Lecture 20

Thus $I_p = I_0 \sum_{j,j'} e^{-i(\vec{k} - \vec{k}_i)\cdot(\vec{r}_j - \vec{r}_{j'})}$

(i) Forward scattering : $\vec{k} = \vec{k}_i$

$$I_p = I_0 \left(\sum_{i=1}^{N} 1\right)\left(\sum_{i'=1}^{N} 1\right) = N^2 I_0$$

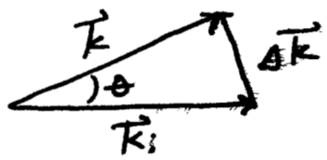
Just as we saw when considering small ($\ll \lambda$) collections of scatterers, coherent scattering goes like N^2

(ii) Side scattering: $\vec{k} \neq \vec{k}_i$

Define $\vec{\Delta k} = \vec{k} - \vec{k}_i$ as the "scattering wavevector"

We've been assuming elastic scattering (i.e. no absorption), so that

$$\left|\vec{k}\right| = \left|\vec{k}_i\right| = \frac{\omega}{c}$$



$$I_p = I_0 \sum_{j=1}^{N} \sum_{j'=1}^{N} e^{-i\vec{\Delta k}\cdot(\vec{r}_j - \vec{r}_{j'})}$$

We'll consider 2 cases-dipole scatterers arranged randomly or in a perfect crystal

(a) Random scatters

⇨ For any atom j, the sum

$$\sum_{j'=1}^{N} e^{-i\vec{\Delta k}\cdot(\vec{r}_j - \vec{r}_{j'})} = e^{-i\vec{\Delta k}\cdot\vec{r}_j} \underbrace{\sum_{j'\neq j}^{N} e^{i\vec{\Delta k}\,\vec{r}_{j'}}} + 1$$

This term oscillates wildly and randomly as the sum over j' is made , and thus averages to zero ($\int_0^{2\pi} e^{i\varphi} d\varphi = 0$)

Thus $I_p = I_0 \sum_{i=1}^{N} = NI_0$

Thus we recover the case of N "independent" scatters when they are randomly positioned.

We see that incoherent scattering goes like $I = NI_0$

For randomly placed molecules/scatters, we do have side scattering and thus attenuation according to the exponential law

$$I(z) = I_{inc} e^{-\sigma N_z}$$

Where N is the number density and σ is the single-molecule scattering cross section

(b) Scattering centers arranged on a perfect lattice
This case can be shown to be equivalent to the perfectly homogeneous medium case:
When $\lambda \gg$ atomic spacing, only coherent formed scattering occurs.

Real life: Thermal fluctuations (defects) => imperfect lattice, in addition to the finite size of the material (or the incident beam)
⇨ Recover exponential attention

Wavelength dependence of scattered light

We must go back to the form of the cross section

$$\sigma(\omega) = \frac{\omega^4 \alpha^2(\omega)}{12\pi\varepsilon_0^2 c^4}$$

Clearly, if $\alpha(\omega)$ shows strong resonances, as it does in the CEO model where

$$\alpha_{CEO}(\omega) = \frac{e^2/m}{\omega_0^2 - \omega^2 + i\omega\gamma}$$

Then the scattering will also be strongly resonant. (e.g. color of metal nanoparticles)

Rayleigh scattering

This situation typically involves nonresonant scattering in transparent media. A classical example is the propagation of light in air. The various molecules in the atmosphere have their absorption resonances in the ultraviolet. We thus have the situation for visible light at ω:

$$\omega \ll \omega_0, \omega\gamma \ll \omega_0^2$$

This gives

$$\alpha_{CEO}(\omega) \simeq \frac{e^2}{m\omega_0^2}$$

Which is independent of frequency.

Thus $$\sigma(\omega) = \frac{e^2\omega^4}{12\pi\varepsilon_0^2 m\omega_0^2 c^4}$$

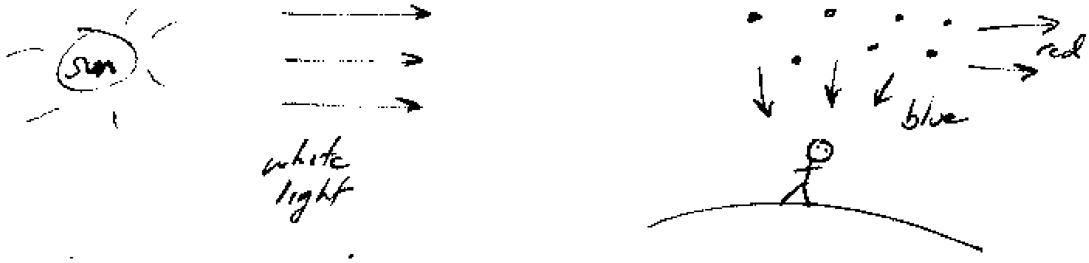
If the polarizability of the scattering molecules is not too strong (as is the case for visible light in transparent media), then the frequency dependence of light scattering is ω^4.

Note that scattering of blue (near UV) light at 400 nm is sixteen times as strong as scattering of red/near-IR light at 800 nm. This gives us the classical argument for the blue sky, similar to that first developed by Lord Rayleigh:

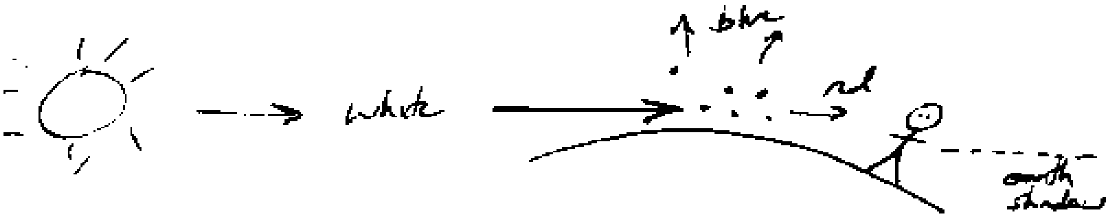
(1) The frequency dependence of the index of refraction is fairly weak across the visible spectrum

(2) The molecules in the atmosphere are completely randomly positioned, and thus show density fluctuations on the scale of a wavelength. These density fluctuations cause light to scatter on propagation through the atmosphere

(3) The ω^4 law means blue is scattered more than red. Thus, when looking in a direction different from that of the source (i.e. the sum), you see blue!

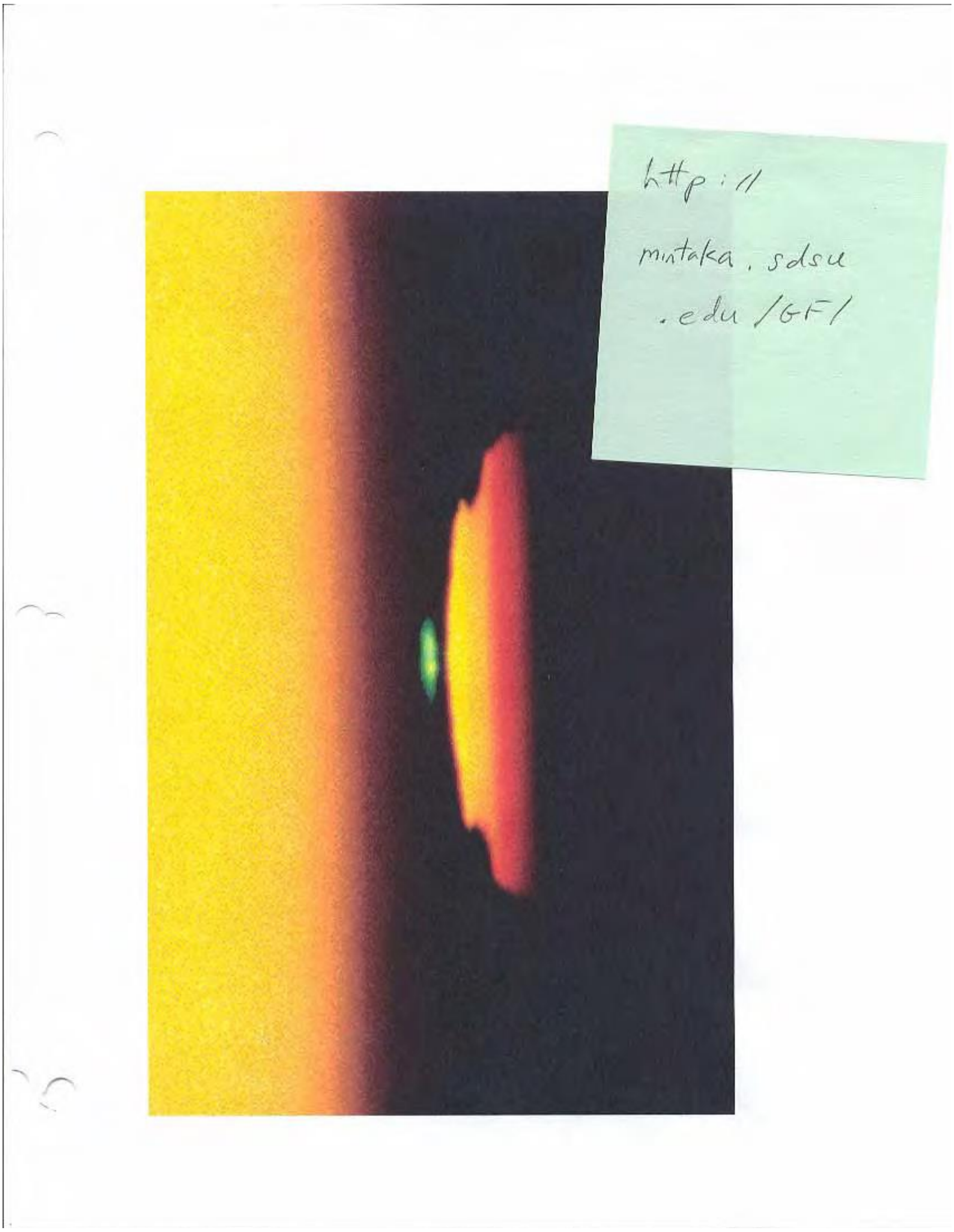


Note that this also explains the red sunset :



As the light travel through a long distance in the atmosphere at sunset, most of the blue is scattered away ,leaving only the red. (If you look carefully after look on a clear evening,you will notice a continuous shift in the color of the sky from red to blue as you look west to east.) note Mount Pinatubo effect !

So why are clouds white (when you are looking at reflected light)?? Clouds are collections of water droplets, which have a size $\geq \lambda$ for visible light .Therefore all visible wavelengths are scattered with roughly equal efficiency, and thus you see white.
Why is the sky gray on a cloudy day? If the sun is behind clouds, you are looking at transmitted light, which is attenuated by scattering. The attenuation is independent of λ , and attenuated white is just gray!



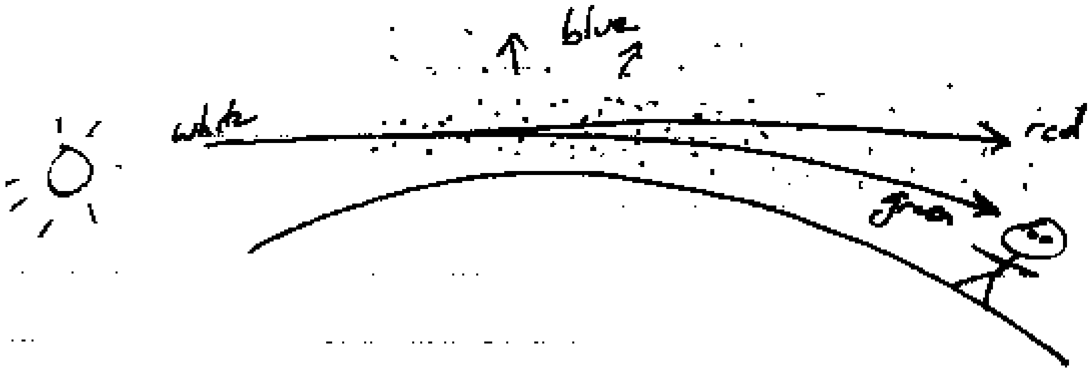
Back to sunsets: at the very moment just after the top of the sun dips below the horizon ,the "<u>green flash</u> "may sometimes be observed .It can only be observed when the sun sets over the ocean or a very flat plain (sometimes from airplanes ,too).

The green flash is caused by a combination of effects:

(1) At sunset, the final rays you see travel a long way through the atmosphere => the blue is scattered away.

(2) The yellow and orange are weakly absorbed by water vapor and possibly oxygen molecules; over a long distance this absorption may be significant $\rightarrow$only red+green left.

(3) As we have seen, rays in the atmosphere can bend, due to the inhomogeneous (radial ,in this case)
Index of refraction. Due to dispersion, the red bends less than the green:



Another practical consequence of Rayleigh scattering: intrinsic losses in optical fibers.
Optical fibers are generally made of glass (e.g. silica), which is a disordered medium. In fact, it is an “amorphous” solid, with the same sort of density fluctuations you find in a gas. These density fluctuations cause light scattering out of the fiber, and since they are unavoidable, they set the lower limit to possible fiber attenuation.

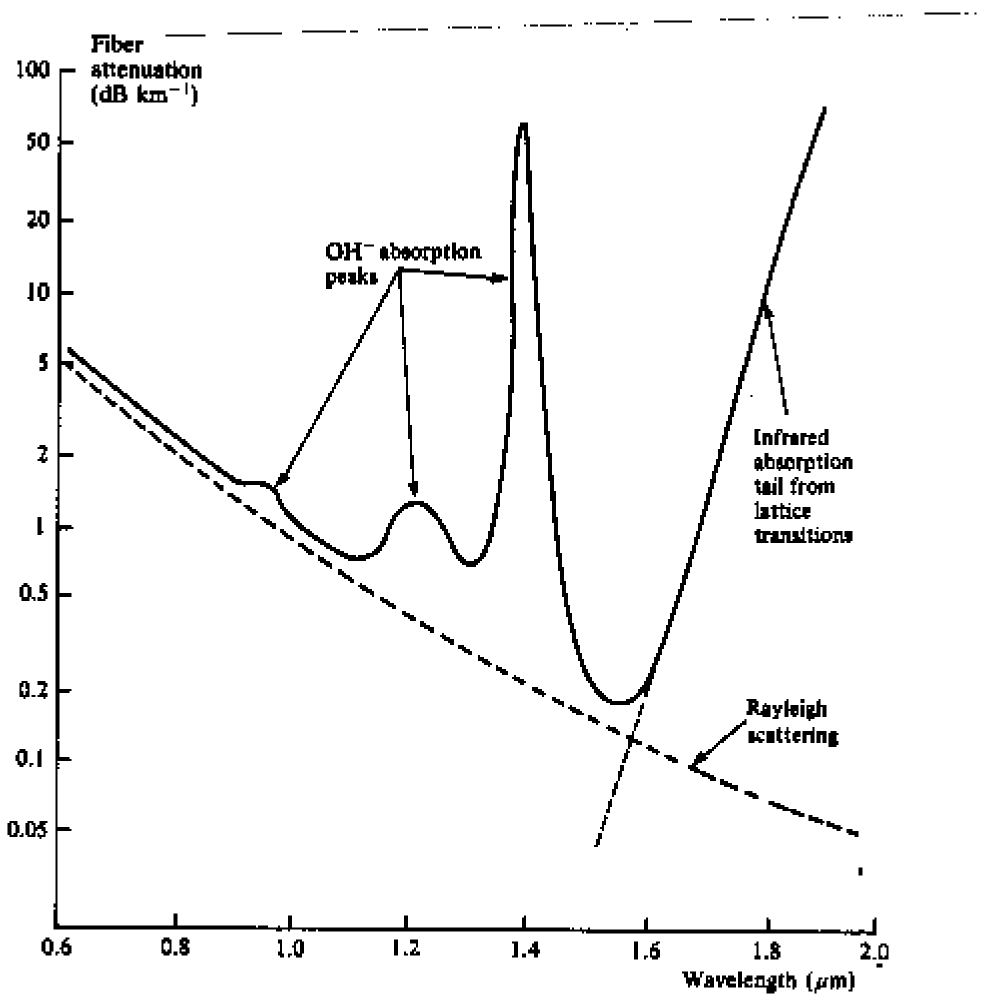
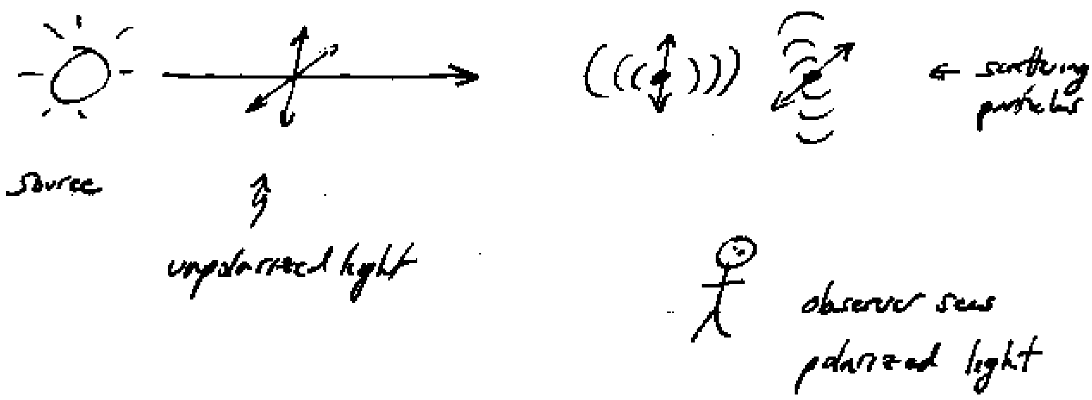


Fig. 8.24 Typical attenuation versus wavelength plot for a silica-based optical fiber. The contribution from Rayleigh scattering is shown, as are the other two main loss mechanisms, namely the infrared absorption tail and the hydroxyl (OH^-) absorption peaks.

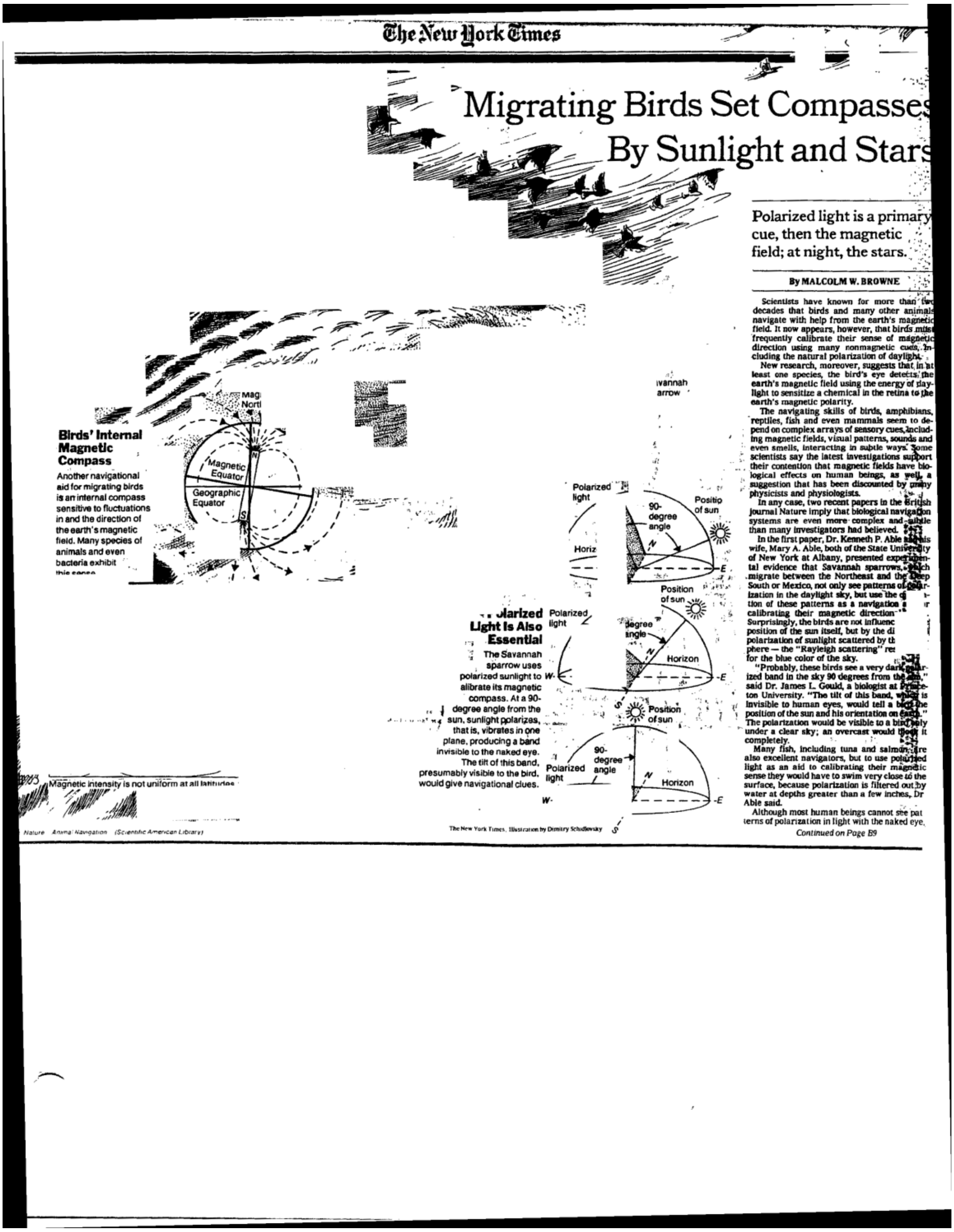
Polarization of scattered light

- A consequence of the dipole na## of the scattering



(you can check this by looking at the sky through Polaroid sunglasses and rotating them. You should look at an angle $\sim 90^\circ$ from the sun.)

Is this of any use? It is if you are a bird! It has long been know that bird navigate with the help of the earth's magnetic field. Now it appears that some birds "calibrate" their magnetic field sense by sensing the polarization of scattered sunlight. (see NY Times 9/29/93).



The New York Times

Migrating Birds Set Compasses By Sunlight and Stars

Polarized light is a primary cue, then the magnetic field; at night, the stars.

By MALCOLM W. BROWNE

Scientists have known for more than two decades that birds and many other animals navigate with help from the earth's magnetic field. It now appears, however, that birds must frequently calibrate their sense of magnetic direction using many nonmagnetic cues, including the natural polarization of daylight.

New research, moreover, suggests that in at least one species, the bird's eye detects the earth's magnetic field using the energy of daylight to sensitize a chemical in the retina to the earth's magnetic polarity.

The navigating skills of birds, amphibians, reptiles, fish and even mammals seem to depend on complex arrays of sensory cues, including magnetic fields, visual patterns, sounds and even smells, interacting in subtle ways. Some scientists say the latest investigations support their contention that magnetic fields have biological effects on human beings, as well, a suggestion that has been discounted by many physicists and physiologists.

In any case, two recent papers in the British journal Nature imply that biological navigation systems are even more complex and subtle than many investigators had believed.

In the first paper, Dr. Kenneth P. Able and his wife, Mary A. Able, both of the State University of New York at Albany, presented experimental evidence that Savannah sparrows, which migrate between the Northeast and the Deep South or Mexico, not only see patterns of polarization in the daylight sky, but use the di... tion of these patterns as a navigation ... calibrating their magnetic direction. Surprisingly, the birds are not influenc... position of the sun itself, but by the di... polarization of sunlight scattered by th... phere — the "Rayleigh scattering" re... for the blue color of the sky.

"Probably, these birds see a very dark polarized band in the sky 90 degrees from the sun," said Dr. James L. Gould, a biologist at Princeton University. "The tilt of this band, which is invisible to human eyes, would tell a bird the position of the sun and his orientation on earth." The polarization would be visible to a bird only under a clear sky; an overcast would block it completely.

Many fish, including tuna and salmon, are also excellent navigators, but to use polarized light as an aid to calibrating their magnetic sense they would have to swim very close to the surface, because polarization is filtered out by water at depths greater than a few inches, Dr Able said.

Although most human beings cannot see patterns of polarization in light with the naked eye,

Continued on Page B9

Nature Animal Navigation (Scientific American Library)

The New York Times; Illustration by Dmitry Schidlovsky

Reading : Guenther chap.5(except resonators + guided waves)