Lecture 10

Classical theory of dispersion

Up to this point, we have taken the linear constitutive relations $\vec{D} = \varepsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$ for granted ,with ε and μ experimental parameters .We have seen that the index $n = \sqrt{\varepsilon_r \mu_r}$ is determined by ε and μ , but we have no microscopic picture of why ε_r might be $\neq 1$.(As usual, we are ignoring magnetic materials and considering $\mu_r = 1$, as we can for most of optics .)

Also , we have considered the propagation of light in free space and linear isotropic media , but we have no picture of <u>why</u> light can be <u>emitted</u> (i.e. generated). The flip side of that is that we also have no microscopic picture of optical <u>absorption</u>.

The so-called <u>Lorentz model</u>, or <u>classical electron oscillator</u> (CEO) model, gives a <u>purely classical</u> picture of both the <u>index of refraction and the emission + absorption</u> of light .This classical model turns out to still have a few "<u>free parameters</u>" which cannot be calculated from first principles , but must be obtained from experiments, as we shall see. Those free parameters ("resonant frequency" and "oscillator strength") can only be predicted on the basis of a quantum-mechanical theory of matters.

The starting point if the model is the recollection that electromagnetic waves are generated by accelerating charge or, equivalently, by the time-varying currents. We all know that antennas (e.g. dipole antennas) are useful for generating e.m. waves in the radio frequency region of the spectrum .However, there are no antennas for optical-frequency e.m. waves!! Light is generated by atoms or molecules, not tiny metal antennas!

Lorentz proposed that atoms be considered to be small ($\ll \lambda$ in size) harmonically oscillating dipoles.



Dipole moment $\vec{P} = e\vec{r}, \vec{r} =$ electron-nucleus separation resonant frequency ω_0 determined from experiment.

In a laser class, you will consider in more detail how real atoms can behave as classical dipole oscillators. For our proposes, we can be very pictorial.

Suppose an atom has two (relevant) quantum states:



 φ_1 = symmetric (e.g. s-like) wavefunction ω / energy E_1

 φ_2 = antisymmetric (e.g. p-like) wavefunction ω / energy E_2

Now suppose that somehow you can put the atom in a superposition of these two states:

$$\varphi_{tot} = \varphi_1 + \varphi_2$$

Consider the average position of the electron as a function of time:

$$<\vec{r}(t)>\equiv\int \varphi_{tot}^{*}\vec{r}\varphi_{tot}dV$$

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And remember that

$$\begin{aligned}
\varphi_{1}\left(\vec{r},t\right) &= \varphi_{1}\left(\vec{r}\right)e^{-iE_{1}t/\hbar} \\
\varphi_{2}\left(\vec{r},t\right) &= \varphi_{2}\left(\vec{r}\right)e^{-iE_{2}t/\hbar} \\
&< \vec{r}\left(t\right) &>= \int \left|\varphi_{2}\right|^{2} \int dV + \int \left|\varphi_{1}\right|^{2} \int dV + \underbrace{\left[\int \varphi_{2}\left(\vec{r}\right)^{*}\vec{r}\varphi_{1}\left(\vec{r}\right)dV\right]}_{0}e^{-i(E_{1}-E_{2})t/\hbar} + c.c. \\
&= 0 = 0
\end{aligned}$$
The average position (i.e. the

dipole moment) oscillates with the energy difference between the two states !!

For a pictorial view of how an atom can look like an oscillating dipole, see Fowles fig.8.8.

