Supersymmetric DBI Inflation

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Introduction

Dirac-Born-Infeld (DBI) inflation

Inflaton = transverse fluctuations of D-branes – DBI inflation

Simple DBI inflation

- Large non-Gaussianity of primordial curvature perturbation by ultra-relativistic motion [Silverstein-Tong (2003)], [Alishahiha-Silverstein-Tong (2004)]
- Difficulty to produce detectable tensor-perturbations
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High energy particle physics – supersymmetric world

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High energy particle physics – supersymmetric world

Supersymmetric DBI inflation is interesting subject

Setup

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1. Appropriate holonomy group in six dim. – SUSY in four dimensions
2. Appropriate warp factor – higher derivative corrections even for zero-slope limit $\alpha' \to 0$
3. Large internal volume with throat – Generalized Lyth bound
4. Geometry has a $U(1)$ internal isometry

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$U(1)$ isometry $= U(1)_R$ symmetry in world-volume field theory

$\longrightarrow$ one of two inflatons are gauged away – single inflation

A typical example – near horizon limit of N coincident D3-branes

$AdS_5 \times S^5$ in CY_3

Supersymmetry

$\mathcal{N} = 1$ supersymmetric D3-brane

Supersymmetric DBI action in flat target space [Rocek-Tseytlin (1998)]

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SUSY DBI action in *warped geometry* [SS-Yamaguchi-Yokoyama]

$$\mathcal{L}_{\mathrm{DBI}} = \int d^4\theta \left[\Phi\Phi^\dagger + \frac{1}{16T} (D^\alpha \Phi D_\alpha \Phi) \left(\overline{D}^{\dot{\alpha}} \Phi^\dagger \overline{D}_{\dot{\alpha}} \Phi^\dagger \right) \frac{1}{1 + A + \sqrt{(1+A)^2 - B}} \right].$$

$$A \equiv \frac{\partial_\mu \Phi \partial^\mu \Phi^\dagger}{T}, \quad B \equiv \frac{\partial_\mu \Phi \partial^\mu \Phi \partial_\nu \Phi^\dagger \partial^\nu \Phi^\dagger}{T^2}, \quad T = H^{-1}(\Phi, \Phi^\dagger).$$

Here $2\pi\alpha' = g_s = 1$

Chiral multiplet:

$$\Phi = \varphi + \theta\psi + \theta^2 F.$$

Introduction of superpotential

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Superpotential by small R-R flux

$$\mathcal{L}_{\rm pot} = \int d^2\theta \; W(\Phi) + \text{h.c.}.$$

R-R fluxes induce superpotentials in Myers term
[Billo-Frerro-Frau-Fucito-Lerda-Morales (2008), Ito-Nakajima-Sasaki (2007)]

Small fluxes – no back reaction on the geometry

Auxiliary field

Elimination of the auxiliary field F $\longrightarrow$ on-shell bosonic action

Equation of motion for the auxiliary field F

$$2G(\varphi)\frac{\partial W}{\partial \varphi}F^3 + \frac{\partial \overline{W}}{\partial \overline{\varphi}}\left(1 - 2G(\varphi)\partial_\mu \varphi \partial^\mu \overline{\varphi}\right)F + \left(\frac{\partial \overline{W}}{\partial \overline{\varphi}}\right)^2 = 0.$$

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Cubic equation – solutions are given by Cardano's method

We can find exact solutions

Three branches

Analytic solutions [SS-Yamaguchi-Yokoyama (2012)]

$$F = \omega^k \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \omega^{3-k} \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}.$$

Here ω is the complex cubic root, $k = 0, 1, 2,$

$$p = \left(\frac{\partial W}{\partial \varphi}\right)^{-1} \frac{\partial \overline{W}}{\partial \overline{\varphi}} \frac{1 - 2G\partial_\mu \varphi \partial^\mu \overline{\varphi}}{2G}, \quad q = \frac{1}{2G} \left(\frac{\partial W}{\partial \varphi}\right)^{-1} \left(\frac{\partial \overline{W}}{\partial \overline{\varphi}}\right)^2.$$

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$\longrightarrow$ We consider the branch $k = 0$ which has smooth canonical limit

Other branches are discussed recently [Koehn-Lehners-Ovrut (2012)]

Inflation

Inflaton dynamics

Dropping the angular direction by $U(1)_R$:

$$2GF^3 + (1 + 2GX)\,F + \sqrt{2}\,\frac{dW}{df} = 0.$$

Approximate solutions are useful

$$\underbrace{2GF^3}_{\text{Domain (i)}} + \underbrace{(1 + 2GX)\,F}_{\text{Domain (iii)}} + \underbrace{\sqrt{2}\,\frac{dW}{df}}_{\text{Domain (ii)}} = 0.$$

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Domain (i)

Subdominance of the first term. The first term is negligible if

$$\frac{8\gamma^2(\gamma+1)}{(3\gamma-1)^3}\frac{1}{T}\left(\frac{dW}{df}\right)^2 \ll 1 \;\longrightarrow\; \gamma \geq 1,\; (dW/df)^2 \ll T,$$

$$\gamma \equiv \frac{1}{\sqrt{1-2X/T}}, \quad X \equiv -\frac{1}{2}(\partial_\mu f)^2,$$

Solution

$$F \simeq -\sqrt{2}\,\frac{\gamma+1}{3\gamma-1}\frac{dW}{df}.$$

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No interesting physics happens

Domain (iii)

Subdominance of the middle term : the most interesting region

$$\frac{8\gamma^2(\gamma+1)}{(3\gamma-1)^3}\frac{1}{T}\left(\frac{dW}{df}\right)^2 \gg 1 \longrightarrow (dW/df)^2 \gg T.$$

Solution

$$F \simeq -\frac{1}{\sqrt{2}}\left(\frac{1+\gamma}{\gamma}\right)^{\frac{2}{3}}\left(T\frac{dW}{df}\right)^{\frac{1}{3}}.$$

On-shell Lagrangian $\mathcal{L}_{DBI} + \mathcal{L}_{\mathrm{aux}}$:

$$\mathcal{L}_{\mathrm{aux}} = -\frac{1}{2^{\frac{2}{3}}}\left(\frac{\gamma+1}{\gamma}\right)^{\frac{2}{3}} V(f), \quad V(f) \equiv \left(\frac{27T}{2}\right)^{\frac{1}{3}}\left(\frac{dW}{df}\right)^{\frac{4}{3}}.$$

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Ultra-relativistic motion of the D-brane is prohibited in the supersymmetric DBI inflation

We consider this domain in the following

Implication of SUSY DBI cosmology

Non-Gaussianities of primordial perturbations

k-inflation [Armendariz-Picon-Damour-Mukhanov (1999), Garriga-Mukhanov (1999)]

$$\mathcal{L}_f = K(f, X)$$

non-Gaussianities of the curvature perturbations are enhanced by $1/c_s^2 = \frac{K_{,X} + 2XK_{,XX}}{K_{,X}}$ [Seery-Lidsey (2005), Chen-Huang-Kachru-Shiu (2006)]

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Tensor perturbations

Field variation and e-folding number

$$\frac{df}{M_{\rm pl}} = \sqrt{\frac{r}{8c_s K_X}} dN.$$

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$$c_s K_X = 1 \quad \text{but} \ldots$$

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Summary and future works

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- Supersymmetric DBI action in warped throat
- Exact solutions for auxiliary field – three branches of on-shell action
- New mechanism producing potential induced terms
- Negligible non-Gaussianities
- Tensor-to-scalar ratio $r \gtrsim 0.01$ even for sub-Planck variation

Future works

- SUSY k-inflation and Galileon models
- Other higher derivative models
- Relation to particle physics