Transverse-momentum resummation for top-quark pairs in hadron colliders

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The top quark

- The particle in the SM that has the largest mass
- plays special role in many new physics model
- large production cross sections at the LHC; background of many new physics signal

Precision predictions for $t\bar{t}$ production

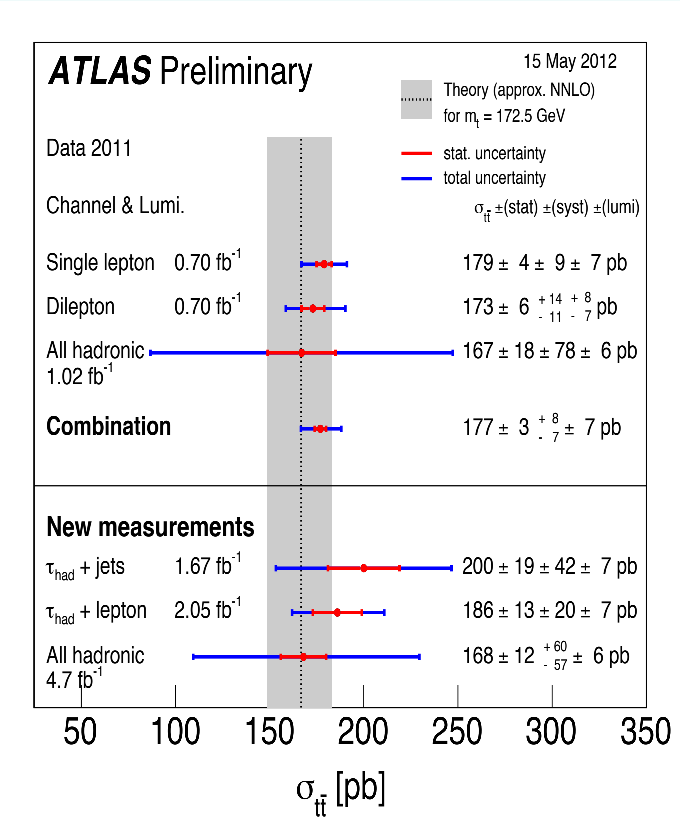
Precision calculation for $\sigma_{t\bar{t}}$ has been an active field of research for many years:

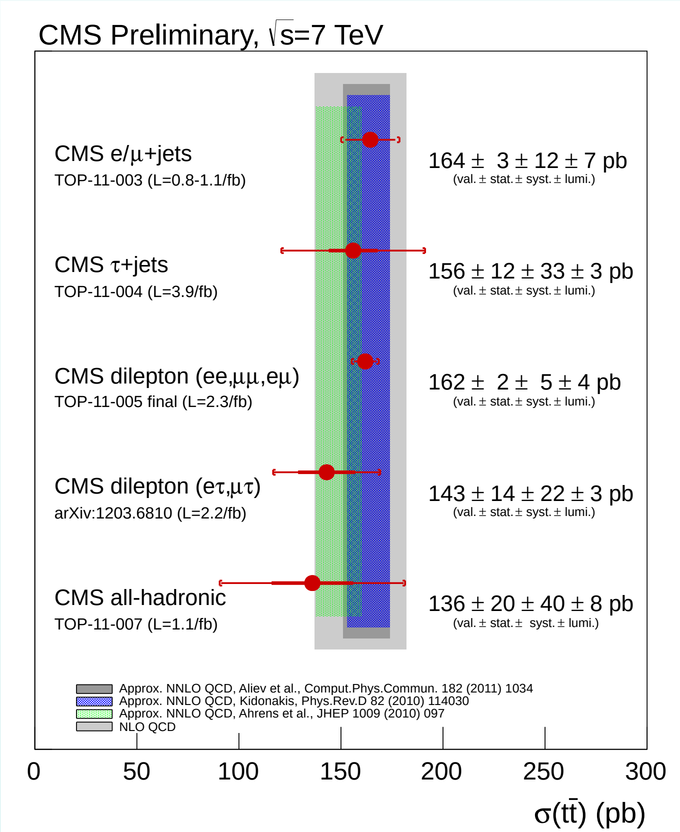
- NLO Nason, Dawson, Ellis, 89'; Beenakker et al, 89'
- EW Beenakker et al, 93'; Bernreuther, Fuecker, Si, 06'; Hollik, Kollar, 07'; Kuhn, Scharf, Uwer, 07'
- NLO spin corr. Bernreuther et al, 04'; Melnikov, Schulze, 09'
- Off-shell effects Denner et al, 10'; Bevilacqua et al, 10'
- NLL threshold resummation Sterman, Kidonakis, 97'; Bonciani, 98
- NNLL threshold resummation Beneke, Falgari, Schwinn, 10'; Czakon, Mitov, Sterman, 10'; Ahrens et al, 10-11'
- Now entering NNLO era Barnreuther, Czakon, Mitov, 12'

–see Si's talk in this conference

Experimental status for top-quark pair

$\sigma_{\rm tot}$ has become a precision observable:

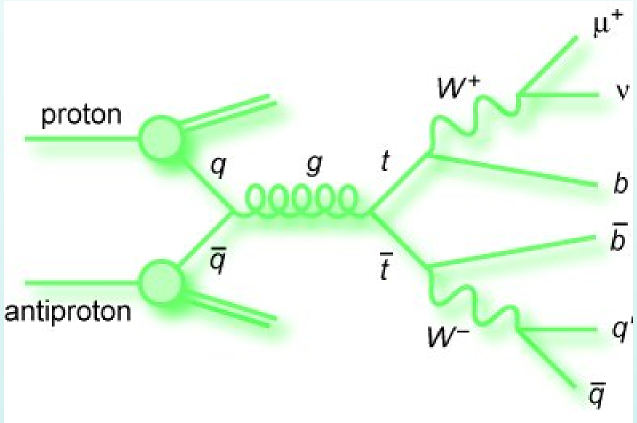




Differential cross sections are now being measured by ATLAS (1207.5644) and CMS (CMS PAS TOP-11-013)

Transverse momentum of top-quark pairs

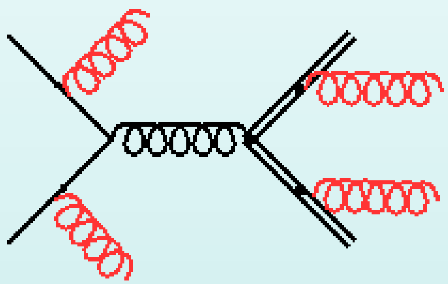
- Experimentally top quark is observed by reconstructing from its decay product



- When includeing top decay, $P_{T,t\bar{t}}$ is unambiguous only if detailed experimental set up for extracting $P_{T,t\bar{t}}$ is specified (Melnikov, Scharf, Schulze, 11')
- To make our lives easier, we consider in this talk the $P_{T,t\bar{t}}$ of stable top-quark pairs

Transverse momentum of top-quark pairs

- At LO, top-quark pairs are produced at exactly zero transverse momentum, $P_{T,t\bar{t}} = 0$.
- Non-trivial $P_{T,t\bar{t}}$ distribution first arises at NLO in α_s:



- The dominant contribution to the cross sections come from small $P_{T,t\bar{t}}$:
  - LO and NLO virtual corrections:

$$\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}P_{T,t\bar{t}}} \sim \delta(P_{T,t\bar{t}}), \quad \frac{\mathrm{d}\sigma^{\mathrm{NLOv}}}{\mathrm{d}P_{T,t\bar{t}}} \sim \delta(P_{T,t\bar{t}})$$

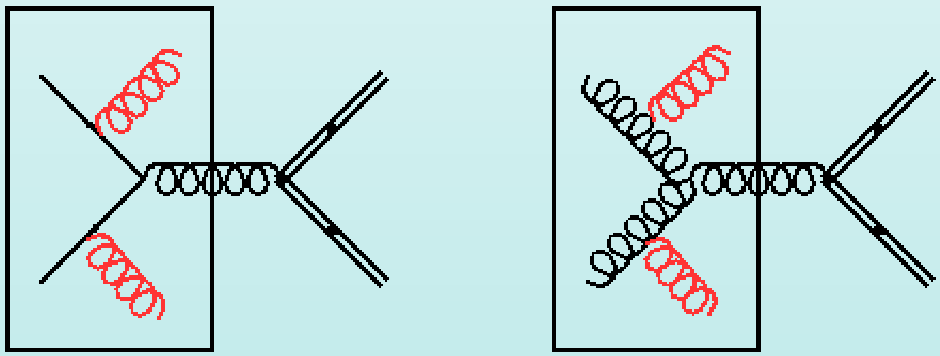
– NLO real corrections:

$$\left.\frac{\mathrm{d}\sigma^{\mathrm{NLOr}}}{\mathrm{d}P_{T,t\bar{t}}}\right|_{P_{T,t\bar{t}}>0} \sim A\frac{\ln\frac{P_{T,t\bar{t}}}{M}}{P_{T,t\bar{t}}} + B\frac{1}{P_{T,t\bar{t}}}$$

– Real corrections are singular when $P_{T,t\bar{t}} \to 0$, a universal property of QCD at soft/collinear limit

– Large logs of $P_{T,t\bar{t}}$ need to be resumed for reliable prediction

– Let's assume for the moment that final-state top quarks do not emit gluon



$+\ t$ and u channel

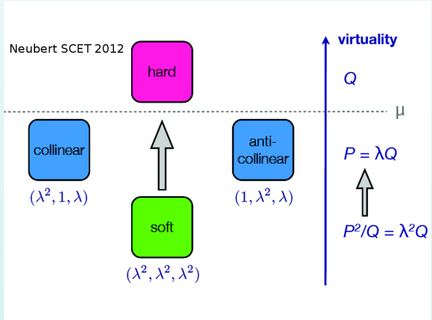
– Then the resummation of $P_{T,t\bar{t}}$ logs resembles q_T resummation of Drell-Yan/Higgs production

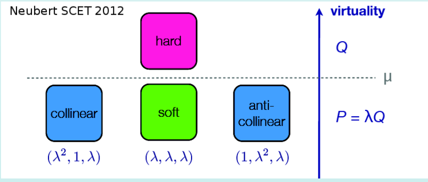
q_T resummation for Drell-Yan and Higgs production

- Early days: Parisi, Petronzio, 79'; Dokshitzer, Dyakonov, Troyan, 80'
- CSS formalism: Collins, Soper, Sterman, 85'
  - Many accurate predictions based on this formalism: RESBOS, C.-P. Yuan et al; HqT, Bozzi et al
- More recently an effective field theory assault on this problem: Y. Gao, C.S.Li, J.J. Liu 05'; Idilbi, Ji, Yuan, 05'; Mantry, Petriello, 09'; Becher, Neubert, 09'; Garcia-Echevarria, Ahmad Idilbi, Ignazio Scimemi, 11'; Chiu et al, 12';
- We use the formalism developed in Becher, Neubert, Wilhelm, 1109.6027 to resum the $P_{T,t\bar{t}}$ logs associated with initial state radiation

q_T resummation in Soft-Collinear Effective Theory

- Resummation in SCET relies on the separation of dynamical D.O.F.
  - In threshold/jet mass/... resummation, different modes are naturally connected by RGE
  - In q_T/jet broadening/... resummation, connections of different modes are not obvious





- Problem solved by understanding collinear anomaly (Becher, Neubert, 1007.4005), or equivalently rapidity RGE (Chiu et al, 1202.0814)

Transverse momentum of $t\bar{t}$ pairs

- The resummation $P_{T,t\bar{t}}$ logs induced by initial state radiations is similar to DY/Higgs production
- However top quark carries color charge and can emit gluons, which also leads to non-trivial $P_{T,t\bar{t}}$ distribution
- Correct resummation of $P_{T,t\bar{t}}$ logs require consistent treatment of final state radiation
- To produce singular $P_{T,t\bar{t}}$ distribution, final state radiations have to be soft; hard raditions are suppressed by power of $\frac{P_{T,t\bar{t}}}{M}$
- In SCET soft radiations are described by vacuum expectation value of soft Wilson loops

The soft function

The soft function describe the emission of soft gluons in the eikonal approximation:

$$\boldsymbol{S}_R(\hat{s}s, \hat{t}, m_t^2, L_\perp) = \int \frac{\mathrm{d}\phi_3}{2\pi} \, \mathrm{d}^2\boldsymbol{q}_\perp \, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_\perp}$$

$$\times \frac{1}{d_R} \sum_{X_s} \left| \langle 0 | (Y_{v_1}(0) Y^\dagger_{v_2}(0) Y^\dagger_{v_3}(0) Y_{v_4}(0)) | X_s \rangle \right|^2 \delta^{(2)}(\boldsymbol{q}_\perp - \hat{\boldsymbol{P}}_{\perp, X_s})$$

$$Y_v(x) = \boldsymbol{P} \exp\left[\int_{-\infty}^{x} \mathrm{d}\lambda \, v \cdot \boldsymbol{A}_s(\lambda v) \right]$$

The $t\bar{t}$ soft function are matrices in color space. For example, when singlet-octet basis is chosen, the $q\bar{q}$-channel soft function at LO is

$$\boldsymbol{S}^{(0)}_{q\bar{q}} = \begin{pmatrix} N_c & 0 \\ 0 & \frac{C_F}{2} \end{pmatrix}$$

Calculating the soft function

The soft function at 1-loop can be written as a combination of color factor and soft integral

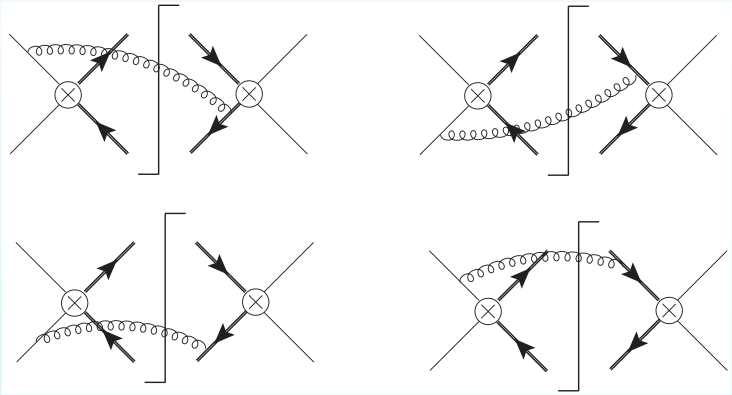
$$\boldsymbol{S}^{(1)}_{R,bare} = \sum_{ij} \boldsymbol{w}^{ij}_{R} I_{ij}(\boldsymbol{b}, \mu)$$

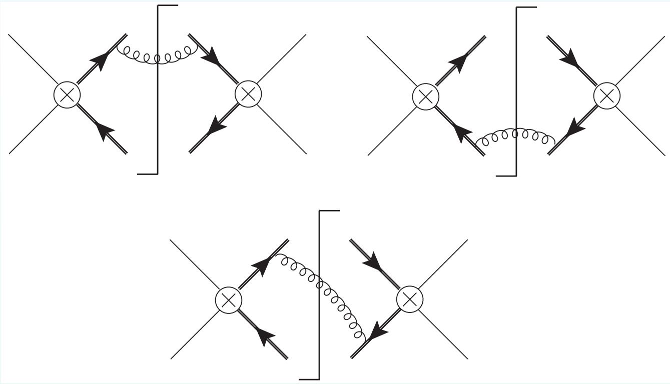
The soft integral is not completely regularized in dimensional regularization. Rapidity divergences show up in some of the integral and extra regulator is need to regulate the integral. We use the regulator propose by Becher, Bell, 11', which is very convenient for actual calculation.

$$\begin{aligned} I_{ij}(\boldsymbol{b}, \mu) \quad &= \quad -\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \frac{1}{(2\pi)^{-2\epsilon}\pi^2} \int_0^{2\pi} \frac{\mathrm{d}\phi_3}{2\pi} \int \mathrm{d}^D k \\ &\quad \times \left(\frac{\nu}{k^+}\right)^{\alpha} (2\pi)\delta(k^2)\Theta(k^0) \frac{v_i \cdot v_j e^{i\boldsymbol{b}\cdot\boldsymbol{k}_\perp}}{v_i \cdot k \, v_j \cdot k} \end{aligned}$$

Results of 1-loop soft integral

The non-vanishing diagrams:





- The soft function satisfies the RGE:

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}\ln\mu} = \boldsymbol{\gamma}_s^\dagger \boldsymbol{S} + \boldsymbol{S}\boldsymbol{\gamma}_s$$

  We check this equation at 1-loop by explicit computation.

- A very interesting property of this soft function: at 1-loop it's completely determined by the RGE. *i.e.*, The undetermined integral constant of the RGE vanishes!

The $P_{T,t\bar{t}}$ distribution in a factorized form

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}P_{T,t\bar{t}}^2\mathrm{d}M^2\mathrm{d}Y\,\mathrm{d}\cos\theta} = \frac{4\pi\beta_t}{3SM}\sum_{i,j=q,\bar{q},g}\int_{\xi_1}^{1}\frac{\mathrm{d}z_1}{z_1}\int_{\xi_2}^{1}\frac{\mathrm{d}z_2}{z_2}$$
$$\times f_{i/N_1}\left(\frac{\xi_1}{z_1},\mu\right) f_{j/N_2}\left(\frac{\xi_2}{z_2},\mu\right)\Theta(z_1z_2S-M^2)$$
$$\times C_{R\leftarrow ij}(z_1,z_2,P_{T,t\bar{t}}^2,\hat{s},\hat{t},m_t^2,\mu)$$

For $q\bar{q}$-channel

$$C_{q\bar{q}\leftarrow ij} = \frac{1}{2}\int_0^\infty \mathrm{d}b\,bJ_0(bP_{T,t\bar{t}})$$
$$\times\exp\left[g_F(\eta,L_\perp,\alpha_s)\right]\bar{I}_{q\leftarrow i}(z_1,L_\perp,\alpha_s)\bar{I}_{\bar{q}\leftarrow j}(z_2,L_\perp,\alpha_s)$$
$$\times\mathrm{Tr}\Big[\boldsymbol{H}_{q\bar{q}}(\hat{s},\hat{t},m_t^2,\mu)\boldsymbol{S}_{q\bar{q}}(\hat{s},\hat{t},m_t^2,L_\perp)\Big]$$

NLO singular $P_{T,t\bar{t}}$ distribution from resummation

- With the factorized cross section and 1-loop beam and soft function, we can easily derive the singular $P_{T,t\bar{t}}$ distribution at NLO:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}P_{T,t\bar{t}}^2\mathrm{d}M^2\mathrm{d}Y\,\mathrm{d}\cos\theta} = \frac{4\pi\beta_t}{3SMd_R}\sum_{i,j=q,\bar{q},g}\frac{\alpha_s}{2\pi}\left\{\mathrm{Tr}\left[\boldsymbol{H}_R^{(0)}\left(\boldsymbol{A}_R\ln\frac{M^2}{P_{T,t\bar{t}}^2}+\boldsymbol{B}_R\right)\right]f_{i/N_1}(\xi_1)f_{j/N_2}(\xi_2)\right.$$

$$\left.+\mathrm{Tr}[\boldsymbol{H}_R^{(0)}\boldsymbol{S}_R^{(0)}]\left[\left(P_{i\leftarrow a}^{(1)}\otimes f_{a/N_1}\right)(\xi_1)f_{j/N_2}(\xi_2)+f_{i/N_1}(\xi_1)\left(P_{j\leftarrow b}^{(1)}\otimes f_{b/N_2}\right)(\xi_2)\right]\right\}\frac{1}{P_{T,t\bar{t}}^2}$$

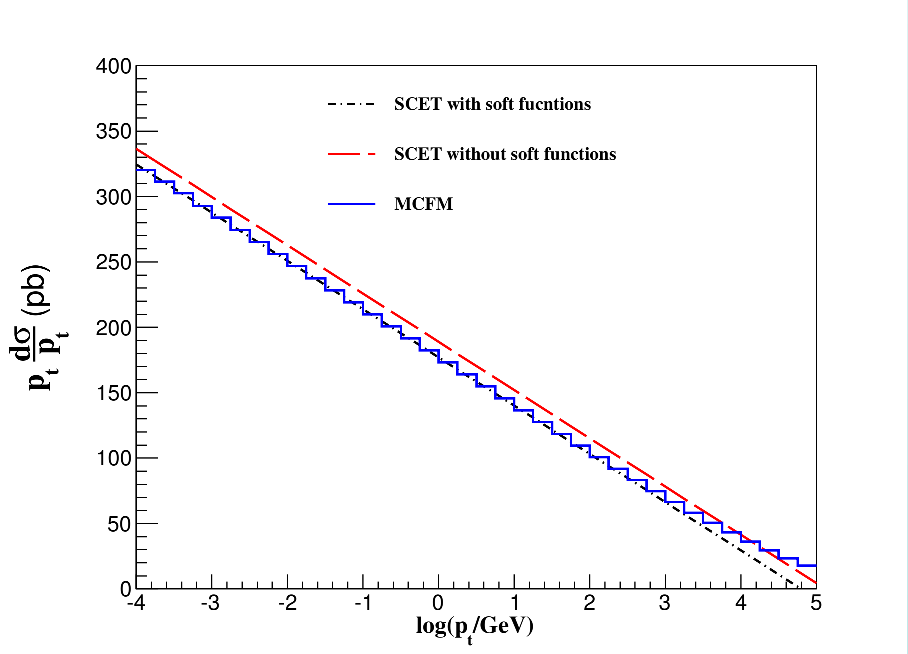
$$\boldsymbol{A}_R = \frac{1}{2}\Gamma_0^R\boldsymbol{S}^{(0)},$$

$$\boldsymbol{B}_R = \gamma_0^R\boldsymbol{S}_R^{(0)} - 2C_F\boldsymbol{S}_R^{(0)} - 2\left(\frac{1+x_s^2}{x_s^2-1}\ln x_s\,\boldsymbol{w}_R^{34} + 2\ln\frac{m_t^2-\hat{t}}{m_tM}\,\boldsymbol{w}_R^{24} + 2\ln\frac{m_t^2-\hat{u}}{m_tM}\,\boldsymbol{w}_R^{23}\right),$$

- Have check the results using MCFM and find complete agreement
- But our results disagree with an old calculation by Berger, Meng, 93'; Mrenna, Yuan, 96'

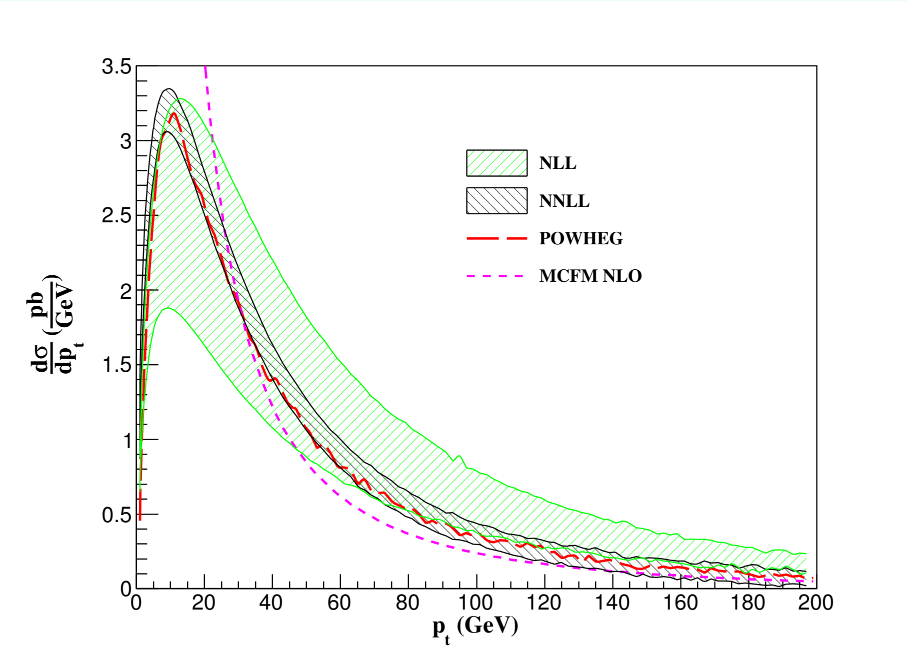
SCET NLO singular distribution v.s. MCFM

- Excellent agreement between SCET NLO and MCFM only when including the soft function!
- A: slope
- B: intercept



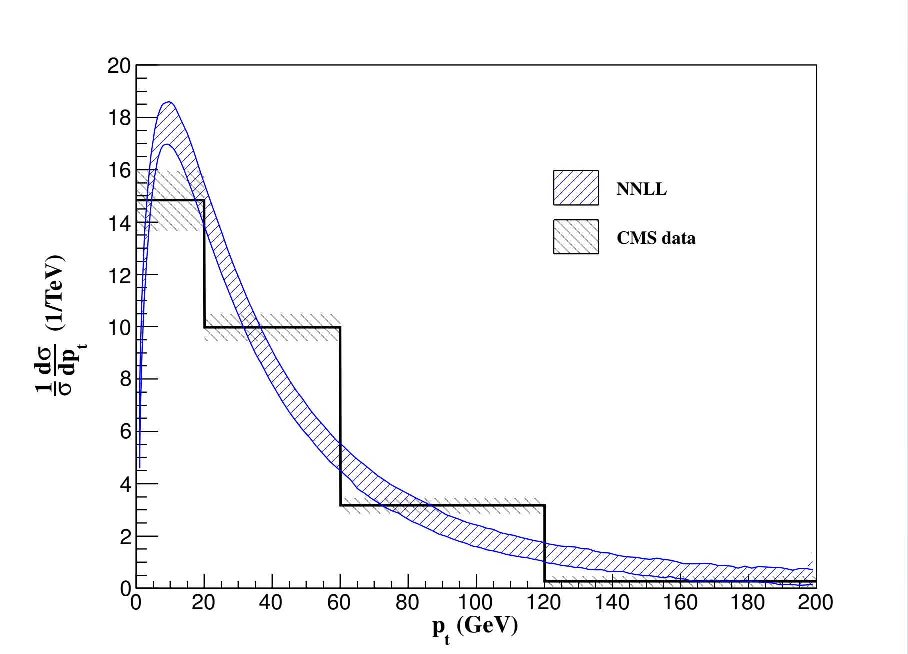
The resummation distribution

- Scale setting: $\mu = q^* + P_{T,t\bar{t}}$, $\mu_h = m_t$, $\mu_s = \frac{2e^{-\gamma_E}}{b}$
- Theoretical uncertainty estimated by varying μ and μ_h by a factor of 2 independently
- Significant reduction of scale dependence from NLL to NNLL



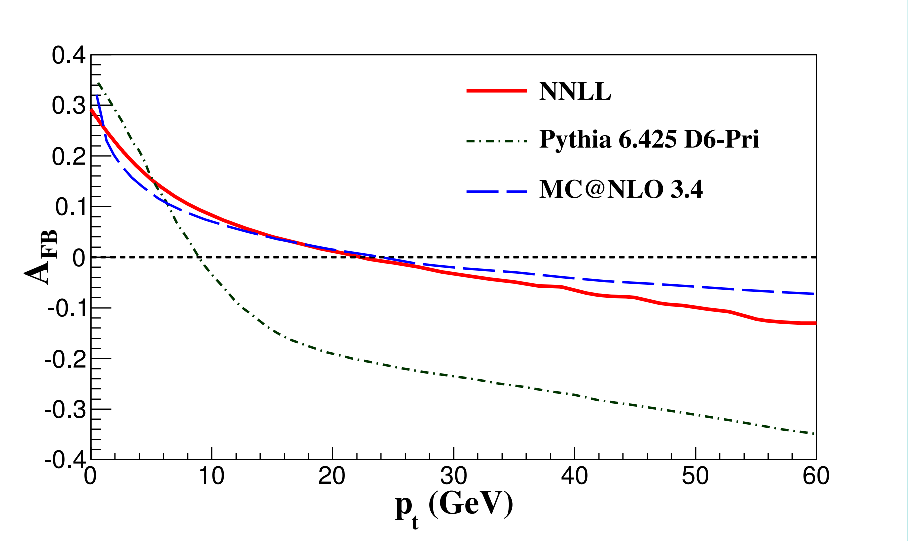
Compare to experimental data

- The $t\bar{t}$ transverse momentum distribution has been measured by DØ, CMS and ATLAS recently
- An arena for precision test of perturbative QCD
- Still too large bin size, but good agreement between data and theory is visible



top quark charge asymmetry

- $A_{\rm FB}$ is sensitive to QCD radiation at small p_T
- Due to color coherence, even a LO event generator could produce non-zero $A_{\rm FB}$ (Skands, Webber, Winter, 12')
- Part of the charge asymmetry is encoded in the soft function, which have asymmetric dependence on $\hat{t}$ and $\hat{u}$
- Very good agreement with MC@NLO, in particular the same cross-over



Summary

- We have developed a formalism to resum transverse momentum logarithms in $t\bar{t}$ production (stable top) using SCET, built upon the q_T resummation for Drell-Yan and Higgs production
- Explicit confirmation of the NLO singular terms by `MCFM`
- Resummation to NNLL; significant scale reduction from NLL to NNLL; good agreement between theory and experiment
- Have calculated the transverse momentum dependent A_{FB}, in good agreement with `MC@NLO`
- We now have full control of $t\bar{t}$ production at small $P_{T,t\bar{t}}$ up to NNLO; following the spirit of `HNNLO` (Catani, Grazzini, 07'), an independent method to compute the completely differential NNLO QCD corrections to $t\bar{t}$ production is at hand

Summary

$$\sigma_{2\to 2}^{\mathrm{NNLO}} = \sigma_{2\to 2}^{\mathrm{NNLO}}\Theta(\Delta - P_{T,t\bar{t}}) + \sigma_{2\to 3}^{\mathrm{NLO}}\Theta(P_{T,t\bar{t}} - \Delta)$$

- $\sigma_{2\to 2}^{\mathrm{NNLO}}\Theta(\Delta - P_{T,t\bar{t}})$: 2-loop hard function, beam function, and soft function
- $\sigma_{2\to 3}^{\mathrm{NLO}}\Theta(P_{T,t\bar{t}}-\Delta)$: NLO corrections to $t\bar{t}$ + jet production (Dittmaier, Uwer, Weinzierl, 07')
- Since the importance of the results of $t\bar{t}$ production at NNLO (Barnreuther, Czakon, Mitov, 12'), it's necessary to have an independent check of their results

Thank you!