

Exploring Grand Unification Scenarios

A.P. Morais¹ D.J. Miller¹

¹School of Physics and Astronomy
University of Glasgow, Scotland, UK

August 14, 2012

Supersymmetry 2012, Peking University, Beijing

- 1 Introduction
- 2 Soft SUSY Breaking Terms
- 3 $SU(5)$ Grand Unification
- 4 $SO(10)$ Grand Unification
- 5 Summary and Conclusions

Introduction: *The Idea of Grand Unification*

- The Standard Model of Strong and Electroweak interactions is described by the gauge group $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
- The main idea is to embed G_{SM} into a larger simple group
 - $SU(N)$, $SO(2N)$, $SO(2N+1)$, Sp_{2N} , G_2 , F_4 , E_6 , E_7 , E_8
- We will consider standard $SU(5)$ and $SO(10)$ candidates

Our Aim

- Most of current work assuming (unjustified) universal parameters at the GUT scale (cMSSM)
 - Hard to get viable points (125 GeV Higgs, light stops, dark matter relic density...)
 - Highly fine tuned
 - Some work done for non-universal gauginos [Caron et. al., 1202.5288]
- **We show that it is possible to get plenty of viable points with non-universal $SU(5)$ masses, keeping universal gauginos**
 - $m_{\bar{5}}, m_{10}, m_{\bar{5}'}$ and $m_{5'}$ instead of a single m_0

$SU(5)$ embedding of G_{SM} : The $\bar{\mathbf{5}} \oplus \mathbf{10}$, $\mathbf{5}'$ and $\bar{\mathbf{5}}'$ reps

- The matter content of G_{SM} is unified in a $\bar{\mathbf{5}} \oplus \mathbf{10}$
- The two Higgs $SU(2)$ doublets are unified in a $\mathbf{5}'$ and a $\bar{\mathbf{5}}'$
 - Doublet-triplet splitting problem assumed to be solved by some mechanism (e.g. orbifold compactification) [Kawamura, 0012125]

The $\bar{\mathbf{5}}$ superpartners

$$\bar{\mathbf{5}} \rightarrow (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} = \tilde{L} \oplus \tilde{d}_R^*$$

The $\mathbf{5}'$ Higgs

$$\mathbf{5}' \rightarrow (\mathbf{1}, \mathbf{2})_{\frac{1}{2}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{1}{3}} = H_u \oplus (T_u)$$

The $\mathbf{10}$ superpartners

$$\mathbf{10} \rightarrow (\mathbf{1}, \mathbf{1})_1 \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} = \tilde{e}_R^* \oplus \tilde{u}_R^* \oplus \tilde{Q}_L$$

The $\bar{\mathbf{5}}'$ Higgs

$$\bar{\mathbf{5}}' \rightarrow (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} = H_d \oplus (T_d)$$

$SO(10)$ embedding of G_{SM} : *The 16 and 10 reps*

Maximal subalgebra of $SO(10)$

$$SO(10) \rightarrow SU(5) \otimes U(1)_x$$

16 and 10 branching rules

$$\mathbf{10} \rightarrow \mathbf{5}_2 \oplus \bar{\mathbf{5}}_{-2}$$

$$\mathbf{16} \rightarrow \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{1}_{-5}$$

From the branching rules of $SU(5)$ down to G_{SM} we see that:

- $\mathbf{10}$ contains the $SU(5)$ Higgs doublets and the colored Higgs triplets
- $\mathbf{16}$ contains the full $SU(5)$ superpartners and an extra singlet $\mathbf{1}_5$
- Extra abelian gauge group $U(1)_x$

Right handed neutrino/sneutrino

$$\mathbf{1}_5 \rightarrow (\mathbf{1}, \mathbf{1})_{(0, 5)} = \tilde{N}_R$$

- **A $SO(10)$ GUT naturally contains a right-handed neutrino/sneutrino**

Soft Supersymmetry Breaking Terms

- If SUSY exists it has to be an exact symmetry spontaneously broken (SSB) in a **Hidden sector** [Martin, 9709356]
- Many breaking scenarios proposed
- Parametrize the unknown realistic scenario of SSB
 - Introduce terms that explicitly break supersymmetry
 - Couplings should be of positive mass dimensions \rightarrow **renormalizable theory**, and given at the **low scale**
 - **SOFT TERMS**

Generic soft SUSY Lagrangian

$$\mathcal{L}_{soft} = - \left(\frac{1}{2} M_{ab} \lambda^a \lambda^b + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + t^i \phi_i \right) + h.c. - (m^2)^i_j \phi^{j*} \phi_i$$

Higgs and Sfermion Soft Masses: 1-Loop RGE example

Third Generation and Higgs Soft Masses RGE

$$16\pi^2 \frac{dm_{\tilde{Q}_3}^2}{dt} = X_t + X_b - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{3} g_1^2 S$$

$$16\pi^2 \frac{dm_{\tilde{t}_R}^2}{dt} = 2X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{3} g_1^2 S$$

$$16\pi^2 \frac{dm_{\tilde{b}_R}^2}{dt} = 2X_b - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{3} g_1^2 S$$

$$16\pi^2 \frac{dm_{\tilde{L}_3}^2}{dt} = X_\tau - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S$$

$$16\pi^2 \frac{dm_{\tilde{\tau}_R}^2}{dt} = 2X_\tau - \frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S$$

$$16\pi^2 \frac{dm_{\tilde{N}_3}^2}{dt} = 2X_\nu$$

$$16\pi^2 \frac{dm_{H_d}^2}{dt} = 3X_b + X_\tau - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S$$

$$16\pi^2 \frac{dm_{H_u}^2}{dt} = 3X_t + X_\tau - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{3}{5} g_1^2 S$$

- $X_t = 2y_t^2 (m_{H_u}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{t}_R}^2 + A_t^2)$
- $X_b = 2y_b^2 (m_{H_d}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{b}_R}^2 + A_b^2)$
- $X_\tau = 2y_\tau^2 (m_{H_d}^2 + m_{\tilde{L}_3}^2 + m_{\tilde{\tau}_R}^2 + A_\tau^2)$
- $X_\nu = 2y_\nu^2 (m_{H_u}^2 + m_{\tilde{L}_3}^2 + m_{\tilde{N}_3}^2 + A_\nu^2)$
- $a_i = y_i A_i$

- S is a D-term contribution

- $S \equiv \text{Tr}(Ym^2) = m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{generations}} (m_{\tilde{Q}_L}^2 - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{L}_L}^2 + m_{\tilde{e}_R}^2)$
- $\frac{dS}{dt} = \frac{66}{5} \frac{\alpha_i}{4\pi} S \Rightarrow S(t) = S(t_G) \frac{\alpha_i(t)}{\alpha_i(t_G)}$

Standard CMSSM or mSUGRA Boundary Conditions

- Common scalar mass: $m_{\tilde{Q}_L}^2(t_G) = m_{\tilde{u}_R}^2(t_G) = m_{\tilde{d}_R}^2(t_G) = m_{\tilde{L}_L}^2(t_G) = m_{\tilde{e}_R}^2(t_G) = \mathbf{m_0^2}$
- Common Higgs mass: $m_{H_u}^2(t_G) = m_{H_d}^2(t_G) = (\mathbf{m_0^2} + \mu^2)$
- Since $S(t_G) = 0$, then $S(t)$ is identically 0 at all scales
- Common gaugino mass: $M_1^2(t_G) = M_2^2(t_G) = M_3^2(t_G) = \mathbf{M_{1/2}^2}$
- Common trilinear coupling: $a_t(t_G) = a_b(t_G) = a_\tau(t_G) = \mathbf{a_0}$
- Universal generations: $m_0^{(1,2,3)} = \mathbf{m_0}$
- **Left with 5 unknown parameters: $\tan\beta$, $\text{sign}(\mu)$, $\mathbf{m_0}$, $\mathbf{M_{1/2}}$, $\mathbf{a_0}$**

SU(5) Grand Unification: *Boundary Conditions*

- Why universal soft terms? → **Simple and clean solutions**
- However → **Not clear which mechanism breaks SUSY**
 - Soft parameters highly (SUSY breaking) model dependent
 - **No reason for universal $M_{1/2}$, A_0 , m_0 ...**

Common m_{10} for matter in a **10**

$$m_{\tilde{Q}_L}^2(t_G) = m_{\tilde{u}_R}^2(t_G) = m_{\tilde{e}_R}^2(t_G) = m_{10}^2$$

Common $m_{\bar{5}}$ for matter in a **$\bar{5}$**

$$m_{\tilde{L}_L}^2(t_G) = m_{\tilde{d}_R}^2(t_G) = m_{\bar{5}}^2$$

Higgs soft masses unrelated

$$m_{H_u}^2(t_G) = m_{\bar{5}'}^2 \text{ and } m_{H_d}^2(t_G) = m_{\bar{5}'}^2$$

S-term non zero

$$S(t_G) = m_{\bar{5}'}^2 - m_{\bar{5}}^2$$

Decoupling generations

No reason for universal masses/trilinears within different generations

$$m_{\bar{5}}^{(1,2)} = K_{\bar{5}} m_{\bar{5}}^{(3)}$$

$$m_{10}^{(1,2)} = K_{10} m_{10}^{(3)}$$

$$1 \leq K_{\mathbf{R}} \leq 10$$

Trilinear couplings ($a_i \equiv y_i A_i$)

$$a_u H_u \tilde{u}_R \tilde{Q}_L \xrightarrow{SU(5)} a_{\bar{5}'} \bar{\mathbf{5}}' \cdot \mathbf{10} \cdot \mathbf{10}$$

$$a_d H_d \tilde{d}_R \tilde{Q}_L \xrightarrow{SU(5)} a_{5'} \mathbf{5}' \cdot \bar{\mathbf{5}} \cdot \mathbf{10}$$

$$a_e H_d \tilde{e}_R \tilde{L} \xrightarrow{SU(5)} a_{5'} \mathbf{5}' \cdot \mathbf{10} \cdot \bar{\mathbf{5}}$$

At GUT scale: $a_{u0} = a_{\bar{5}'}$ and
 $a_{d0} = a_{e0} = a_{5'}$

Gauginos (adjoint rep): $\mathcal{L}_{G-K} = -\frac{1}{M_p} F_{ab} \lambda^a \otimes \lambda^b \xrightarrow{\langle F_{ab} \rangle} M_{ab} \lambda^a \lambda^b$

$$24 \otimes 24 = \mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75} \oplus \mathbf{200}$$

Universal $M_{1/2}$ at GUT scale only if $F_{ab} \in \mathbf{1} \rightarrow F_{ab} = F_0 \delta_{ab} \rightarrow \langle F_{ab} \rangle = F_0$

If $F_{ab} \in \mathbf{24}, \mathbf{75}, \mathbf{200}$ or combinations \rightarrow **non-universal gaugino mass**

19 FREE PARAMETERS

$m_{\bar{5}}^i, m_{10}^i, M_{1/2}^\alpha, m_{\bar{5}'}^i, m_{5'}^i, a_{\bar{5}'}^i, a_{5'}^i, \tan \beta, \text{sign}(\mu)$

$\alpha, i = 1, 2, 3$

SU(5) Parameter Space Scan: *Strategy*

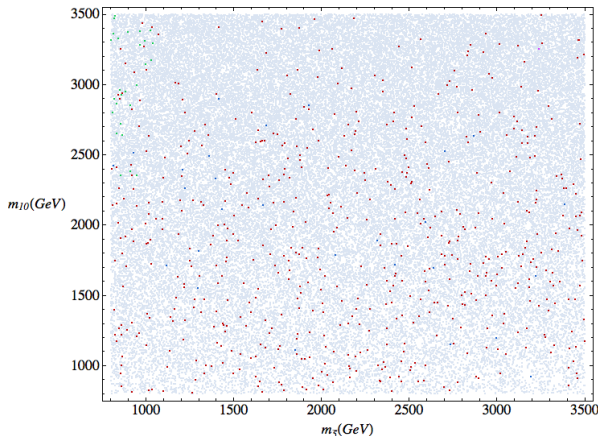
- 1 Universal trilinear couplings \mathbf{a}_0 , universal gauginos $\mathbf{M}_{1/2}$ and universal generations $\mathbf{m}_{\bar{5}}$ and \mathbf{m}_{10}
 - 8 free parameters
- 2 *SU(5)* trilinear couplings $\mathbf{a}_{\bar{5}'}^{\prime}$, $\mathbf{a}_{5'}^{\prime}$, universal gauginos $\mathbf{M}_{1/2}$ and non-universal generations $\mathbf{m}_{\bar{5}}^{(1,2)} = \mathbf{K}_{\bar{5}} \mathbf{m}_{\bar{5}}^{(3)}$, $\mathbf{m}_{10}^{(1,2)} = \mathbf{K}_{10} \mathbf{m}_{10}^{(3)}$
 - $1 \leq K_{\mathbf{R}} \leq 10$
 - 11 free parameters (first and second generation trilinears negligible)

Low Scale Constraints

- Higgs mass: $m_{h^0} \in [122.6, 127.0]$ GeV (ATLAS + CMS)
- First and second generation sfermions: $m_{\tilde{f}_{1,2}} > 1.4$ TeV (ATLAS + CMS)
- Gluino mass: $M_{\tilde{g}} > 800$ GeV (ATLAS + CMS)
- $4 \leq \tan\beta \leq 30$
- $\text{sign}(\mu) = +$
- Dark matter relic density: $0.0941 < \Omega h^2 < 0.131$ (WMAP)

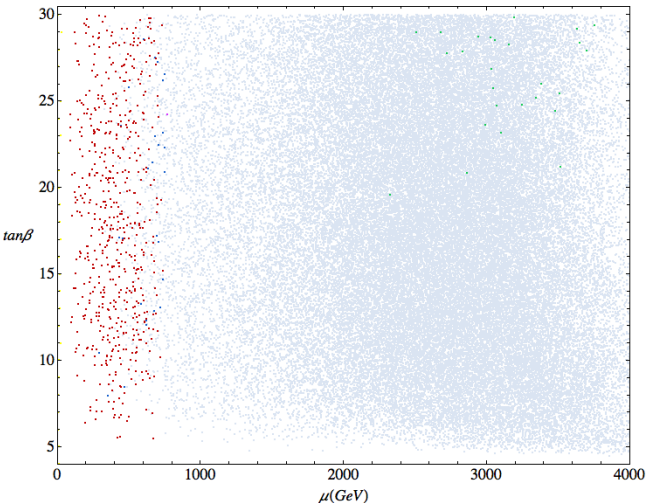
- Spectrum calculator: SOFTSUSY (v3.3.0)
 - 2-loop RGE and 2-loop radiative corrections
- Relic density calculator: micrOMEGAs (v2.4.5)

$(m_{\bar{5}}, m_{10})$ - Plane Scan: Scenario 1



- 1 Blue points: $\tilde{\chi}_1^0$ DM within RD bounds
- 2 Red points: $\tilde{\chi}_1^0$ DM with RD below bounds
- 3 Light blue points: $\tilde{\chi}_1^0$ DM with RD above bounds (ruled out)

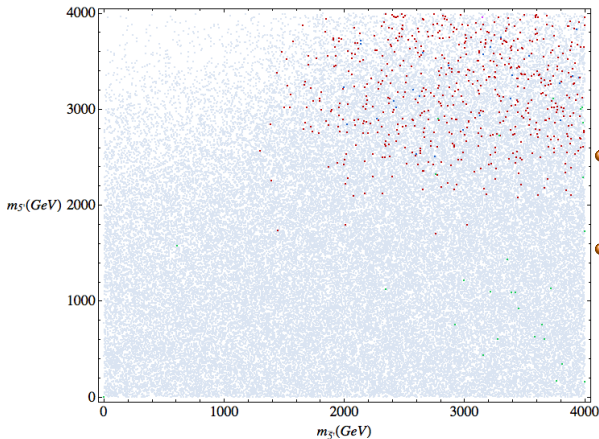
$(\mu, \tan\beta)$ - Plane Scan: Scenario 1



- Small $\mu \rightarrow$ Higgsino dominated DM
- Large $\mu \rightarrow$ Bino dominated DM

$(m_{\bar{5}'}, m_{5'})$ - Plane Scan: Scenario 1

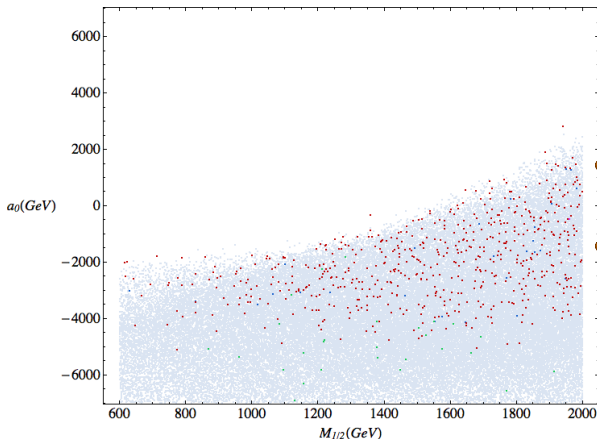
Note first that $\mu^2 = \frac{1}{2} \left[\tan 2\beta \left(m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta \right) - M_Z^2 \right]$



- $(m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta) \ll 0 \Rightarrow$
large μ
- $(m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta) < 0 \Rightarrow$
small μ

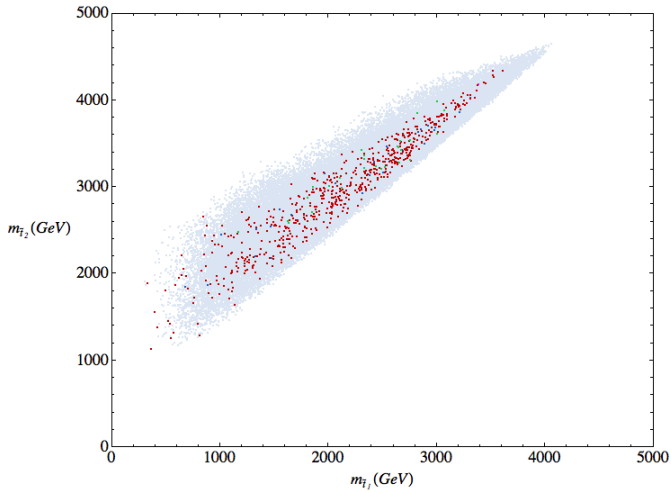
$(M_{1/2}, a_0)$ - Plane Scan: Scenario 1

One loop correction to m_{h^0} has the contribution $2 \log \frac{m_{\tilde{t}}^2}{m_{\tilde{\tau}}^2} + \frac{X_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{X_t^2}{12m_{\tilde{t}}^2} \right)$, $X_t = A_t - \mu \cot \beta$

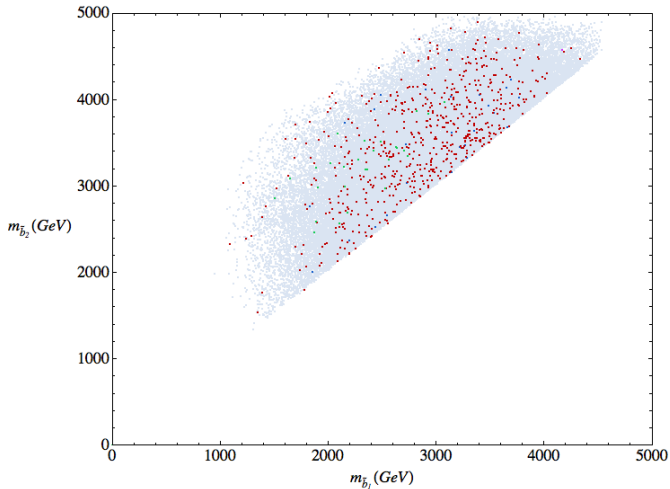


- Large X_t if $A_t \ll 0$ (favours 125 GeV h^0)
- Larger $m_{\tilde{t}}$ for larger $M_{1/2}$ (favours 125 GeV h^0)

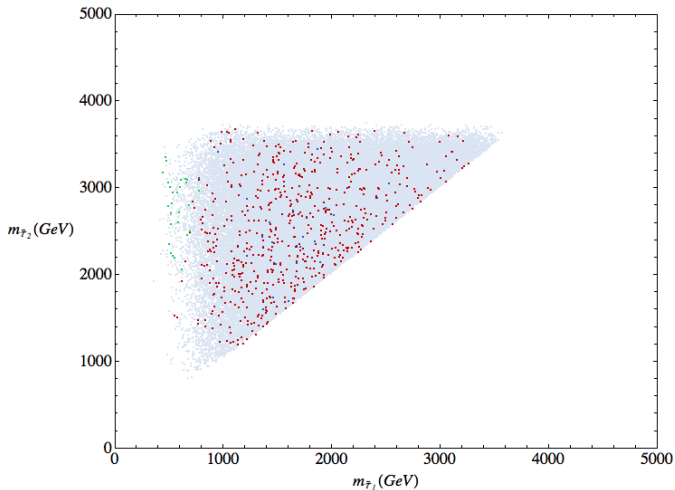
Stops: Scenario 1



Sbottoms: Scenario 1

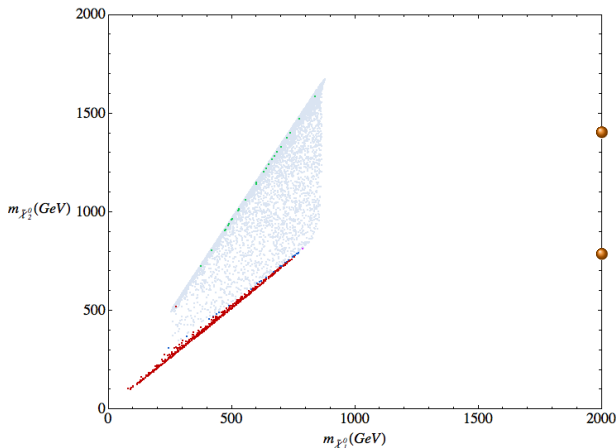


Staus: Scenario 1



Neutralinos: Scenario 1

For universal gauginos, EW scale relation: $M_3 : M_2 : M_1 = 6 : 2 : 1$



● Higgsino dominated DM:

● $\tilde{\chi}_{1,2}^0 \approx (\tilde{H}_u^0 \pm \tilde{H}_d^0) / \sqrt{2}$

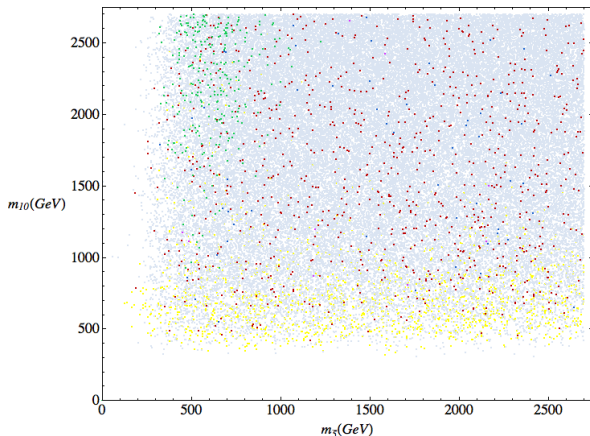
● $M_{\tilde{\chi}_1^0} \approx M_{\tilde{\chi}_2^0}$

● Bino dominated DM

● $\tilde{\chi}_1^0 \approx \tilde{B}, \tilde{\chi}_2^0 \approx \tilde{W}^0$

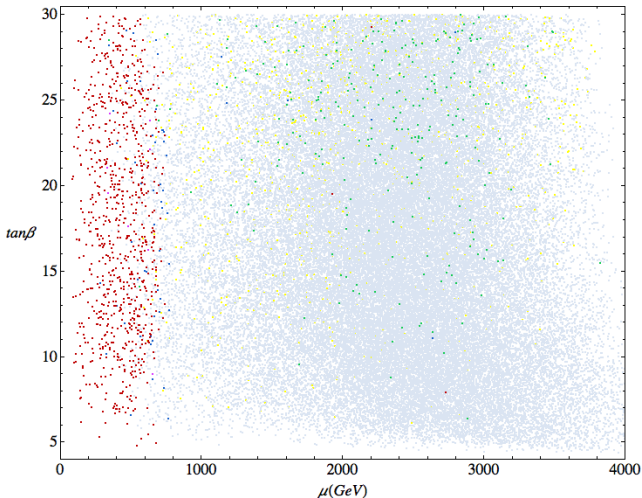
● $M_{\tilde{\chi}_1^0} \approx \frac{1}{2} M_{\tilde{\chi}_2^0}$

$(m_{\bar{5}}, m_{10})$ - Plane Scan: Scenario 2



- Yellow points: Charged LSP (ruled out)
- $\tilde{L}_3 \subset \bar{5} \Rightarrow \tilde{\nu}_L$ LSP when $m_{\bar{5}} \rightarrow$ small
- m_{10} small favours charged LSP, $\tilde{\tau}_1 (\approx \tilde{\tau}_R)$
- White region \rightarrow excluded by LHC

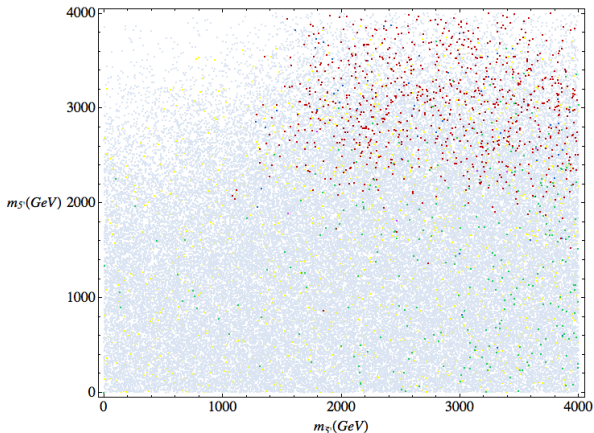
$(\mu, \tan\beta)$ - Plane Scan: Scenario 2



- Small $\mu \rightarrow$ Higgsino dominated DM
- Large $\mu \rightarrow$ Bino dominated DM

$(m_{\bar{5}'}, m_{5'})$ - Plane Scan: Scenario 2

Note first that $\mu^2 = \frac{1}{2} \left[\tan 2\beta \left(m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta \right) - M_Z^2 \right]$

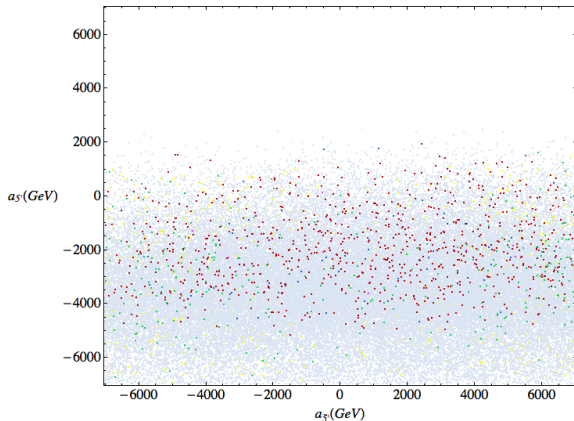


● $\left(m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta \right) \ll 0 \Rightarrow$
 large μ

● $\left(m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta \right) < 0 \Rightarrow$
 small μ

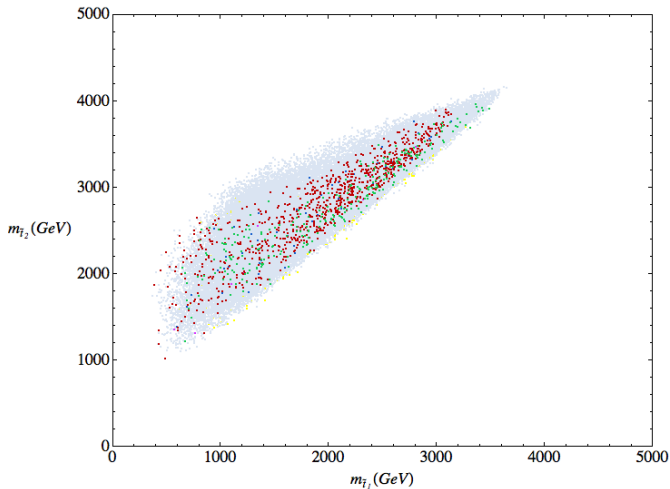
$(a_{\bar{5}'}, a_{5'})$ - Plane Scan: Scenario 2

One loop correction to m_{h^0} has the contribution $2 \log \frac{m_t^2}{m_{\bar{t}}^2} + \frac{X_t^2}{m_{\bar{t}}^2} \left(1 - \frac{X_t^2}{12m_{\bar{t}}^2} \right)$, $X_t = A_t - \mu \cot \beta$

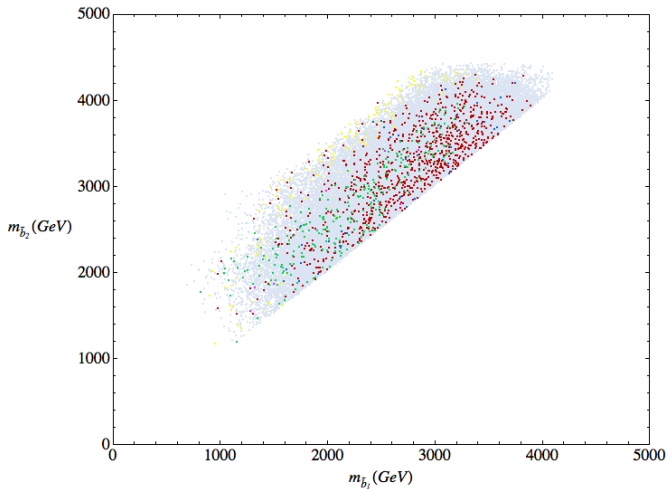


- Large X_t if $A_t \ll 0$ (favours 125 GeV h^0)

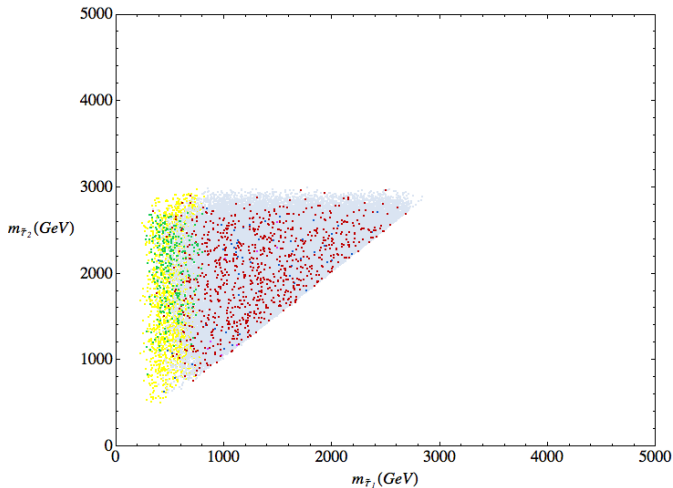
Stops: Scenario 2



Sbottoms: Scenario 2

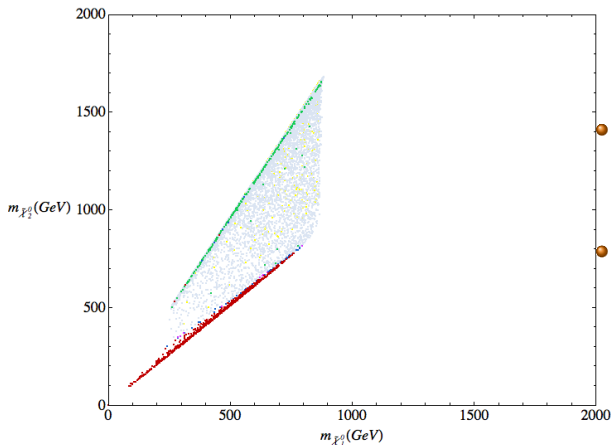


Staus: Scenario 2



Neutralinos: Scenario 2

For universal gauginos, EW scale relation: $M_3 : M_2 : M_1 = 6 : 2 : 1$



● Higgsino dominated DM:

- $\tilde{\chi}_{1,2}^0 \approx (\tilde{H}_u^0 \pm \tilde{H}_d^0) / \sqrt{2}$
- $M_{\tilde{\chi}_1^0} \approx M_{\tilde{\chi}_2^0}$

● Bino dominated DM

- $\tilde{\chi}_1^0 \approx \tilde{B}, \tilde{\chi}_2^0 \approx \tilde{W}^0$
- $M_{\tilde{\chi}_1^0} \approx \frac{1}{2} M_{\tilde{\chi}_2^0}$

$SO(10)$ Grand Unification: *Boundary Conditions*

WORK IN PROGRESS...

- Breaking $SO(10) \rightarrow SU(5) \otimes U(1)_x \rightarrow G_{SM}$ the rank is reduced from 5 to 4
 - D-term contributions from the additional $U(1)_x$ of the form $\Delta m_a^2 = -\sum_k Q_{ka} g_k^2 D_k$
 [Kolda and Martin, 9503445]

- Consider that the Higgs are embedded in a **10** of $SO(10)$

Common sfermion mass m_{16}

$$m_{\tilde{Q}_L}^2(t_G) = m_{\tilde{u}_R}^2(t_G) = m_{\tilde{e}_R}^2(t_G) = m_{16}^2 + g_{10}^2 D$$

$$m_{\tilde{L}_L}^2(t_G) = m_{\tilde{d}_R}^2(t_G) = m_{16}^2 - 3g_{10}^2 D$$

$$m_{\tilde{N}_e}^2(t_G) = m_{16}^2 + 5g_{10}^2 D$$

- $S(t_G) = -4g_{10}^2 D$

Common Higgs mass m_{10}

$$m_{\tilde{H}_u}^2(t_G) = m_{10}^2 - 2g_{10}^2 D$$

$$m_{\tilde{H}_d}^2(t_G) = m_{10}^2 + 2g_{10}^2 D$$

Decoupling generations

$$m_{16}^{(1,2)} = K_{16} m_{16}^{(3)}$$

$$1 \leq K_{\mathbf{R}} \leq 10$$

Trilinear couplings unified ($a_i \equiv y_i A_i$)

$$a_u H_u \tilde{u}_R \tilde{Q}_L \xrightarrow{SO(10)} a_{10} \mathbf{10} \cdot \mathbf{16}_R \cdot \mathbf{16}_L$$

$$a_d H_d \tilde{d}_R \tilde{Q}_L \xrightarrow{SO(10)} a_{10} \mathbf{10} \cdot \mathbf{16}_R \cdot \mathbf{16}_L$$

$$a_e H_d \tilde{e}_R \tilde{L} \xrightarrow{SO(10)} a_{10} \mathbf{10} \cdot \mathbf{16}_R \cdot \mathbf{16}_L$$

Gauginos (adjoint rep): $\mathcal{L}_{G-K} = -\frac{1}{M_p} F_{ab} \lambda^a \otimes \lambda^b \xrightarrow{\langle F_{ab} \rangle} M_{ab} \lambda^a \lambda^b$

$$45 \otimes 45 = \mathbf{1} \oplus \mathbf{54} \oplus \mathbf{210} \oplus \mathbf{770}$$

Universal $M_{1/2}$ at GUT scale only if $F_{ab} \in \mathbf{1} \rightarrow F_{ab} = F_0 \delta_{ab} \rightarrow \langle F_{ab} \rangle = F_0$
 If $F_{ab} \in \mathbf{54}, \mathbf{210}, \mathbf{770}$ or combinations \rightarrow **non-universal gaugino mass**

13 FREE PARAMETERS

m_{16}^i , $g_{10}^2 \mathbf{D}$, $M_{1/2}^\alpha$, m_{10} , a_{10}^i , $\tan \beta$, $\text{sign}(\mu)$

$\alpha, i = 1, 2, 3$

Summary and Conclusions

- Review of the $SU(5)$ and $SO(10)$ G_{SM} embedding
- Studied the $SU(5)$ and $SO(10)$ boundary conditions
- Scan on the $SU(5)$ parameter space
 - LHC bounds and Dark Matter constrains
- **Plenty of viable points for $100 \leq \mu \leq 800$ GeV (Higgsino dominated DM)**
- Fine tuning estimate: [Athron, Miller, 0705.2241]
 - Scenario 1: $\Delta_1 \approx 106.1$
 - Scenario 2: $\Delta_2 \approx 70.7$ (improved)
- **Non universality of the scalar masses favours good points**
- Relaxing further constraints, eg. gaugino masses, might improve further (work in progress...)