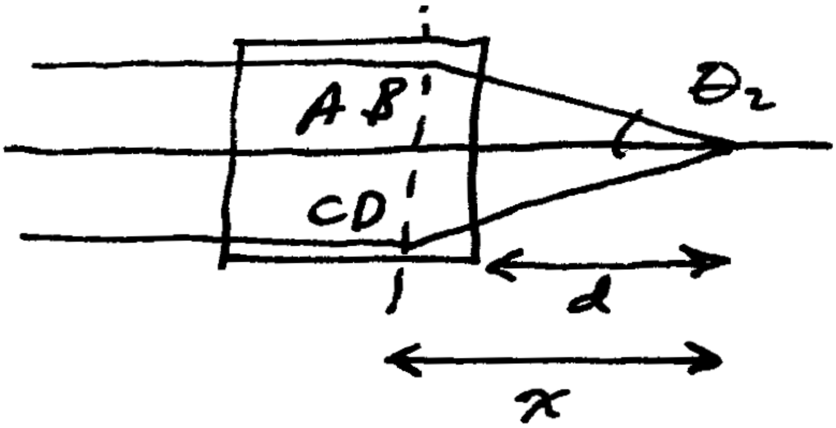
Lecture 26

Let's see how this works with an example.

Consider an object at $-\infty$ on axis, so $\theta_1 = 0$

Q1: at what distance d behind the back reference plane of the ABCD system do the rays come to a focus?

Q2: where is 2^{nd} principal plane—let's call it a distance x from the focal point?



A: consider propagation a distance d from the 2^{nd} reference plane:

$$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A+dC & B+dD \\ C & D \end{bmatrix}$$

For $\theta_1 = 0, y_2 = (A+dC)\, y_1$

At focus, $y_2 = 0 \Rightarrow A + dC = 0 \Rightarrow d = -\dfrac{A}{C}$

Now, the angle the rays converge on the focal point is $\theta_2 = Cy_1$, when $\theta_1 = 0$

Now $\tan\theta_2 \simeq \theta_2 = -\dfrac{y_1}{x}$

$$x = -\frac{y_1}{\theta_2} = -\frac{y_1}{Cy_1} = -\frac{1}{C}$$

Therefore, the rays look like they refract at the 2^{nd} principal plane where

$$\mathrm{h}_2 = d - x = -\frac{A}{C} + \frac{1}{C}$$

$\boxed{\mathrm{h}_2 = \dfrac{1-A}{C}}$ as it should be

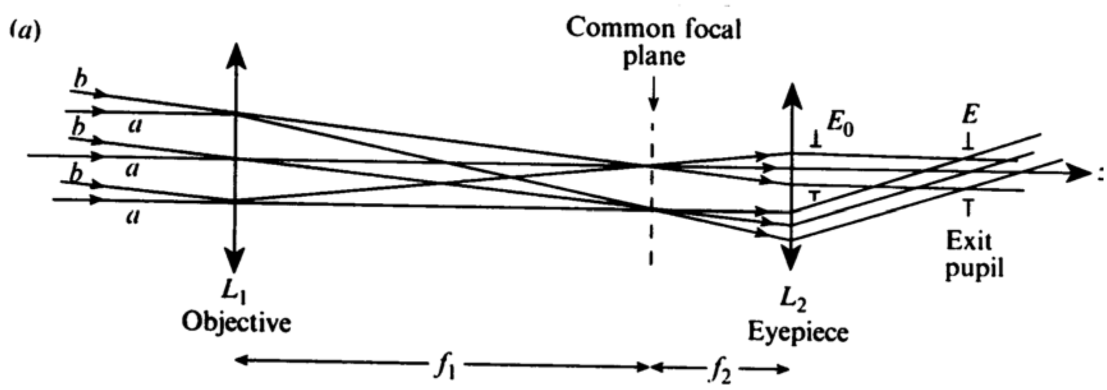
Steps and Pupils

It is also necessary in designing optical systems to understand

(a) What determines the amount of light in an image, i.e. its brightness, and

(b) What is the field of view, i.e. how much of the object plane is imaged?

It is not necessary for us to consider the problem in general, but only to understand the basic idea, which is most easily discussed in terms of a specific optical system, namely the telescope. We consider the optical system, with both object and image of infinity.



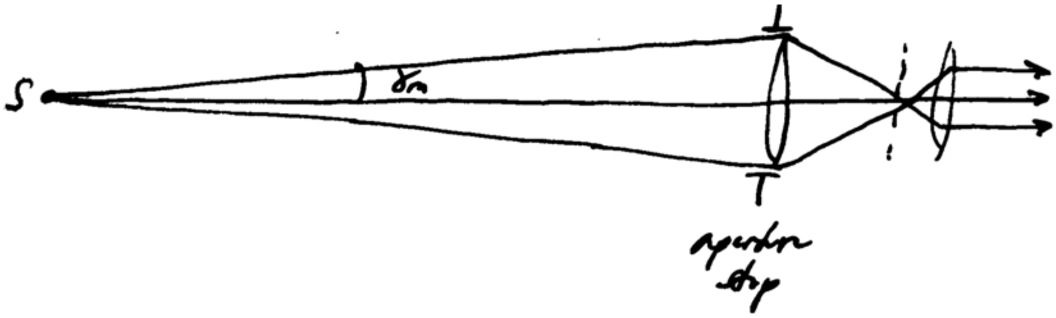
aaa = rays form object point on-axis

bbb=rays from object point off-axis, at an angle θ

"stop" = an obstruction (usually a circular aperture; often the edge of a lens mount) which blocks rays beyond some value of y from propagating through the system.

Aperture stop: the stop which limits or determines the amount of light passing through the system from a point on axis (i.e. at $\theta = 0$).

- as drawn in the figure, the aperture stop is the entrance lens diameter (as is common in telescopes; after all, most optical systems are designed so that the most expensive /difficult-to –fabricate element serves as the aperture stop).
- Guenther's definition is equivalent : maximum angle γ_m of a ray from object to image is defined by the aperture stop

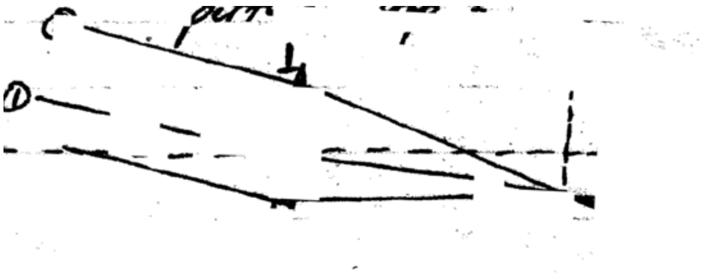


- Clearly the image of S is unaffected by the size of the aperture stop; however , the number of rays and hence the brightness of the image obviously depends on the size of the aperture stop

Principal ray (or chief ray)= any ray that propagates through the center of the aperture stop.= ①

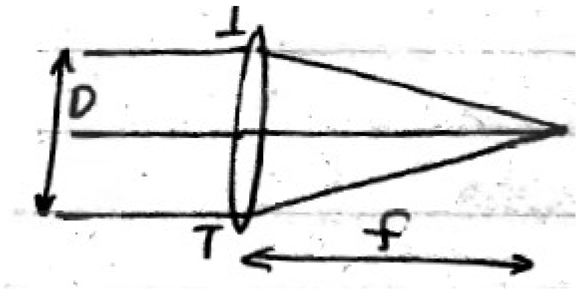
below

Marginal ray = a ray pacing at the edge of the aperture stop =② below



Clearly all rays between the principal and marginal rays will propagate through the system.

There is an easy way to see how the aperture stop determines the image brightness:



Fraction of solid angle $d\Omega \sim \frac{dA}{r^2} \propto \frac{D^2}{f^2}$

⇨ Amount of light collected $\sim \frac{D^2}{f^2} = \frac{1}{(f/\#)^2}$

Where the <u>f-number</u> is defined as

$$\boxed{f/\# = \frac{f}{D} = \frac{\text{focal length}}{\text{diameter}}}$$

<u>Lens Aberrations</u> (see Guenther Appendix 5-B)

It is quite straightforward to calculate any optical system within the paraxial approximation, via the ABCD matrix formalism. However, we know from our introductory discussion that a spherical surface dose not form a perfect image. Only a Cartesian oval does (and a given oval only works for specific conjugates, so that's not very useful).The <u>deviations</u> from <u>perfect</u> point-to-point <u>imaging</u>

(within the geometrical optics approximation that rays are sufficient to describe the propagation) are called <u>aberrations</u> .

There are basically two kinds we will be concerned with:

(i) Chromatic aberrations : n = n(w)

⇨ Rays corresponding to different wavelengths travel different paths

(ii) <u>Monochromatic aberrations</u>: image is blurred or deformed due to breakdowns in the paraxial approximation

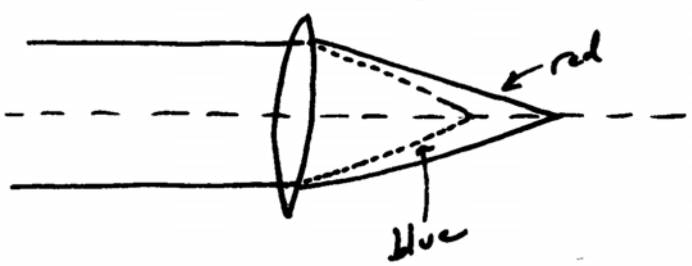
Unfortunately, while paraxial optics is simple, the derivation of aberrations and how to minimize them is very complicated. Our purpose here is to understand them from a pictorial point of view, to obtain some insight into their physical origin + thus be able to recognize them in the laboratory.

Chromatic aberration

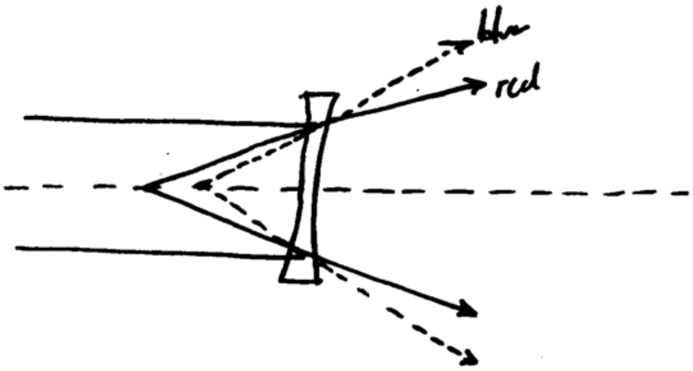
- Even of the paraxial approximation holds, we still have the problem for a simple lens that

$$\frac{1}{f(\lambda)} = \left[n(\lambda) - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

-normal dispersion: $n \uparrow$ as $\lambda \downarrow$ $\left(n_{blue} > n_{red}\right)$

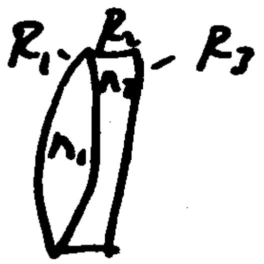


Blue to left of red $\leftrightarrow$ "positive" chromatic aberration



=>"negative" chromatic aberration

- How to correct? You need combinations of simple lenses to do it. The most common case is the achromatic doublet (often called just an "achromat" for short).



n_1 = e.g. float glass, n_2 = e.g. crown glass

Radii of curvature are chosen to yield desired f (and minimize monochromatic aberrations!)

The basic idea is to make the focal lengths the same for both red and blue. It is easy to show that this is satisfied if

$$(n_{b1}-n_{r1})\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+(n_{b2}-n_{r2})\left(\frac{1}{R_2}-\frac{1}{R_3}\right)=0$$

Where we have assumed that the two lenses share a common radius (i.e. a cemented doublet).

$n_{b(1,2)}$ = index in the blue and glass (1, 2)

$n_{r(1,2)}$ = index in the red for glass (1, 2)

If you want a lens of focal length f, it is useful to consider the two lenses in the doublet separately with focal lengths f_1 and f_2, so that

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$ (where the f's are in the yellow)

And the achromatism condition above can be written

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

Where $\omega_i=\dfrac{n_{bi}-n_{ri}}{n_{yi}-1}$ = dispersive power of lens j

And n_{yj} = index of glass j in the yellows.

From the point of view of wavefront aberrations, the procedure usually followed is to consider (for rotationally symmetric lens systems, or one usually has)

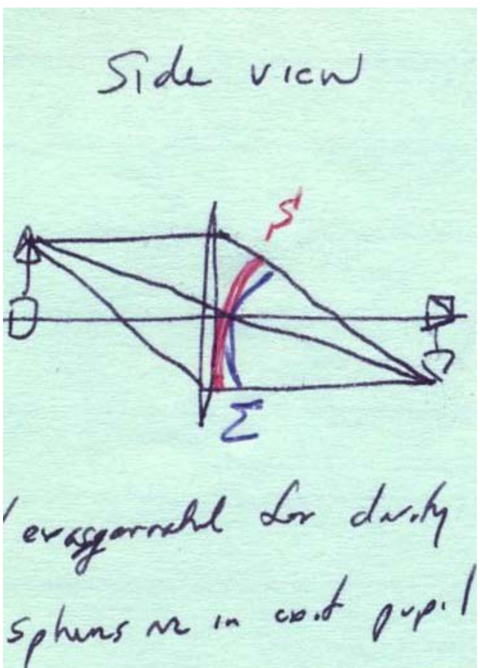
$$h=\frac{\text{position in object plane }(\text{y}-\text{direction})}{\text{object radius}}$$

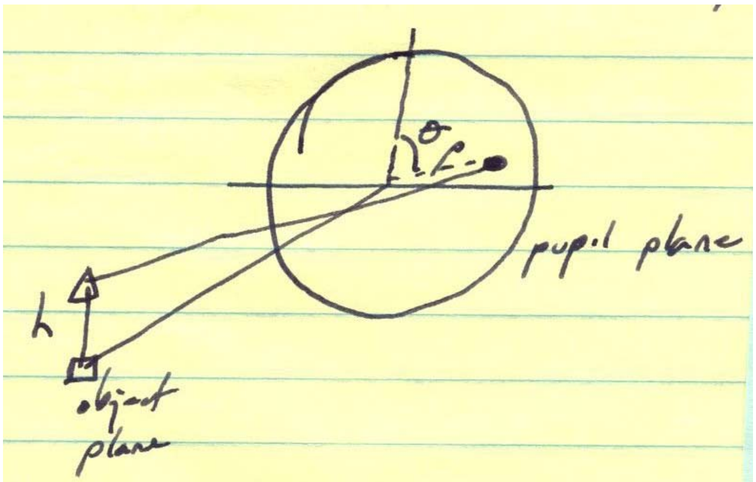
ρ,θ = position coordinate in exit pupil (normalized to pupil radius)

$S(\rho,\theta,h)$ = reference sphere (converging on paraxial image of h) in exit pupil

= wavefront expected from paraxial theory

$\sum(\rho,\theta,h)$ =actual wavefront in exit pupil

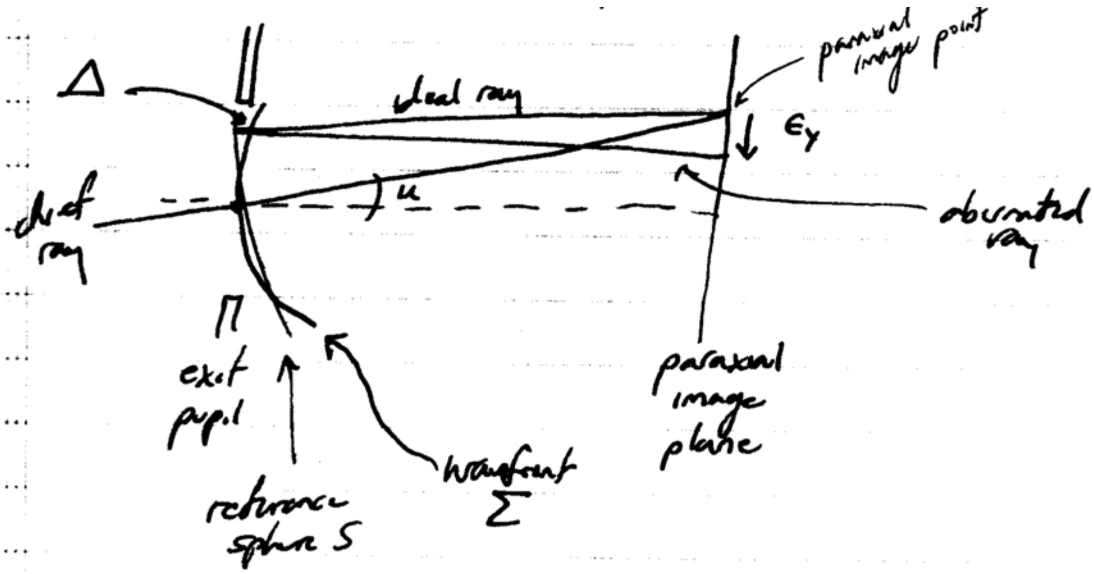




Def. $\Delta = n_2(\sum - S)$ = aberration function

(n_2 = index in image space)

- Note that Δ is an optical path difference, so one often speaks of the aberrations in terms of the "number of waves" between the actual and paraxial wavefronts
- From the following figure we can see how the wavefront deviation gives rise to ray position errors:



The general forms of the aberration functions for the five primary monochromatic aberrations are shown graphically in Born+Wolf fig.5.3(y. $\leftrightarrow$ h)

It is not easy to see directly from these wavefronts what the aberrated image of a point object looks like, but it is useful to return to these wavefronts after considering the ray propagation.