particle physics contents of $N=2$ SGM (NLSUSY GR) we take Riemann-flat space-time, where NL/L SUSY relation gives:

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\mathrm{suface\ terms}] = L_{N=2\mathrm{SUSYQED}}. \tag{49}$$

- We can study **true vacuum structures of $N=2$ LSUSY QED action in stead of $N=2$ SGM**.

The true vacuum is given by the minimum of the potential $V(A,\phi,B^i,D)$ of $L_{N=2\mathrm{LSUSYQED}}$,

$$V(A,\phi,B^i,D) = -\frac{1}{2}D^2 + \left\{\frac{\xi}{\kappa} - f(A^2-\phi^2) + \frac{1}{2}e(B^i)^2\right\} D + \frac{e^2}{2}(A^2+\phi^2)(B^i)^2. \tag{50}$$

Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2-\phi^2-\frac{e}{2f}(B^i)^2-\frac{\xi}{f\kappa}\right\}^2 + \frac{1}{2}e^2(A^2+\phi^2)(B^i)^2 \geq 0. \tag{51}$$

• The low energy particle spectrum is obtained by expanding the fields (A, ϕ, B^i) around the vacuum field configurations.

• Adopt following expressions ($k \equiv (\frac{\xi}{f\kappa})^{\frac{1}{2}}$)

$$
\begin{aligned}
A &= -(k+\rho)\cos\theta\cos\varphi\cosh\omega, \\
\phi &= (k+\rho)\sinh\omega, \\
\tilde{B}^1 &= (k+\rho)\sin\theta\cosh\omega, \\
\tilde{B}^2 &= (k+\rho)\cos\theta\sin\varphi\cosh\omega.
\end{aligned}
$$

• Substituting these expressions into $V(A, \phi, \tilde{B}^i)$
and expanding them around the true vacuum configuration:

$\rho \ll 1$ and angles for $\tilde{B}^i = 0$ or $A = \phi = 0$

we obtain the physical particle contents.

• We obtain

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \rho)^2 - 4f^2k^2\rho^2\} \\
& +\frac{1}{2}\{(\partial_a \theta)^2 + (\partial_a \varphi)^2 - e^2k^2(\theta^2 + \varphi^2)\} \\
& +\frac{1}{2}(\partial_a \omega)^2 \\
& -\frac{1}{4}(F_{ab})^2 \\
& +\frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& +\frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/ \chi + \bar{\nu}\partial\!\!\!/ \nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots . \qquad (52)
\end{aligned}
$$

and following mass spectra:

$$
\begin{aligned}
&m_\rho^2 = m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
&m_\theta^2 = m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
&m_{v_a} = m_\omega = 0,
\end{aligned} \tag{53}
$$

which produces mass hierarchy by the factor $\frac{e}{f}$.

• The vacuum breaks both SUSY and $O(2)(U(1))$ for $(A, \tilde{B}^2)$ and restores(maintains) $O(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which gives soft masses $< A >$ to λ^i and produces NG-Boson ω and massless photon v_a, respectively.

• We have shown explicitly that N=2 LSUSY QED, i.e. the matter sector (in flat-space) of $N = 2$ SGM, possesses **a unique true vacuum with** $V_{P.E} = 0$.

The resulting model describes:

one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\phi^c \sim \theta + i\varphi$),
one neutral complex scalar ($\phi^0 \sim \rho(+i\omega)$),

which are **composites of superons (or functions of** ψ).

• Remakably the lepton-Higgs sector of SM analogue $SU(2)_{gl} \times U(1)$ appears from $N = 2$ LSUSY QED **without manifest superpartners**. **R-parity ?**

$\Longleftrightarrow$ **LHC**

4.2 SGM scenario for Cosmology and Low Energy Particle Physics

- The vacuum of N=2 SGM explains naturally observed mysterious relations:

$$\underline{\text{(dark) energy density of the universe} \sim {m_\nu}^4 \sim (10^{-12} GeV)^4 \sim {g_{sv}}^2},$$

provided ${\lambda_D}^0$ is identified with neutrino,
which gives a new insight into the origin of neutrino mass.

5. Summary

NLSUSY GR(SGM) scenario:

• Ultimate entity; **New unstable $d = 4$ space-time:**$[x^a, {\psi_\alpha}^N; x^\mu]$ described by $[L_{NLSUSYGR}(w)]$ **:** NLSUSY GR with Λ: Mach principle is realized.

$\Longrightarrow$ **Big Decay** **to** $[L_{SGM}(e.\psi)]$;

• The creation of Riemann space-time $[x^a; x^\mu]$ and massless fermionic matter $[{\psi_\alpha}^{\mathbf{N}}]$ $[L_{SGM} = L_{EH}(e) - \Lambda + T(\psi.e)]$ **:** Einstein GR with N superon and ρ_D inducing the present accelerating expansion of universe.

• Superfluidity of space-time and matter $[{e^a}_\mu, {\psi_\alpha}^N]$

• Phase transition proceeding to the true vacuum achieved by forming massless composite LSUSY $\Longrightarrow L_{LSUSY} \longleftrightarrow$ BCS vs GL

$\Longrightarrow$ **Big Bang**:

• Subsequent oscillations around the true vacuum.

• In flat space-time, broken N-LSUSY theory emerges from the N-NLSUSY cosmological term of $L_{SGM}(e, \psi)$ **[NL/L SUSY relation].** $\longleftrightarrow$ **BCS / GL**

The cosmological constant is the origin of everything!

Predictions and Speculations:

@ Group theory of SO(10) sP with $\underline{10} = \underline{5} + \underline{5}^*$.
$\underline{5} = \underline{5}_{SU(5)GUT}$ interpreted as superon-quintet(SQ):

- Spin-$\frac{3}{2}$ lepton-type doublet (Γ^-, ν_Γ)
- Doubly charged spin 1/2 particles $E^{2\pm}$
- Neutral $J^P = 1^-$ boson S.
- Proton decay diagrams in GUTs are forbidden by selection rules. $\Rightarrow$ stable proton
- K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, B^0-$\bar{B}^0$ mixings look natural.

@Field theory via Linearization:

- NLSUSY GR(SGM) scenario predicts **4 dimensional space-time**.
- Neutral scalar particle $\phi^0 \sim O(m_\nu) \Longleftrightarrow$ dark matter?
- The bare gauge coupling constant is determined.
- N-LSUSY from N-NLSUSY $\Longleftrightarrow$ SQ hypothesis for all particles (except gravity)

cosmological constant $\leftrightarrow$ dark energy density $\leftrightarrow$ SUSY Br. $\rightarrow m_\nu$

Many Open Questions ! e.g.,

- d=4 case

- Large N case(especially N=5 and N=10), Is SQM realistic and minimal?
- SGM scenario suggests $N \geq 2$ low energy MSSM, SUSY GUT

- Meanings of Chiral symmetry, V-A, Yukawa and gauge coulings

in SGM composite scenario

- Direct linearization(NL/L relation) of SGM action in curved space-time.
- Superfield systematics of NL/L SUSY relation for SGM action.

- Complete Detour of No-Go Th.! (Massive high-spins in linearized theory)

- Superfluidity of $L_{SGM}(e, \psi)$ for sapce-time and matter?

- Inflation in NLSUSY GR and Pauli principle?

- Physical consequences of spin 3/2 NLSUSY GR.