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# To detect the looped Bloch bands of Bose–Einstein condensates in optical lattices

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**Abstract**

A loop structure was predicted to exist in the Bloch bands of Bose–Einstein condensates in optical lattices recently in [Phys. Rev. A 61 (2000) 023402]. We discuss how to detect experimentally the looped band with an accelerating optical lattice through extensive and realistic numerical simulations. We find that the loop can be detected through observing either nonlinear Landau–Zener tunneling or destruction of Bloch oscillations.


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## 1. Introduction

The simple system of Bose–Einstein condensates (BECs) in optical lattices is of amazingly rich physics, as shown in recent theoretical and experimental studies. With certain choices of densities of BEC and strengths of optical lattice, this system exhibits various interesting phenomena, ranging from the dynamics of BEC Bloch waves [1–6], Josephson effect [7], squeezed states [8], and quantum phase transition between superfluidity and Mott-insulator [9]. One can only expect more interesting physics to be discovered in this system, considering the rich physics that we have known in the condensed-matter physics, where the prototype system is electrons in a crystal lattice.

One very surprising finding for the system of BECs in optical lattices is the loop structure appeared in the Bloch bands as found in Ref. [10] (see Fig. 1). This finding was later confirmed in further theoretical studies [6,11,12]. This unusual and unique loop structure has very interesting physical consequences: First, it leads to the nonlinear Landau–Zener tunneling that is characteristically different from the linear Landau–Zener tunneling [13], in particular, the nonzero tunneling in the adiabatic limit [10]. Second, it destroys Bloch oscillations [2,6,11].

However, experimental exploration of this looped band and its related physical phenomena is yet to come

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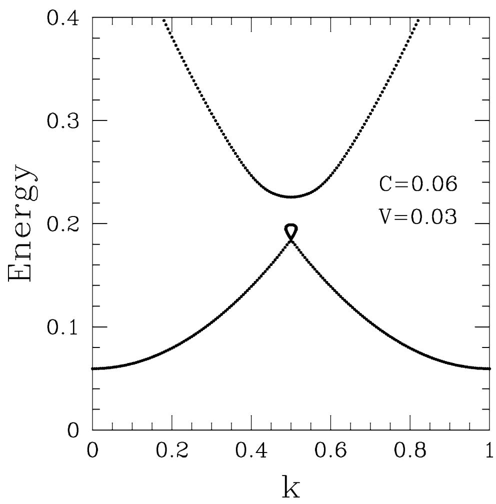


Fig. 1. Two lowest Bloch bands of a BEC in an optical lattice when $C > V$. The energy is in units of $4\hbar^2 k_L^2/m$ and the wave number $k$ in units of $2k_L$, where $k_L$ is the wave number of the laser light that generates the optical lattice.

forth. To our judgment, this experimental stalemate does not come from the lack of experimental techniques; it is the lack of enough theoretical guidance. On the one hand, it is not clear what signals to look for in an experiment to confirm the existence of the loop structure. On the other hand, one may be concerned that the unavoidable inhomogeneity of the BEC used in a real experiment may wash away all the interesting physics since the predicted loop structure and its related physics is based on the analysis of homogeneous BECs.

The purpose of this Letter is to dismiss these concerns. Based on extensive numerical calculations, we argue that one can confirm the existence of the loop structure in an experiment that involves dragging a high density BEC with an accelerating optical lattice. Similar experiments with low density BECs [5] or cold-atoms [14] have been carried out to observe Bloch oscillations and Landau–Zener tunneling. As we will see later, there are two ways to look for the signs of the loop structure: the destruction of Bloch oscillations and the observation of nonlinear Landau–Zener tunneling that shows a very distinct behavior from the well-known Landau–Zener tunneling.

We focus on the experimental situations similar to what is described in Refs. [5,7,14], where a realized BEC is loaded into an optical lattice and there is no trapping potential. These experiments can be well described by the mean-field Gross–Pitaevskii equation in one dimension

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V_0\cos\left[2k_L\left(x - \frac{at^2}{2}\right)\right]\psi + \frac{4\pi\hbar^2 a_s}{m}|\psi|^2\psi, \tag{1}$$

where $m$ is the atomic mass, $k_L$ is the wave vector of the laser light that generates the optical lattice, $a_s$ is the $s$-wave scattering length between atoms, $a$ is the acceleration, and $V_0$ is the strength of the potential which is proportional to the laser intensity. In our calculations, Gaussian functions are used to simulate the inhomogeneous BECs loaded in optical lattices in the real experiments. Strictly, the lateral expansion of the BEC has certain effects on the longitudinal motion [15]. In this Letter we only consider the case where the lateral motion is negligible [1]. In experiments, another possible setup for quasi-one-dimensional dynamics is to confine the lateral motion [16,17].

Without the acceleration, $a = 0$, the system becomes a nonlinear periodic system for which we can define Bloch waves as for a linear periodic system

$$\psi(x) = e^{ikx}\psi_k(x), \tag{2}$$

where $\psi_k$ is periodic, $\psi_k(x + \pi/k_L) = \psi_k(x)$. The Bloch state satisfies

$$\mu(k)\psi_k = -\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial x} + ik\right)^2\psi_k + V_0\cos(2k_L x)\psi_k + \frac{4\pi\hbar^2 a_s}{m}|\psi_k|^2\psi_k. \tag{3}$$

The eigenenergies (or more precisely, the chemical potentials) $\mu(k)$ then form Bloch bands in the Brillouin zone. As shown in Fig. 1, when the interaction between atoms gets larger than a certain critical value, a loop structure is formed in the Bloch bands.

In the following, we first briefly describe our numerical methods. Then, we present our numerical results on nonlinear Landau–Zener tunneling and Bloch oscillations, and explain how to look for signs of the loop structure through these two phenomena. In the end, we discuss the relevance of our results to the experiments.

## 2. Numerical method

For the convenience of numerical calculations, we cast Eq. (1) into a dimensionless form

$$i\frac{\partial\phi}{\partial t} = -\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + V\cos\left(x - \frac{1}{2}at^2\right)\phi + C|\phi|^2\phi, \tag{4}$$

where we have used the following set of scaled variables,

$$\tilde{x} = 2k_L x, \qquad \tilde{t} = \frac{4\hbar k_L^2}{m}t, \qquad \tilde{a} = \frac{m^2}{8\hbar^2 k_L^3}a,$$

$$\phi = \frac{\psi}{\sqrt{n_0}}, \qquad V = \frac{m}{4\hbar^2 k_L^2}V_0, \qquad C = \frac{\pi n_0 a_s}{k_L^2},$$

with $n_0$ being the peak density of the BEC cloud. In writing down Eq. (4), we dropped the tildes (replacing $\tilde{x}$ by $x$, etc.) without causing confusion. We use the Crank–Nicholson method for the numerical solution of Eq. (4). This method preserves the unitarity of the time-evolution, and yields good convergence of the solutions for moderate values of the coupling strength $C$. Note that for the experiment where the lateral motion is confined, $C$ has different definitions, see one example in Ref. [17].

In many experiments [4,5], a BEC cloud has a typical size of order 10 µm, covering 100–200 wells of an optical lattice. To model such inhomogeneous BEC clouds, we use a Gaussian wave packet as the initial state and then turn it into a inhomogeneous BEC Bloch wave by adiabatically turning on the optical lattice.

The lattice strength $V$ is taken to be smaller than 0.4 (or 3.2 in units of the recoil energy $\hbar^2 k_L^2/2m$). This choice serves two purposes. First, it guarantees that there be only one bound state inside each well so that we can use the separation of a BEC cloud to measure the tunneling probability as we will explain below. Second, it means that the mean field theory (1) is a good description of the BEC system.

## 3. Nonlinear Landau–Zener tunneling

The nonlinear Landau–Zener tunneling has been studied quite extensively in Ref. [10], where it is found to be very different from the linear Landau–Zener tunneling [13]. In particular, when $C > V$, the tunneling probability no longer depends on the sweeping rate exponentially (sweeping rate is the acceleration for the system of a BEC in an accelerated lattice). Moreover, in the adiabatic limit where the acceleration approaches zero, the tunneling probability approaches a finite value, instead of zero as in the linear case. This nonzero tunneling probability in the adiabatic limit is the direct result of the loop structure: when a Bloch state is driven to the edge of the loop, it has to split, resulting in tunneling [10]. Therefore, experimental observation of this adiabatic tunneling, along with the nonexponential dependence of tunneling probability on the acceleration, can be viewed as a direct evidence of the looped band.

However, the analysis in Ref. [10] is based on a simplified two-level model derived with the assumption of homogeneity of BEC. This may leave experimentalists wonder whether the unique characteristics of nonlinear Landau–Zener tunneling can be observed in real experiments, where BECs are inhomogeneous and span only a finite range of space. This concern is legitimate, but our numerical results show that the inhomogeneity does not blur up the essential physics.

In our numerical simulations, we calculate the tunneling probability through the separation of a BEC cloud. As we mentioned earlier, the lattice strength is chosen such that there is a only one band below the well barrier. As a result, the part of a wave packet tunneled into the upper band will not be dragged along the lattice while the part remained in the lower band will be dragged along. This leads to a separation of a BEC cloud after a certain time of acceleration. By integrating the left-behind wave packet, we obtain the tunneling probability. This technique was actually used in experiments to measure the tunneling probability [5,14].

Fig. 2 shows our results of the nonlinear Landau–Zener tunneling probability with $V = 0.2$ for various values of $C$. The initial Gaussian wave packet is $\phi(x, t = 0) = e^{-x^2/\omega^2}$, where $\omega$ is the width of the condensate and its typical values used is $\omega = 325$. Optical lattice potential is turned on from $V(x) = 0$ to $V(x) = V$ for $t \leqslant 40$ to achieve an inhomogeneous BEC Bloch wave. Then it is boosted with an acceleration $a$ for two Bloch periods, and moves with a constant velocity afterward.

In general, tunneling probability is greater for larger $C$ at a fixed acceleration $a$, and greater for

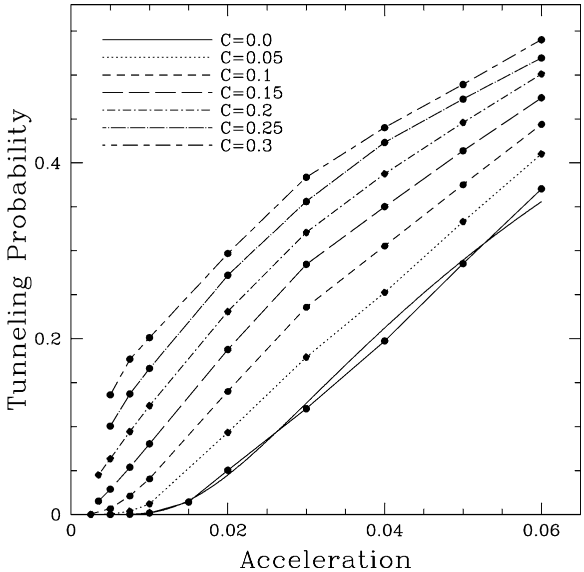


Fig. 2. Tunneling probability as a function of acceleration $a$ for various values of $C$ and $V = 0.2$. The case of $C = 0$ is compared to the exponential function $e^{-0.061/a}$ indicated by solid line without (•).

large accelerations $a$ at a fixed $C$. For $C < V$, the tunneling probability changes with acceleration $a$ rather exponentially and in particular, it goes to zero when $a \to 0$. This is exactly what is predicted in Ref. [10]. Therefore, this result implies that the inhomogeneity has only negligible impact on the tunneling probability when $C < V$.

When $C > V$, we also do not see much impact of the inhomogeneity of BEC: the tunneling probability does not depend on the acceleration $a$ exponentially and in particular, when the data is extrapolated to the adiabatic limit, $a \to 0$, the tunneling probability seemingly tends to a nonzero value. This clearly shows that these two unique characteristics of nonlinear Landau–Zener tunneling are not washed away by the inhomogeneity of the BEC cloud. The experimental observation of them will be viewed as the confirmation of the loop structure. However, since the acceleration cannot be made arbitrarily small in a real experiment and our simulations, one may argue the validity and confidence on the extrapolation of the data to the adiabatic limit of zero acceleration. We believe that this problem can be partially solved by repeating the experiment with many different densities of BECs. Consistency among different sets of data will be a strong support of the validity of extrapolation. On the other hand, the observation of nonexponential dependence of the tunneling probability on the acceleration will provide an unambiguous evidence.

## 4. Bloch oscillations

Bloch oscillations occur when a Bloch state is driven across the Brillouin zone by a small external field [14]. For a BEC in an optical lattice, Bloch oscillations can be achieved by dragging the lattice with a small acceleration as reported experimentally in Refs. [5,7]. In order to observe these oscillations, a key requirement is that the acceleration must be small enough so that the tunneling to the upper band is negligible. If tunneling probability into the upper band is increased, say, by large accelerations, Bloch oscillations can be destroyed.

What is interesting with BECs is that we have another way to increase the tunneling probability besides increasing the acceleration. It is to increase the density of the BEC. Especially, as discussed in the last section, when the density is high enough such that $C > V$, there is a nonzero lower limit on the tunneling probability as the result of the loop structure in the energy band. In other words, no matter how small the acceleration is, we will not be able to observe Bloch oscillations due to the nonzero adiabatic tunneling when $C$ is big enough. Therefore, the observation of breakdown of Bloch oscillations will provide another way to detect the loop structure.

In an experiment, one can repeat the measurements with increasing densities of BECs for a fixed small acceleration. One expects to observe Bloch oscillations when the density is low; as the density increases, the oscillations will deteriorate and eventually be destroyed. This is indeed the case as shown in Fig. 3 from our simulations. In this figure, we show currents of the condensate, $j = \int (\hbar/m)\,\mathrm{Im}(\phi^* d\phi/dx)\,dx$, as a function of time in the accelerating frame for a small $a$. For $C < V$, Bloch oscillations are preserved during the first Bloch period; however, for $C > V$, Bloch oscillations are disrupted and completely destroyed during the first Bloch period. Even for the case of $C = V$, Bloch oscillations are seriously disrupted.

In our view, this is a much better way to detect the loop structure shown in Fig. 1 than the method

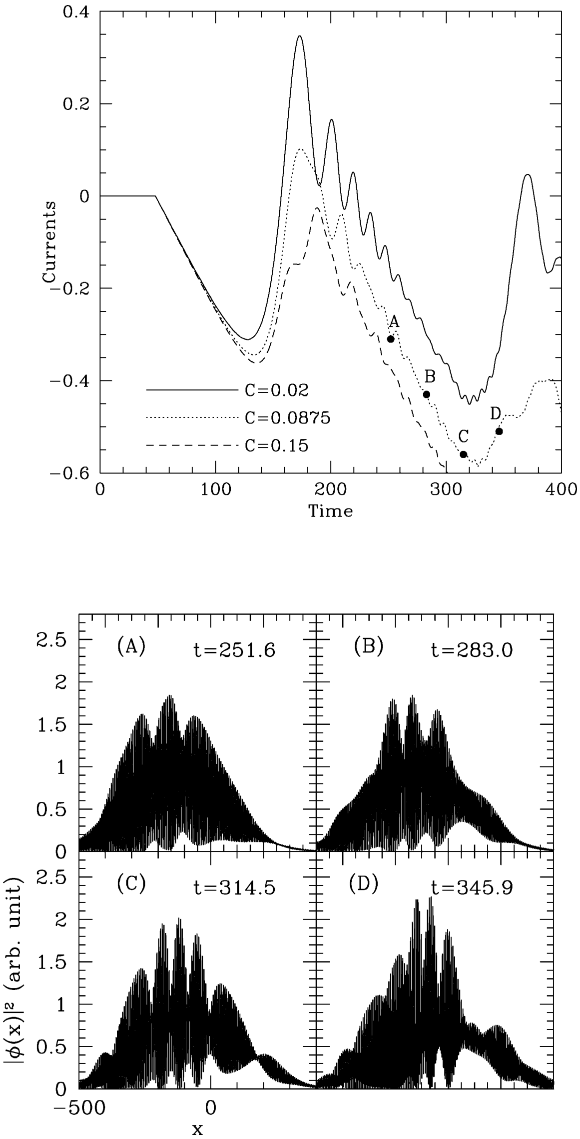


Fig. 3. Top panel: Currents as a function of time for various values of $C$ at $V = 0.0875$ and $a = 0.005$. Bloch oscillations are destroyed clearly when $C > V$. Results are shown for $C = 0.02, 0.0875$, and 0.15 in the accelerating frame. Bottom panel: The wave functions at different points of breakdown of Bloch oscillations, as indicated in the top panel at $t = 251.6$ (A), 283.0 (B), 314.5 (C), and 345.9 (D). The smoothness of the wave functions shows no signs of dynamical instability, implying the destruction of Bloch oscillations is the result of nonlinear Landau–Zener tunneling.

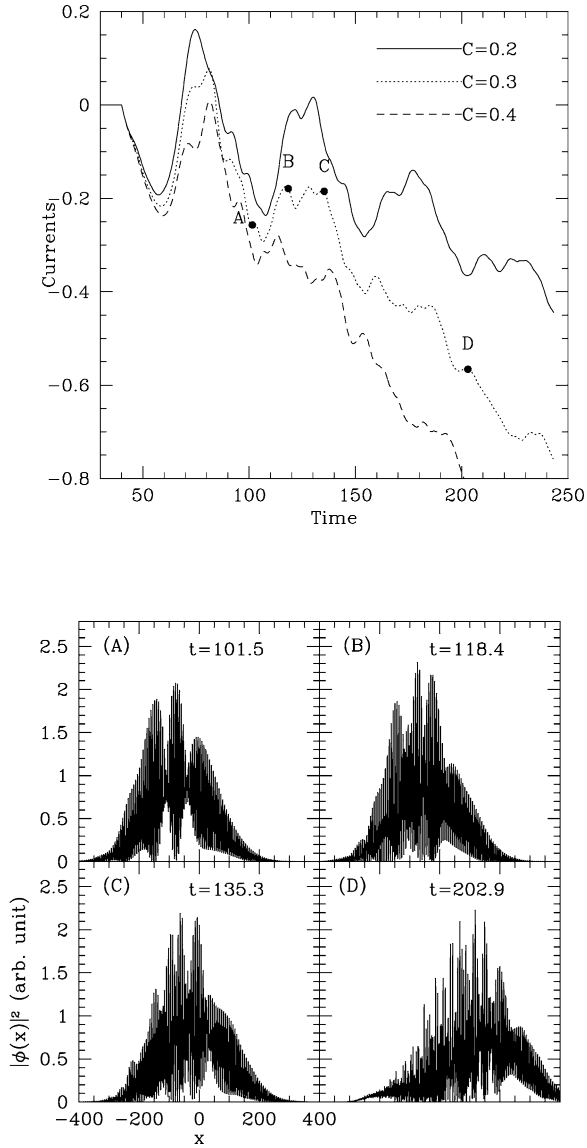


Fig. 4. Top panel: Currents as a function of time with $V = 0.3175$ and $a = 0.02$. Results are shown for $C = 0.2, 0.3$, and 0.4 in the accelerating frame, and we see the destruction of Bloch oscillations even when $C < V$, indicating that it is caused by the dynamical instability. Bottom panel: Densities of the wave function, $|\phi(x)|^2$, for $C = 0.3$ at $t = 101.5$ (A), 118.4 (B), 135.3 (C), and 202.9 (D). The "spiky" or "messy" wave functions at points C and D are the result of dynamical instability.

discussed in the last section regarding the nonlinear Landau–Zener tunneling. With increasing densities, the measurement of the tunneling probability based on the separation of a BEC cloud will become more difficult because the higher the density the faster the cloud expands. This makes the separation more difficult. With Bloch oscillations, we do not need to worry about this difficulty.

One caution we need to take is that there is another mechanism for breakdown of Bloch oscillations, the dynamical instability of Bloch waves discussed in Refs. [2,6,12]. The instability will cause the system stray away from the Bloch states with very small amount of perturbations or noises. This is demonstrated in Fig. 4, where we see the destruction of Bloch oscillations even when $C < V$ for a very small acceleration. The "spiky" or "messy" wavefunctions in the bottom panel signal the onset of dynamical instability.

The dynamical instability always exists in the neighborhood of $C \sim V$ [6,12], where the breakdown of Bloch oscillations starts to take place. As a result, one must make sure that the breakdown of Bloch oscillations is the result of the loop structure, instead of the dynamical instability. One way is to use weak optical lattices (small values of $V$). In this case, the growth rates of unstable modes are small therefore the dynamical instability will not dominate in the first few oscillations as seen in the bottom panel of Fig. 3. These smooth wave functions indicate that the dynamical instability is yet to come into play. Therefore, the destruction of Bloch oscillations in the upper panel is purely due to the nonzero adiabatic tunneling resulted from the loop structure.

## 5. Conclusion

In summary, using extensive and realistic calculations we have demonstrated experimental feasibility to detect the loop structure in the BEC Bloch bands. We suggested two possible scenarios: the observation of breakdown of Bloch oscillations and nonlinear Landau–Zener tunneling in an accelerating lattice. Experiments similar to what we suggest have already been carried out with low density BECs [5], where the corresponding coupling strength is in the range of $C = 0.026$–$0.04$. The values used in our calculations cover the range of $C = 0.05$–$0.3$. This means that the signatures of loop structure studied in this Letter can be observed by increasing the density of BECs by two to ten times. This is certainly possible with the current experimental set-ups. The density of the BEC in Ref. [5], $10^{14}$ cm$^{-3}$, can be increased to meet the requirement considering the BECs of density as high as $3 \times 10^{15}$ cm$^{-3}$ have already been achieved [18].

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## References

[1] D.-I. Choi, Q. Niu, Phys. Rev. Lett. 82 (1999) 2022.
[2] B. Wu, Q. Niu, Phys. Rev. A 64 (2001) 061603.
[3] O. Zobay, B.M. Garraway, Phys. Rev. A 61 (2000) 033603.
[4] S. Burger, F.S. Cataliotti, C. Fort, F. Minadri, M. Inguscio, M.L. Chiofalo, M.P. Tosi, Phys. Rev. Lett. 86 (2001) 4447.
[5] O. Morsch, J.H. Müller, M. Cristiani, D. Ciampini, E. Arimondo, Phys. Rev. Lett. 87 (2001) 140402;
M. Cristiani, O. Morsch, J.H. Müller, D. Ciampini, E. Arimondo, Phys. Rev. A 66 (2002) 043409.
[6] B. Wu, R.B. Diener, Q. Niu, Phys. Rev. A 65 (2002) 025601.
[7] B.P. Anderson, M.A. Kasevich, Science 282 (1998) 1686.
[8] C. Orzel, A.K. Tuchman, M.L. Fenselau, M. Yasuda, M.A. Kasevich, Science 291 (2001) 2386.
[9] M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, I. Bloch, Nature 415 (2002) 39;
D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys. Rev. Lett. 81 (1998) 3108;
V.A. Kashurnikov, N.V. Prokof'ov, B.V. Svistunov, cond-mat/0205510.
[10] B. Wu, Q. Niu, Phys. Rev. A 61 (2000) 023402.
[11] D. Diakonov, L.M. Jensen, C.J. Pethick, H. Smith, Phys. Rev. A 66 (2002) 013604.
[12] M. Machholm, C.J. Pethick, H. Smith, Phys. Rev. A 67 (2003) 053613.
[13] L.D. Landau, Phys. Z. Sov. 2 (1932) 46;
G. Zener, Proc. R. Soc. London, Ser. A 137 (1932) 696.
[14] M. Ben Dahan, E. Peik, J. Reichel, Y. Castin, C. Salomon, Phys. Rev. Lett. 76 (1996) 4508;
M. Raizen, C. Salomon, Q. Niu, Phys. Today 50 (1997) 30.
[15] P. Massignan, M. Modugno, Phys. Rev. A 67 (2003) 023614.
[16] A. Görlitz, J.M. Vogels, A.E. Leanhardt, C. Raman, T.L. Gustavson, J.R. Abo-Shaeer, A.P. Chikkatur, S. Gupta, S. Inouye, T. Rosenband, W. Ketterle, Phys. Rev. Lett. 87 (2001) 130402.
[17] R.B. Diener, B. Wu, M.G. Raizen, Q. Niu, Phys. Rev. Lett. 89 (2002) 070401.
[18] D.M. Stamper-Kurn, M.R. Andrews, A.P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, W. Ketterle, Phys. Rev. Lett. 80 (1998) 2027.