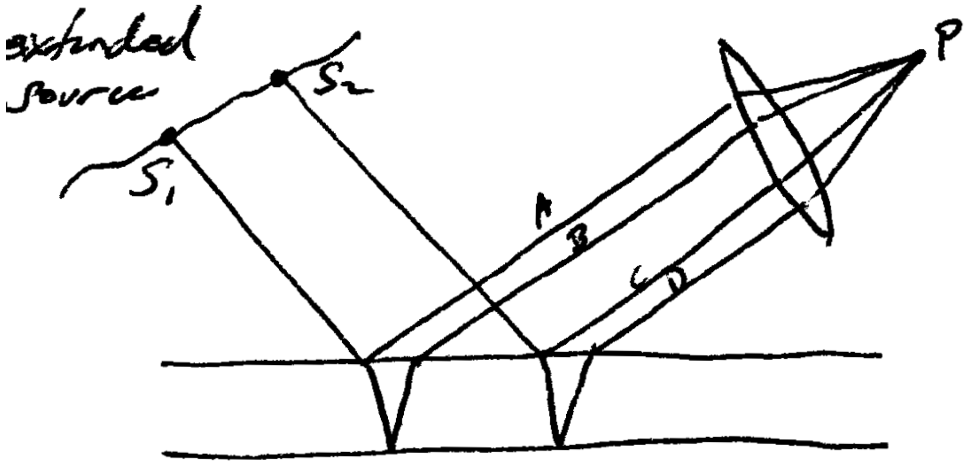
Lecture 29

Consider now the following geometry :



The points on the source S_1 and S_2 emit light incoherently with respect to each other if the source is a classical (thermal or fluorescent) source. (Only if the source is the output of a laser will S_1 and S_2 have a definite phase relationship. For a classical source, there is no way for the emitting dipoles at S_1 and S_2 to "talk to each other" and thereby establish any definite phase relationship.) So, what happens at P?

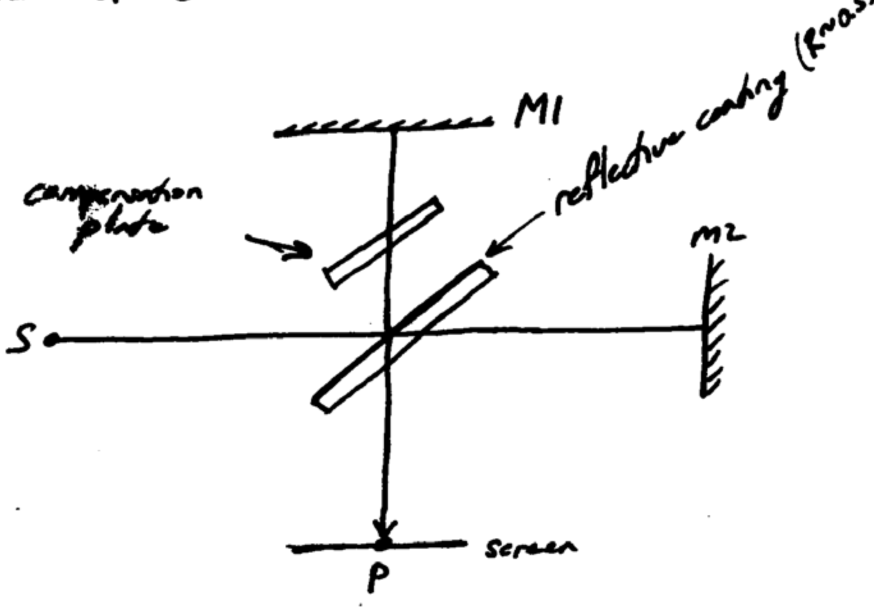
Both S_1 and S_2 contribute to the fringe at P, because the angle for constructive (or destructive) interference is the same.

- The rays from S_1, taking paths A and B) add coherently at P
- The rays from c ,taking paths C and D)add coherently at P
- However, S_1 adds incoherently to S_2 at P (each source point produces its own fringes in the observation plane, and the intensities of these add .

Michelson Interferometer

These are many interferometer which in essence result in the coherent addition of two waves (for a discussion of many of them, see Born+Wolf chap.7).The final one we will consider is the Michelson interferometer. This interferometer will be particularly useful for us in our consideration of coherence. The basic optical set up is as follows:

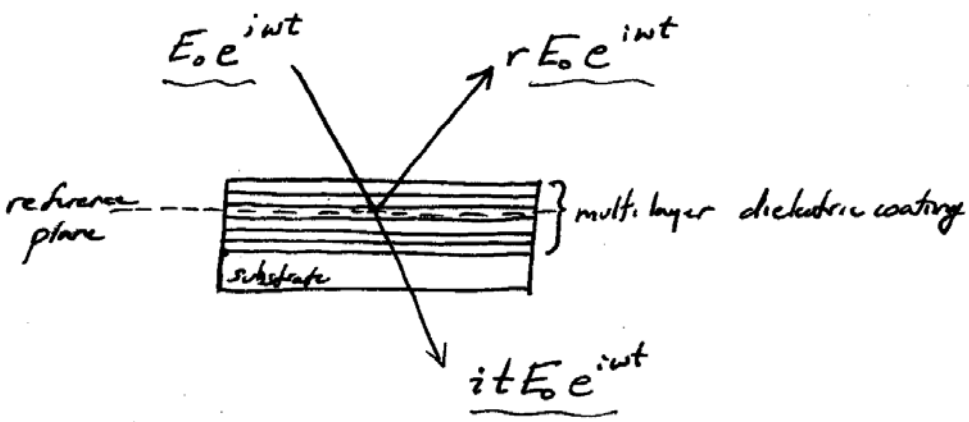
as follows .



m_1 , m_2 are identical, R=1 reflectors.

The partially reflecting coating on the beam splitter is designed to reflect 50% of the light, so equal intensities propagate in each "arm" of the interferometer.
The compensation plate is included so that the light passing through each arm goes through the thickness of glass of the beam splitter exactly three times.

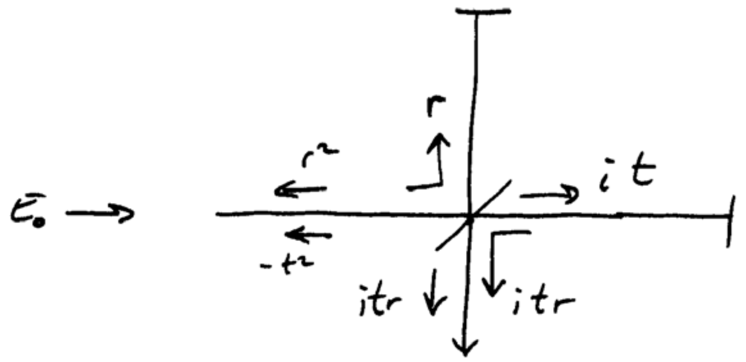
- Each beam also reflects off the beam splitter once and is transmitted once. Thus if the mirrors m_1 and m_2 have equal distances from the beam splitter, then the optical paths are identical for the two arms , so there must be constructive interference at P.
- A note on mirrors and phase shifts :
  It can be shown from quite general considerations (power conservation and time reversal symmetry in particular), that for a loss less dielectric mirror, the reflected and transmitted waves has a 90° phase shift with respect to each other .Here the wave is reflected from an effective single surface (or "reference plane") with amplitude reflectivity r and transmissivity t , where $t=\sqrt{1-r^2}$ (r, t real)



Extensive discussion and proof of this points can be found in Siegman, Lasers, chap.11.1

Haus, Wave+ Fields in Optoelectrances chap.3.

Thus to calculate the reflected or transmitted fields in a Michelson Interferometer, consider the following diagram:



Reflected field:

$$E_R = r(rE_0) + it(itE_0) = (r^2 - t^2)E_0 = \left[r^2 - \left(1 - r^2\right)\right]E_0 = \left[2r^2 - 1\right]E_0$$

If $r^2 = 0.5$ (50% beam splitter), then $E_R = 0$ (no reflection!)

Transmitted field:

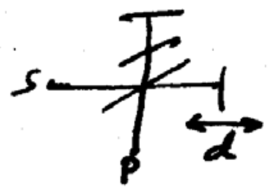
$$E_T = r(itE_0) + it(rE_0) = 2irtE_0$$

If $r^2 = 0.5$, then r=t=$\frac{1}{\sqrt{2}}$ and 2rt=1

So $E_T = iE_0$ and $|E_T| = |E_0|$ (complete transmission!)

The consequence of this is simple. When the mirrors are equidistant from the beam splitter, the optical paths for light propagating from S to P in each arm are identical. However, there is a $90° + 90° = 180°$ phase shift between the waves from the two arms propagating back toward the source S. Thus there is complete destructive interference in the "reflected" direction and complete contructive interference in the "transmitted" direction,i.e. towards point P. Thus all the light from S (along the axis of the interferometer) ends up at P; more is reflected back towards the source.

Now consider the intensity at P as one of the mirrors is moved a distance d:



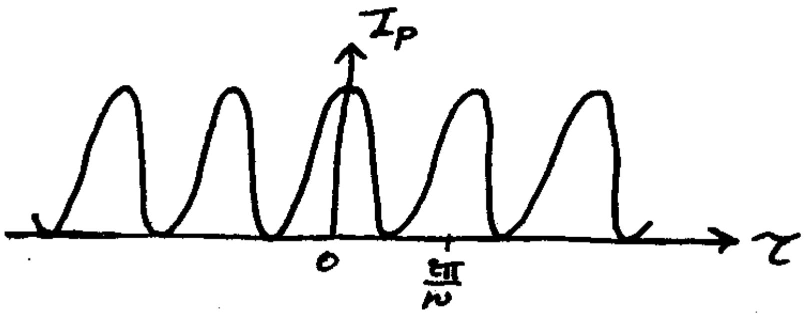
The optical path length in that arm changes by an amount 2d (assuming an index n=1 , as is usually the case). Thus the phase difference is $\delta = \frac{2\pi}{\lambda_0} \cdot 2d = \frac{2\omega d}{c}$

⇨ $I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\frac{2\omega d}{c}$

If the beam splitter is exactly 50% , $I_1 = I_2$ =>

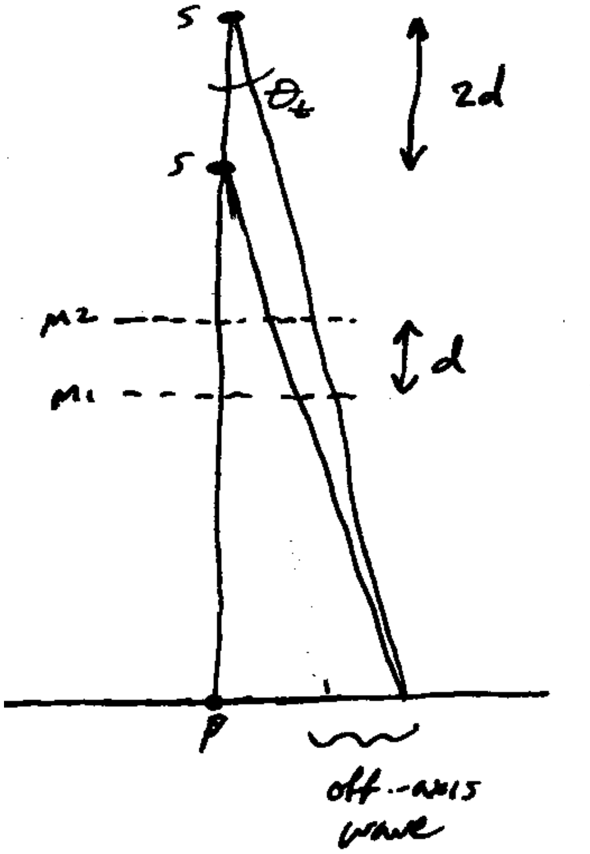
$$I = I_1\left[1+\cos\frac{2\omega d}{c}\right] = \boxed{I_1\left[1+\cos\omega\tau\right]}$$

Where $\tau = \frac{2d}{c}$ is the retardation time, or optical delay caused by displacing the mirror by distance d.



<u>Intensity pattern</u> on <u>observation screen</u>:
Note that the point source S puts out spherical waves.

The easiest way to understand the intensity pattern is to realize that mirrors m_1 and m_2 give rise to <u>virtual images</u> of S as seen from the observation screen:



(see Moore pattern and 2 point sources displaced vertically)

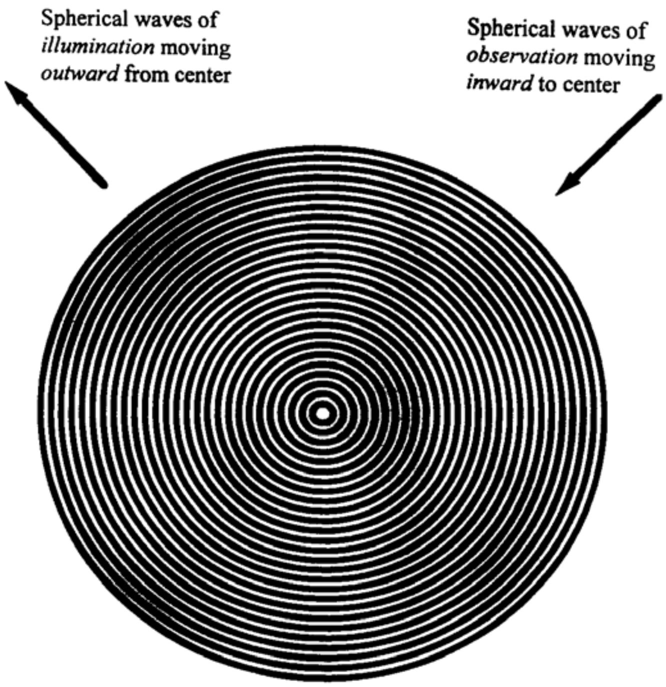


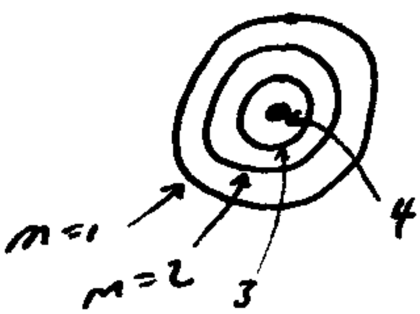
Figure 2.18 ILLUMINATION: A point source radiates monochromatic coherent light. The spherical light waves move *outward* with the speed of light like the concentric rings on water after you throw a stone. We call these outward-moving waves *waves of illumination.*

OBSERVATION: The figure could also be used to visualize how spherical light waves move *inward,* e.g., to a point of observation. Other waves that do not move toward the center will not reach the observer. If the detection method is phase sensitive, like holography, only coherent light will be detected. Thus the observer sees only the concentric waves shown in this figure. We call these inward-moving waves *waves of observation.*

The off-axis waves will also give rise to constructive interference when

$$\delta = \frac{2\pi}{\lambda} \cdot 2d\cos\theta_t = 2\pi m \qquad \text{m= integer}$$

- ⇨ Get fringes of equal inclination
- ⇨ Rings appear when you look from P towards S

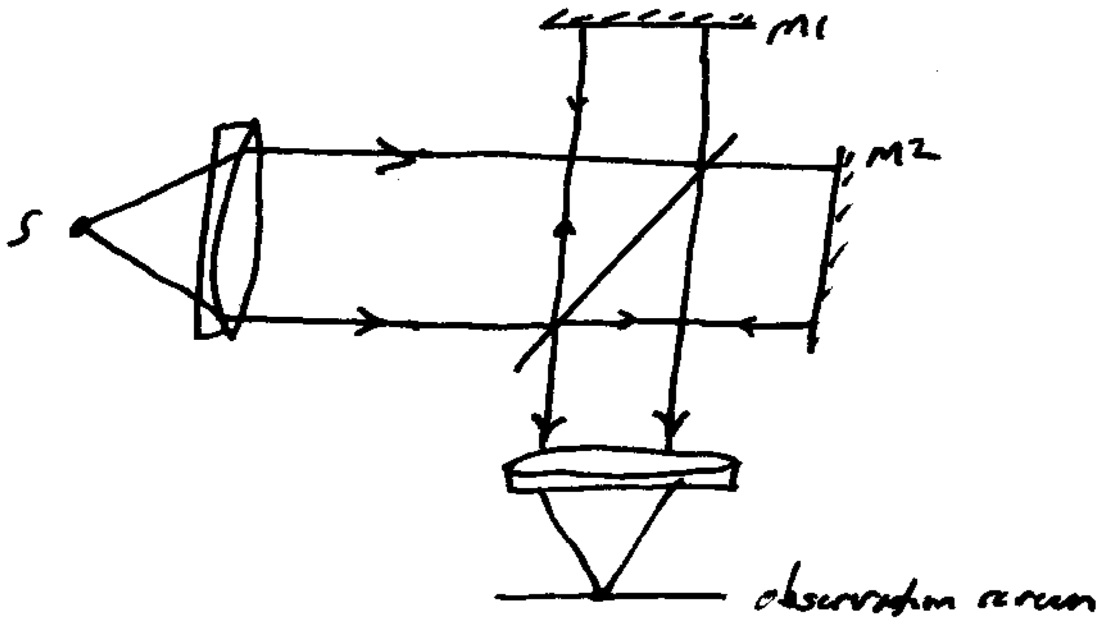


e.g. for $2d = 4\lambda_0$:

if the source emits several wavelength, the rings will appear colored, since different wavelengths satisfy the constructive interference condition at different angles. A nice picture is given in Guenther color plate 4-19.

Tyman-Green Interferometer

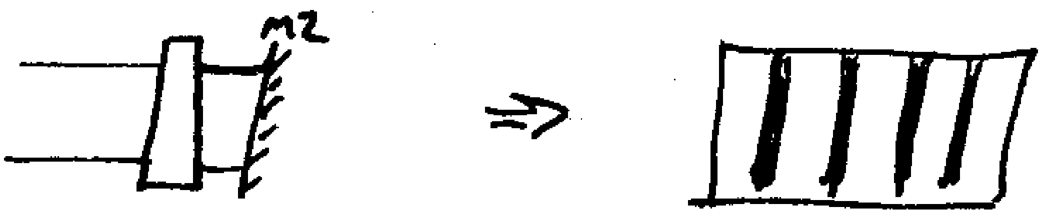
This is an important variation on the Michelson interferometer. The basic idea is to put a collimating lens in front of the source so that plane waves rather than spherical waves are incident on the interfetometer.



The value of the T-G interferometer is that a single fringe is observed when m_1 and m_2 are flat (i.e. the screen is uniformly bright or dark if the conditions for constructive on destructive interference are met).

Now, optical components may be placed in one arm of the interferometer in order to see their effect on the wavefront.

e.g. wedge (in what might normally be an optical flat)



e.g. test lens aberrations

- see Born+ Wolf figs.7.41 and 7.42

(+ compare of Born+ Wolf figs 5.31)

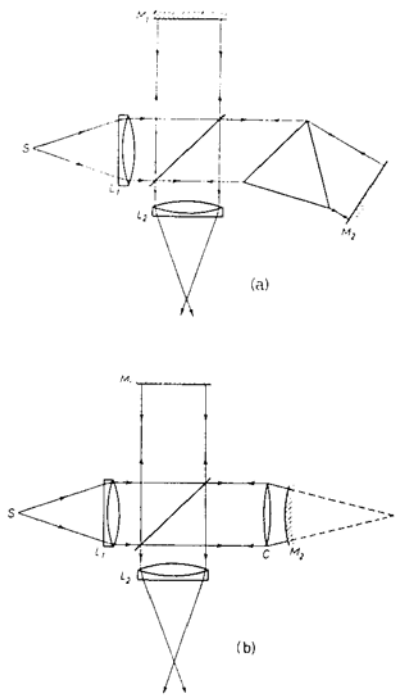
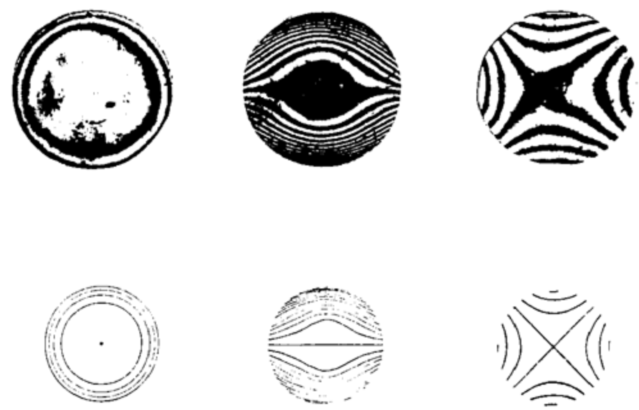


Fig. 7.41. The Twyman–Green interferometer.
(a) Arrangement for testing a prism.
(b) Arrangement for testing a camera lens.



(a) *Spherical aberration* $\Phi_{max} = 0{\cdot}4\lambda_0$ Gaussian focal plane

(b) *Coma* $\Phi_{max} = 2\lambda_0$ Gaussian focal plane

(c) *Astigmatism* $\Phi_{max} = 2{\cdot}1\lambda_0$ Central plane

Fig. 7.42. Twyman–Green interference patterns from lenses showing primary aberrations. Φ_{max} is the maximum wave aberration in the exit pupil. The patterns above are observed, those below are calculated.

(After R. Kingslake, *Trans. Opt. Soc., London*, **27** (1927), 94.)