Lecture 28

Interference

Reading: Lipson S 8.3, chap.9 (all)
Guenther chap.4 (all) or Born+Wolf Ch.7

"Interference" refers to the general phenomena that result when multiple waves are added together coherently, i.e. with specific phase relationships between the waves.

Simplest case: summation of two waves

$$\vec{E}_1 = \vec{E}_{01} e^{i(\omega t - \vec{k}_1 \cdot \vec{r} + \varphi_1)}$$

$$\vec{E}_2 = \vec{E}_{02} e^{i(\omega t - \vec{k}_2 \cdot \vec{r} + \varphi_2)}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = e^{i\omega t}\left[\vec{E}_{01} e^{-i(\vec{k}_1 \cdot \vec{r} - \varphi_1)} + \vec{E}_{02} e^{-i(\vec{k}_2 \cdot \vec{r} - \varphi_2)}\right]$$

As we know, all optical photo detectors are "square-law" detectors, with a time response slow compared to an optical cycle. Thus the time-averaged intensity is proportional to

$$<\left|\vec{E}\right|^2> = <\left|\vec{E}_1\right|^2> + <\left|\vec{E}_2\right|^2> + <\vec{E}_1^* \cdot \vec{E}_2> + <\vec{E}_2^* \cdot \vec{E}_1>$$

So $$I(\vec{r}) = I_1 + I_2 + \sqrt{I_1 I_2}\left[e^{i\left[\left(\vec{k}_1 - \vec{k}_2\right)\cdot\vec{r} + \left(\varphi_1 - \varphi_2\right)\right]} + e^{-i\left[\left(\vec{k}_1 - \vec{k}_2\right)\cdot\vec{r} + \left(\varphi_1 - \varphi_2\right)\right]}\right]$$

Where we have assumed parallel polarizations (only parallel polarization components can interfere, since the waves are transverse)

Def. δ = phase difference between the two waves

$$\delta = \left(\vec{k}_1 - \vec{k}_2\right)\cdot\vec{r} + \left(\varphi_1 - \varphi_2\right)$$

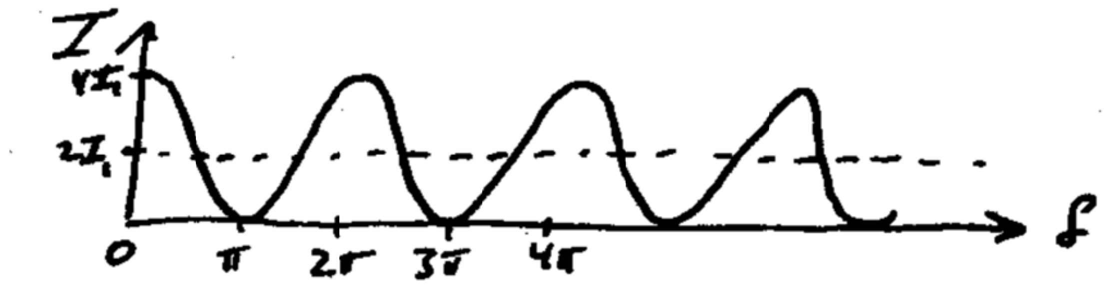
Then $$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$$

Thus the time-averaged intensity at some position $\vec{r}$ is just the sum of the wave intensities plus an "interference term" which oscillates with the phase difference between the two waves.

$\delta = 0, \pm 2\pi, \pm 4\pi, \cdots \Rightarrow$ constructive interference

$\delta = \pm\pi, \pm 3\pi, \cdots \Rightarrow$ destructive interference

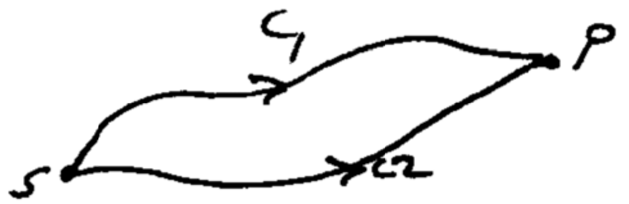
Suppose $I_1 = I_2$. Then $I = 4I_1 \cos^2\frac{\theta}{2}$



In a sense, this is all there really is to interference. The solution of interference problems usually boils down to finding the phase difference δ .
Note that

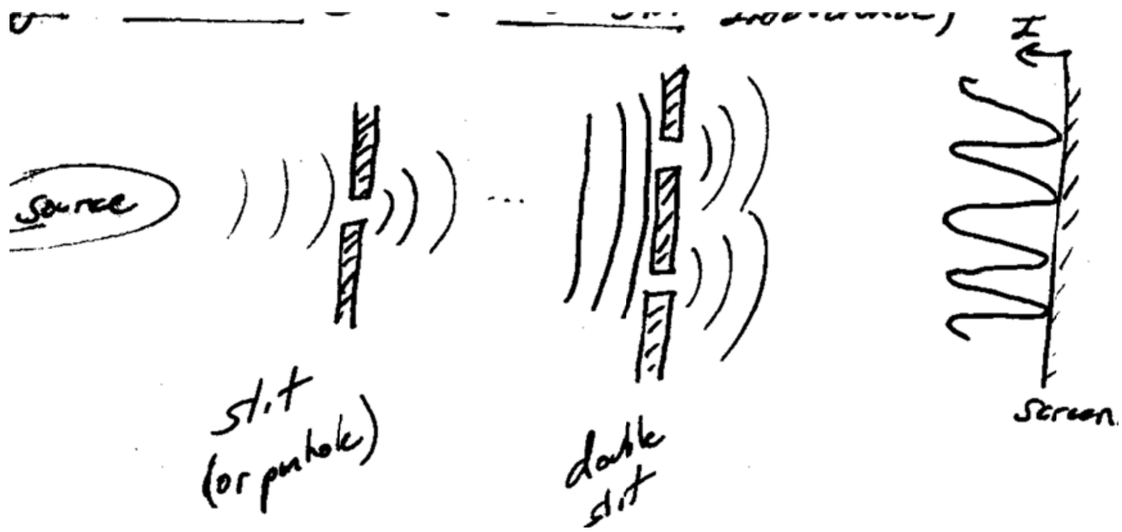
$$\delta = \underbrace{\left(\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r}\right)}_{\substack{\text{phase difference due to} \\ \text{a difference in optical path} \\ \text{length that the two waves} \\ \text{have traversed}}} + \underbrace{\left(\varphi_1 - \varphi_2\right)}_{\substack{\text{phase difference at the start} \\ \text{(in many problems } =0\text{)}}}$$

In general, we may have



$$\delta = \int_{C_1} \vec{k}_1\left(\vec{r}\right) \cdot d\vec{s} - \int_{C_2} \vec{k}_2\left(\vec{r}\right) \cdot d\vec{s}$$

Young's Interference (Double-Slit Interference)

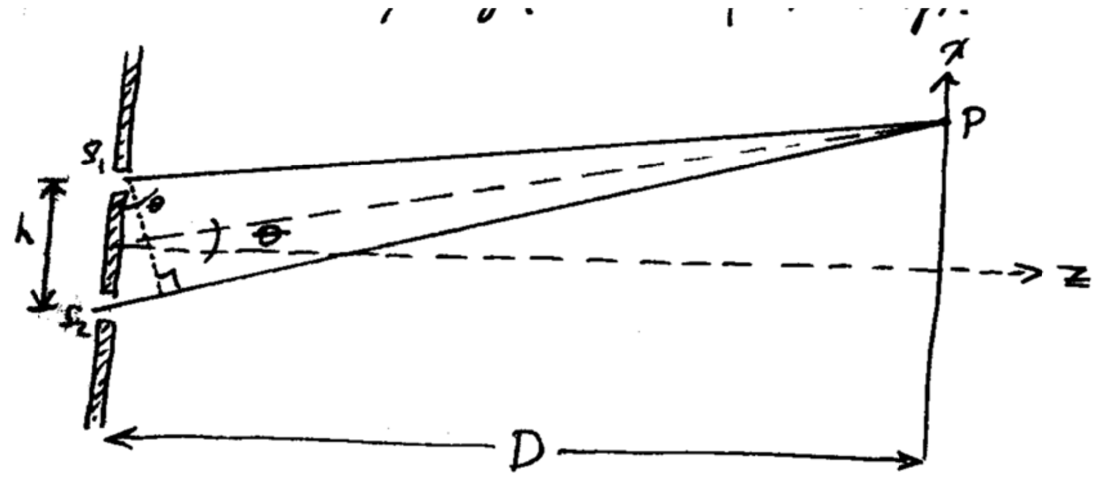


The purpose of the first slit or pinhole is to provide a harmonic plane wave at the double slit. (It's unnecessary if a laser is used to illuminate the double slit.)

If the first slit is centered between the two slits in the double-slit arrangement, then the phases of the two waves emitted from the two slits are the same: $\underline{\varphi_1 - \varphi_2}$

Such an arrangement is often called interference by division of wavefront, since the double slit divides up a single wavefront into two sources which subsequently are made to interfere.

Each slit acts as a line source, if the slit width is $< \lambda$, so cylindrical waves are emitted from each. They propagate to the screen, which we will assume to be much farther away than the slit spacing (i.e. effectively at infinity).



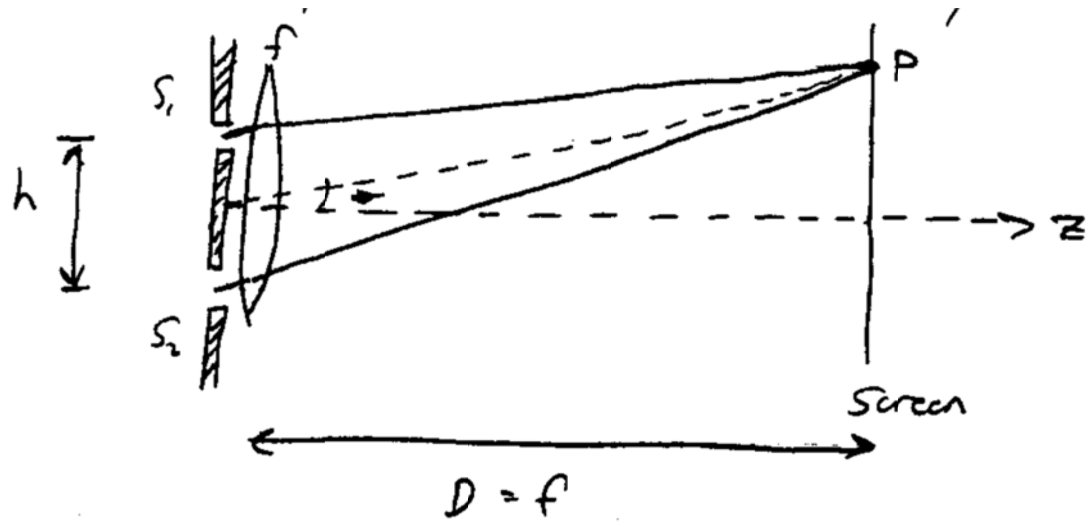
If $D \gg h$, then $\theta_1 \simeq \theta_2 \simeq \theta$ is the angle of the rays w.r.t. the z-axis .Clearly, the optical path difference

O.P.D.= $\overline{S_2P} - \overline{S_1P} = h \sin\theta \simeq h\theta$ for paraxial rays

And $\theta \simeq \tan\theta = \dfrac{x}{D}$

Of course, $\theta_1 \neq \theta_2$ exactly ; this is true only in the limit $D \to \infty$.

We can use a slightly different implementation of the double-slit experiment that does not require this limit:



The lens is immediately behind the slits, and the screen is at the back focal plane of the lens.

We still have O.P.D= $h\sin\theta$, but clearly $\theta_1 = \theta_2 = \theta$ .Note that from the lens onward, the OPD between the two waves if <u>zero</u> (since the screen is at focus , by Fermat's Principle!)

One often says that the screen is effectively "at infinity" by using the lens. (The same trick will turn out to be very useful in diffraction problems)

As always, the phase difference is $k \times$ OPD $\rightarrow$

We have <u>constructive interference</u> whenever

$$\delta = \frac{2\pi}{\lambda}\cdot\frac{hx}{D} = 2\pi m \quad , m = 0, \pm 1, \pm 2, \cdots$$

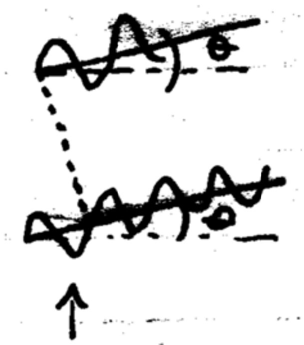
Note that a perfectly equivalent may of finding the positions of constructive interference is to demand that

$$OPD = m\lambda$$

⇨ In either case , the positions of interference maxima on the screen are

$$\boxed{\chi_m = m\frac{\lambda D}{h}} \qquad \text{m="order of the interference"} = \frac{OPD}{\lambda}$$

Recall the simple physical picture of the position of the first maximum:

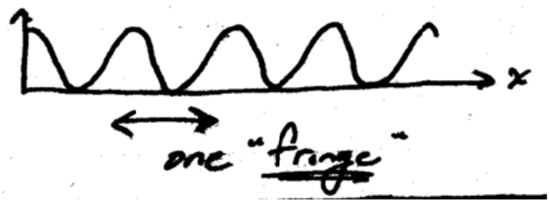


Exactly one wave fits in the OPD

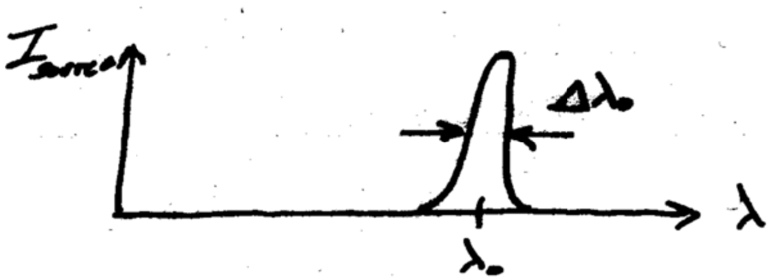
<u>Intensity pattern</u>:

- Equal intensities and phases at double slits =>

$$I = 4I_1 \cos^2\frac{\delta}{2} = 4I_1\cos^2\left(\pi\frac{hx}{\lambda D}\right)$$



Note that cosine- squared fringes are obtained only if the source is perfectly monochromatic. Of course, any real source will have a spectrum of nonzero width $\equiv \Delta\lambda_0$, centered on a wavelength λ_0 .



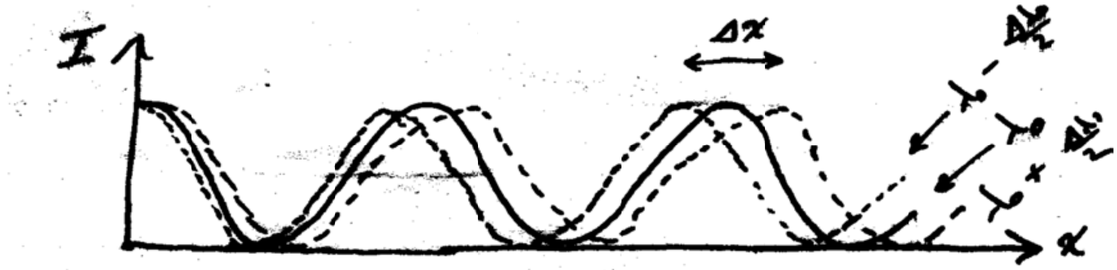
If the width of the source spectrum is comparatively small, so that

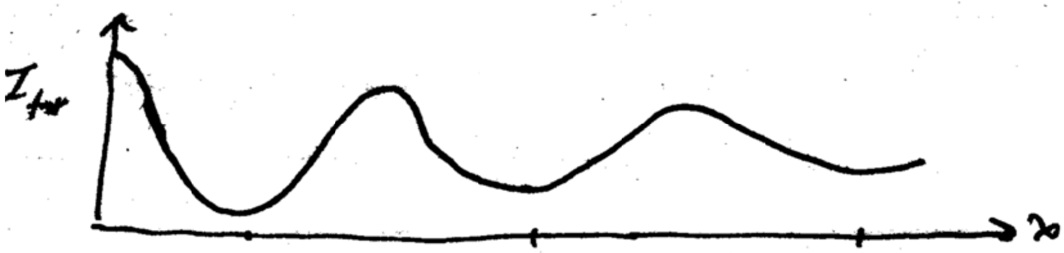
$$\Delta\lambda_0 \ll \lambda_0$$

Then the source is said to be "<u>quasi-monochromatic.</u>"

Intuitively, it is easy to see how the interference pattern will be washed out by the nonzero spectral width: all frequency components may be considered separately and each yields constructive interference at x=0 (corresponding to m=0), but the positions of the maxima for $|m| > 0$ are slightly different:

=>





Clearly the fringe wash at more and more as the order m increases. The m=0 fringe, however, is unaffected.

The maxima are spread act by an amount

$$|\Delta\chi| = |m|\frac{D}{h}\Delta\lambda_0$$

We can compare this with the <u>mean spacing</u> between fringes:

$$|\delta x| = \frac{D}{h}\lambda_0 \quad \text{(i.e. } \delta m = 1\text{)}$$

The maxima are spread out by a negligible amount if

$$\frac{|\Delta x|}{|\delta x|} \ll 1$$

Or $$\frac{|m|\Delta\lambda_0}{\lambda_0} \ll 1 \Rightarrow \boxed{|m| \ll \frac{\lambda_0}{\Delta\lambda_0}}$$

Thus for highly monochromatic light, many orders of interference can be seen. For broad-band light, only a few fringes can be seen.

It is interesting to consider in a qualitative way what happens if "white light" is used as the source. Recall that the eye is sensitive in the region of 400-700 nm

⇨ $$\frac{\Delta\lambda_0}{\lambda_0} \sim \frac{300nm}{550nm} \sim 0.545$$

Or $$|m| < \frac{\lambda_0}{\Delta\lambda_0} \sim 1.8$$

FIGURE 2-11. Propagation of light in an absorbing medium. Light from a HeCd laser propagates through first a layer of xylene and then water containing the dye, Rhodamine 6G, in solution. The blue laser light is absorbed by the Rhodamine as the wave propagates through the water. The Rhodamine reemits the energy in the form of a red fluorescence. The reemitted light is radiated in all directions, making the beam appear diffuse. Some of the absorbed energy is not reemitted but instead heats the Rhodamine.

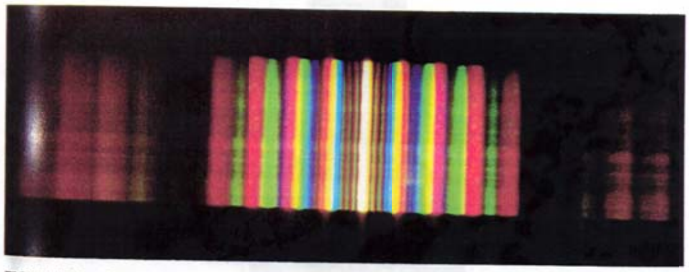


FIGURE 4.7. Fringes from a Young's two-slit experiment.

Thus there will be roughly three regions on the screen:

(i) Near x=0 , there will be a bright white fringe corresponding to m=0 constructive interference for all wavelengths

(ii) For x corresponding to $m = \pm 1, \pm 2$, there will be <u>colored fringes</u> (see the color plate of

Guenther fig.4-7 for a photo of a similar example but with a somewhat narrower bandwidth)

(iii) For x corresponding to $m \gg 2$, the screen will be ~uniformly white, since all colors will overlap.

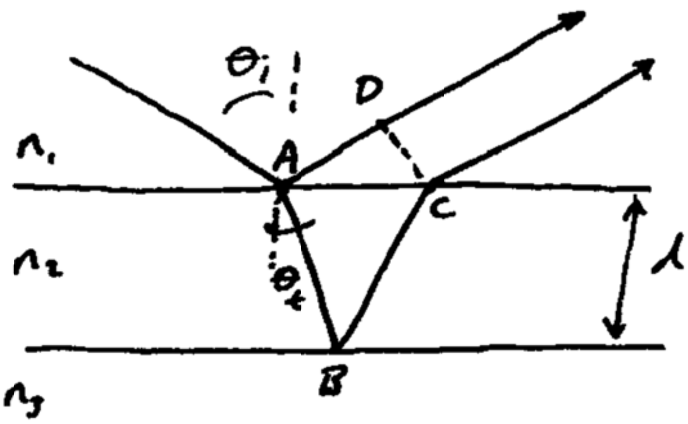
Clearly, the interference pattern actually could be used to tell us something about the width of the source spectrum .In our discussion of coherence, we will consider this problem in detail.

Dielectric Layer

A second kind of interference is often called interference by division of amplitude. In this case, a partial reflector divides an incident beam into two parts, which are recombined elsewhere.

The simplest example is refection form a dielectric layer of thickness d.

- We will assume for now that the reflection amplitude at each interface is small => we need consider only the first reflection at each interface. (Recall that the air-glass reflection is R $\sim 4\%$, so this is a good approximation.)

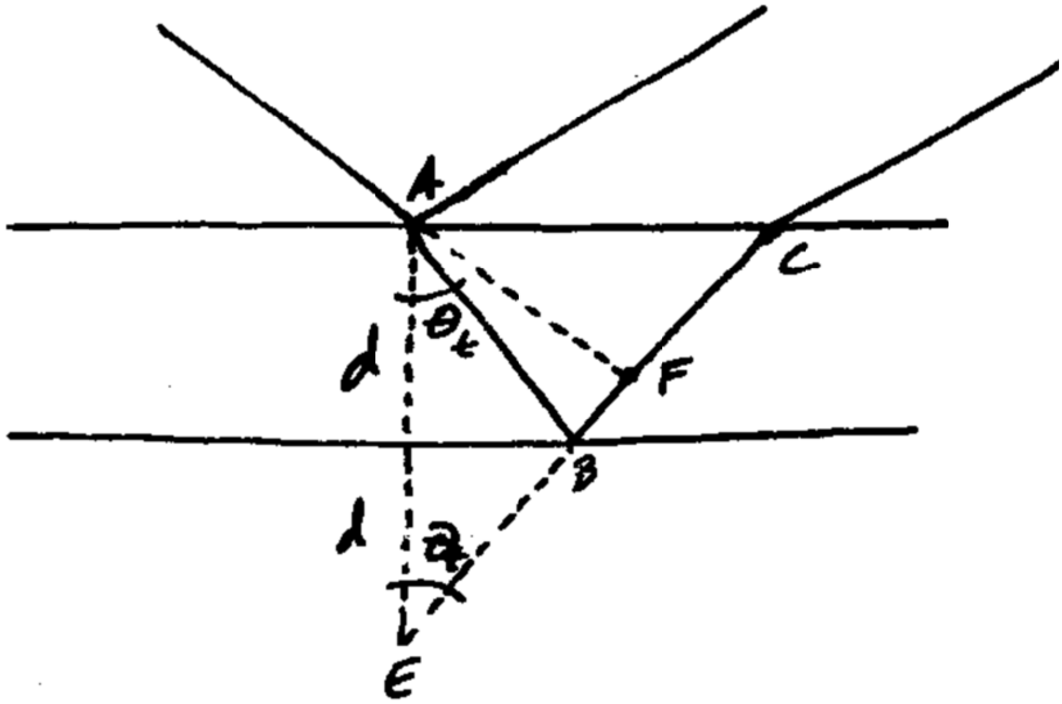


← Look at interference between the two plane waves determined by these rays.

Guenther's derivation: Since DC is a wavefront ($\perp$ to rays), the required phase is

$$n_2\left(\overline{AB}-\overline{BC}\right)-n_1\left(\overline{AD}\right)$$

Here is a slightly different derivation:



Note that AF is also a wavefront ($\perp$ to ray)

$$\rightarrow OPD = n_2 \overline{ABF} = n_2(\overline{AB} + \overline{BF})$$

Geometrical trick: extend line AE

$$\overline{AB} = \overline{BE}$$

$$\Rightarrow \cos\theta_t = \frac{\overline{BE} + \overline{BF}}{2\text{d}} = \frac{\overline{AB} + \overline{BF}}{2\text{d}}$$

$$\Rightarrow OPD = 2n_2 d \cos\theta_t$$

The phase shift due to the difference in propagation distances is therefore

$$\delta = k \times OPD = \frac{2\pi}{\lambda_0} \cdot 2n_2 d \cos\theta_t$$

(λ_0 = free-space wavelength).

Important modification: as we saw early in the semester, you can get phase shifts on reflection.(see notes P.35)

- If go from medium n_i to medium n_t then

$n_i < n_t \Rightarrow -\pi$ phase shift

$n_i > n_t \Rightarrow 0$ phase shift

If we now assume $n_1 < n_2 > n_3$, then there is a phase shift only on reflection at point

A .Therefore the condition for observing constructive interference is

$$\delta = \frac{2\pi}{\lambda_0} \cdot 2n_2 d \cos\theta_t - \pi = 2\pi m$$

We can rewrite this in the following way

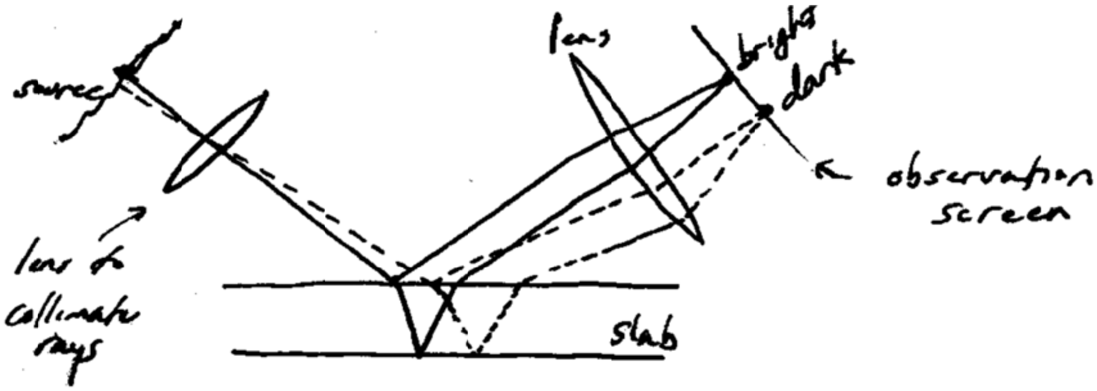
$$4\pi n_2 d \cos\theta_t = 2\pi(m + \frac{1}{2})\lambda_0$$

$$\underbrace{n_2 d \cos\theta_t}_{\text{"effective? path \#\#\# in layer}} = \underbrace{(2m+1)\frac{\lambda_0}{4}}_{\text{odd-integral number of quarter waves}} \qquad m= \qquad 0,1,2,\cdots$$

} bright fringes when $n_1 < n_2 > n_3$

It is trivial to show that , when $n_1 < n_2 < n_3$, so that there is an additional π phase shift , the above condition yields the position of destructive interference. The condition of destructive interference in reflection corresponds to maximum transmission through the film. If $n_1 = 1$, then $n_2 = \sqrt{n_3}$ yields zero refelction, in which case the dielecrtric layer is called an anti-reflection coating.

A note on the geometry:
As drawn on P.252, the geometry is for parallel rays $\leftrightarrow$ plane waves incident on + reflected from the slab. Suppose one has a polychromatic source:



——— = λ_1 satisfying constructive interference condition

…… = λ_2 satisfying constructive interference condition

(note that the lens could be the lens of your eye! Also, the collimating lens is not necessary if the source is very far away so that the rays incident on the slab are essentially collimated. The classical example here is a thin film of oil on a puddle of water, or a soap bubble.)

It is the angle that determines whether the interference is constructive if d is fixed, so these are often called fringes of equal inclination.