0.1 Prism GVD

The scheme is similar to gratings:(Figure 1) the first prism provides angular (and hence group delay) dispersion, and the second prism is adjusted to compensate the angular dispersion so the phase fronts of the various frequency components of the pulse are parallel:

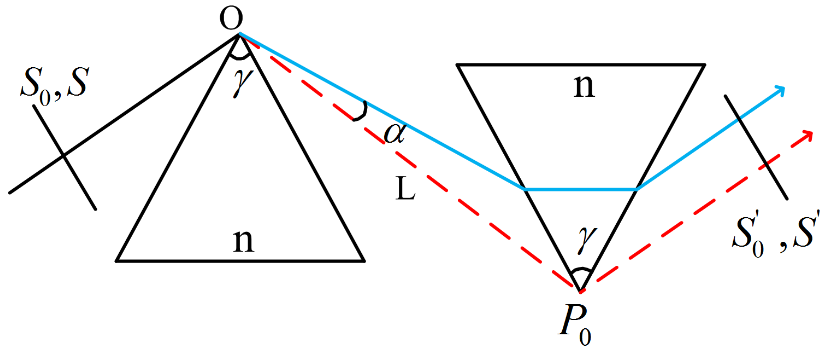

Figure 1: Prism GVD.

1. **Brewster prism** are usually used in conjunction with p-polarised light to essentially **eliminate reflection losses.** This is a **huge** advantage that the prism pair has over the grating pair. (Recall that a Brewster prism is one in which the **minimum deviation** condition corresponds to an angle of incidence= Brewster's angle.)

2. Output has spatial chirp, as with gratings. Solution: double pass using a mirror in the S' plane, or (frequently done in dye lasers) use 4 prisms.(Figure 2)

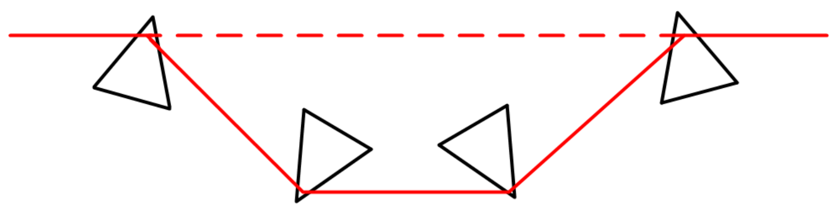

Figure 2: use 4 prisms to deal with spatial chirp.

Note that this arrangement does **not displace** the beam from its original path. This has been a useful feature in some systems (especially dye lasers with ring cavities), in which alighment is more easily accomplished **without** the prisms, and then the prisms are inserted as a unit to perform dispersion compensation.

3. Note that the ray goes the apex of the prisms. This is to minimize the **material** contribution to the total GDD of the prism pair.

4. Before we go on to calculate the prism GDD, we can get a basic idea of the prism operation graphically:(Figure 3)

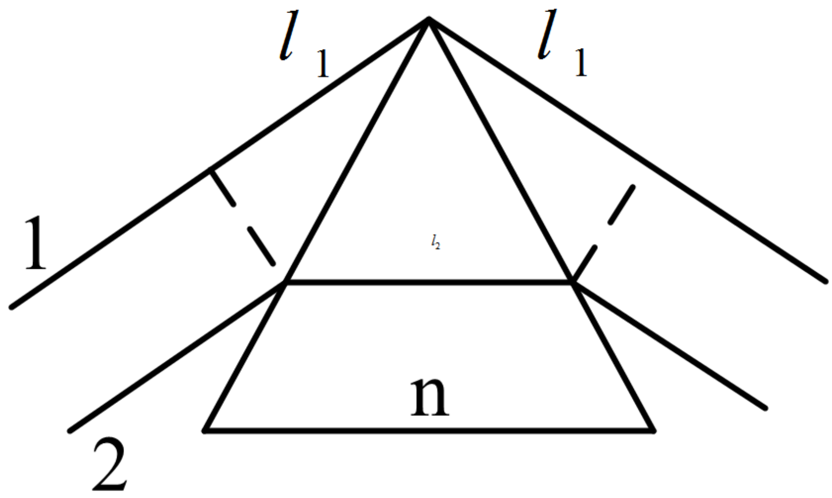

Figure 3: basic idea of the prism operation.

a

The dashed lines are wavefronts, so the optical paths:

$$2l_1 = nl_2 \text{ (equal phase delays)}$$

However, beam **2** has a larger **group** delay, since it goes through more glass, and $v_p > v_g$. Thus (for any given frequency component of the pulse), the pulse acquires a **tilt.** (Figure 4)

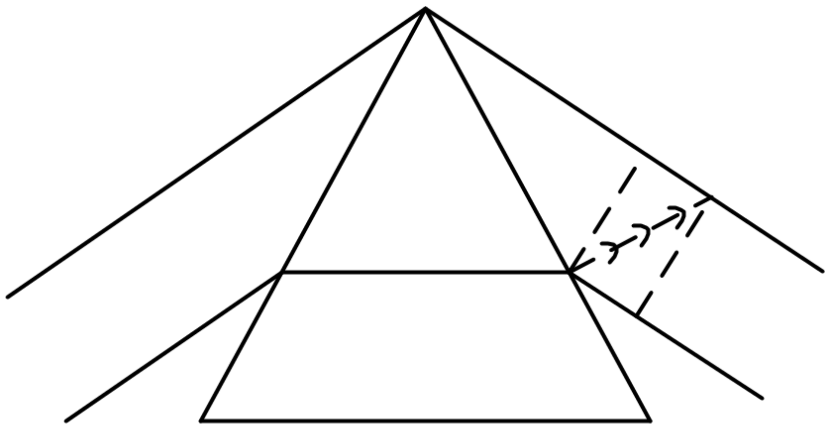

Figure 4: the pulse acquires a tilt.

The second prism of the pair does **two** things:

- it undoes the **angular dispersion** of the first prism, so that the **wavefronts** of the different frequency components are parallel (Figure 5)

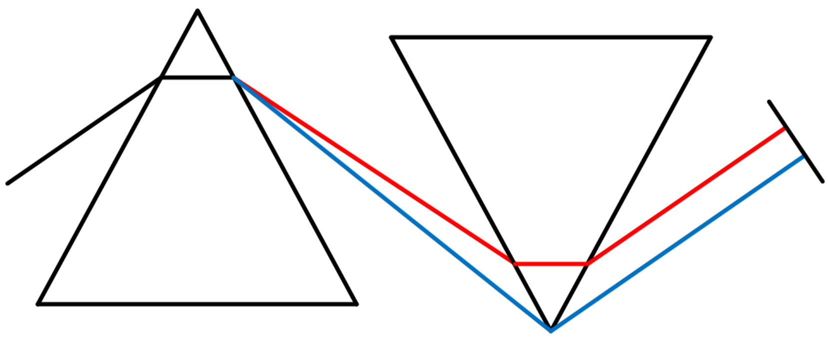

Figure 5: the wavefronts of the different frequency components are parallel.

- It undoes the pulse **tilt** from the first prism. Thus for each frequency component, there is no pulse tilt after the second prism, only a group delay. (Figure 6)

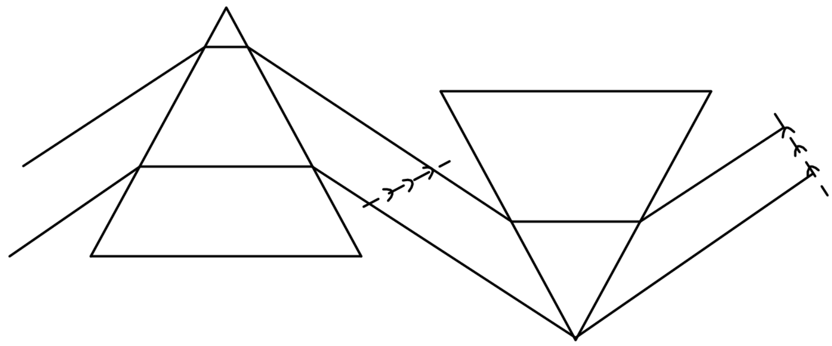

Figure 6: for each frequency component, there is no pulse tilt after the second prism, only a group delay.

We have shown that angular dispersion in general gives negative GDD. Thus although the pulse tilt is undone for both the red and glue components of the pulse, there is a negative group delay:(Figure 7)

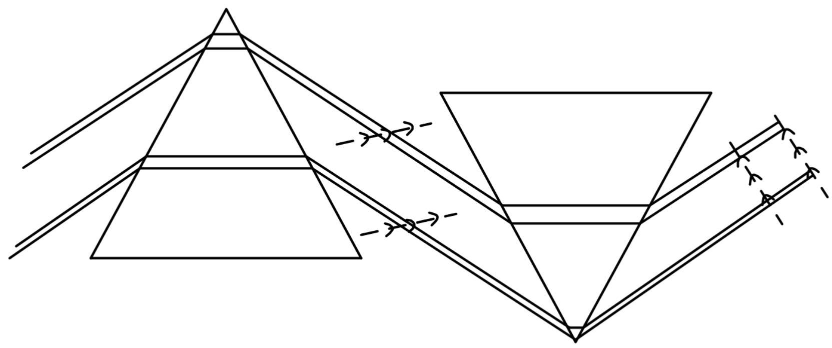

Figure 7: although the pulse tilt is undone for both the red and glue components of the pulse, there is a negative group delay.

Note that (for the case of positive dispersion) the red is redshifted less than the blue $\Rightarrow$ goes through more glass in the second prism $\Rightarrow$ final pulse front has a larger group delay for red than blue $\Rightarrow$ negative GDD.

5. Suppose L $\rightarrow$ 0 (prisms in contact).(Figure 8) Then the GDD of the material would imply that the red would come out before the blue (positive GDD).

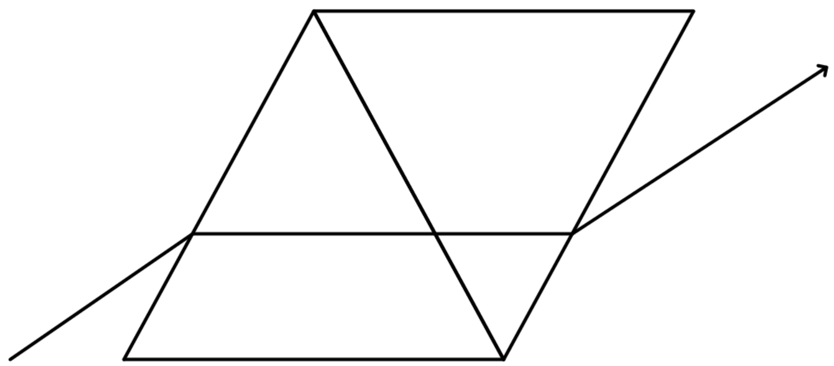

Figure 8: L $\rightarrow$ 0 (prisms in contact).

Thus the prisms must be sufficiently **far apart** that the negative GDD due eto angular dispersion is greater than the positive GDD due to material dispersion.

6. We are now in a position to calculate the prism GDD.

Consider a ray at the carrier frequency ω_0, and calculate the dispersion of other frequencies within the pulse bandwidth that propagate at an angle α with respect to the central frequency.

$\Rightarrow$ we are after

$$\frac{d\alpha}{d\omega} = \frac{d\alpha}{dn}\frac{dn}{d\omega}$$

We know

$$\frac{dn}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{dn}{d\lambda}$$

$$= \text{prism material dispersion}$$

$\Rightarrow$ we need $\frac{d\alpha}{dn}$ for the Brewster prism

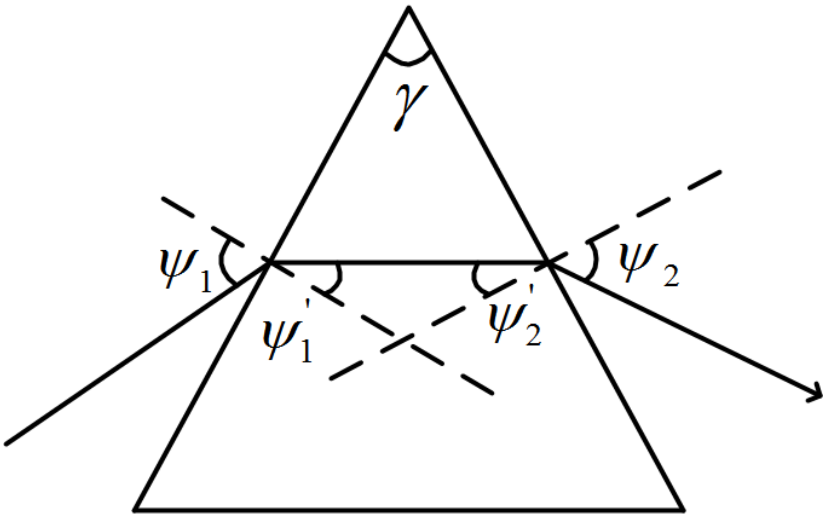

Figure 9: minimum deviation condition: $\psi_1 = \psi_2$**.**

minimum deviation condition: $\psi_1 = \psi_2$ (so $\psi_1' = \psi_2'$) (at ω_0 only!) (Figure 9)

Brewster prism:

$$\tan\psi_2 = n \text{ (fixes all } \psi)$$

apex angle:

$$\gamma + (90^0 - \psi_1') + (90^0 - \psi_2') = 180^0$$

$$\Rightarrow \gamma = \psi_1' + \psi_2'$$

$$\Rightarrow \frac{d\psi_1'}{dn} = -\frac{d\psi_2'}{dn}$$

Apply Snell's law to 1st interface:

$$\sin\psi_1 = n\sin\psi_1'$$

$$\cos\psi_1 \tfrac{d\psi_1}{dn} = 0 \text{ since input angle is fixed}$$

$$\Rightarrow \sin\psi_1' + n\cos\psi_1' \frac{d\psi_1'}{dn} = 0$$

$$n\frac{d\psi_1'}{dn} = -\tan\psi_1' = -n\frac{d\psi_2'}{dn}$$

Snell's law at 2nd interface

$$\sin\psi_2 = n\sin\psi_2'$$

$$\cos\psi_2 \tfrac{d\psi_2}{dn} = \sin\psi_2' + n\tfrac{d\psi_2'}{dn}\cos\psi_2'$$

$$= \sin\psi_2' + \tan\psi_1'\cos\psi_2'$$

$$\frac{d\psi_2}{dn} = \frac{1}{\cos\psi_2}(\sin\psi_2' + \tan\psi_1'\cos\psi_2')$$

(true in general, but now we can apply the min. dev. condition)

$$\tfrac{d\alpha}{dn}|_{\lambda_0} = -\tfrac{d\psi_2}{dn} = -\tfrac{1}{\cos\psi_2}(\sin\psi_2' + \sin\psi_2') \text{ since } \psi_1' = \psi_2'$$

$$= -\frac{2\sin\psi_2'}{\cos\psi_2} = -\frac{2(\frac{1}{n}\sin\psi_2)}{\cos\psi_2} = -\frac{2}{n}\tan\psi_2$$

$$= -\frac{2}{n}\cdot n = -2$$

Now we can get the GDD:

$$\varphi'' = -\frac{L\omega_0}{c}(\frac{d\alpha}{d\omega})^2$$

$$= -\frac{L\omega_0}{c}(\frac{d\alpha}{dn}\frac{dn}{d\omega})^2$$

$$= -\frac{L\omega_0}{c}(4)(\frac{\lambda^2}{2\pi c}\frac{dn}{d\lambda})^2$$

$$\varphi_2'' = -4L\frac{\lambda^3}{2\pi c^2}(\frac{dn}{d\lambda}|_{\lambda_0})^2$$

(angular dispersion condition to prism dispersion)

We would not go through the derivation, but the thrid order term is

$$\frac{d^2\alpha}{dn^2} = \frac{2}{n^3} - 4n$$

$\Rightarrow$

$$\varphi_2''' = \frac{12L\lambda^4}{2\pi^2 c^3}\{(\frac{dn}{d\lambda})^2[1 - \lambda\frac{dn}{d\lambda}(\frac{1}{n^3} - 2n)] + \lambda(\frac{dn}{d\lambda}\frac{d^2n}{d\lambda^2})\}|_{\lambda_0}$$

ratio of 3rd order to 2nd order

$$R_{32} = |\frac{\varphi'''\Delta\omega}{3\varphi''}| = [1 - \lambda\frac{dn}{d\lambda}(\frac{1}{n^3} - 2n) + \lambda\frac{\frac{d^2n}{d\lambda^2}}{\frac{dn}{d\lambda}}]_{\lambda_0}\frac{\Delta\lambda}{\lambda}$$

$\Rightarrow$ both **material parameters** and the pulse **bandwidth** will determine the relatie importane of the 3rd order.

e.g. $R_{32} \simeq 0.1$ for 20 fs pulses at 620 nm in fused silica. (This may not be too big a problem for a single pass application. In laser cavities, however, the pusle makes repetitive passes through the prisms, and small phase errors build up, so 3rd order phase considerations are much more crucial.)

So far, we have considered only the **angular** dispersion contribution. There is also a material contribution.(Figure 10) (e.g. for L˜0, it is like the dispersion in a plane parallel piece of glass!)

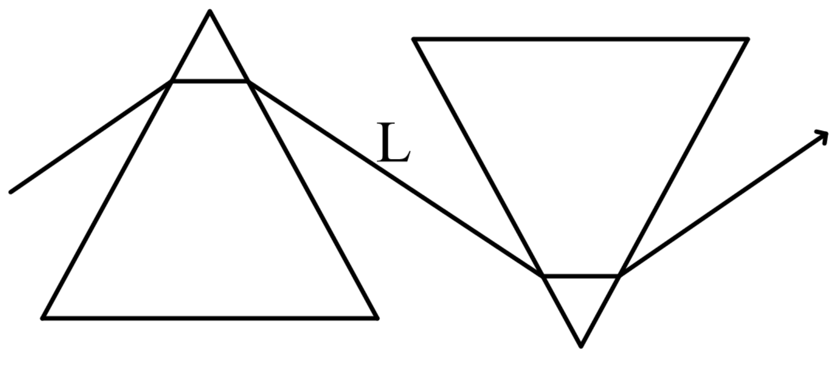

Figure 10: material dispersion contribution.

def. d= mean accumulative path length in glass (i.e. length at ω_0)

$\Rightarrow$ must add $\varphi''_d = \frac{\lambda^3}{2\pi c^2} d \frac{d^2 n}{d\lambda^2}|_{\lambda_0}$

in 3rd order

$$\varphi'''_d = -\frac{\lambda^2}{4\pi^2 c^3}[3\lambda^2 \frac{d^2 n}{d\lambda^2} + \lambda^3 \frac{d^3 n}{d\lambda^3}]d$$

So that the **total** prism dispersion is

$$\varphi'''_{tot} = \frac{\lambda^3}{2\pi c^2}[d\frac{d^2 n}{d\lambda^2} - 4L(\frac{dn}{d\lambda})^2]_{\lambda_0}$$

The third order term is obtained b y adding $\varphi'''_d + \varphi'''_l$.

1. Note that

angular dispersion < 0

material dispersion > 0 (in visible+ near IR; $\lambda < 1.3\mu m$)

$\Rightarrow$ the **sum** can be **positive or negative**

$\Rightarrow$we can have an **adjustable** GDD by simply **varying** the amount of **material** d (this is easier than adjusting the spacing L, since varying d does not change the beam position by much) (Figure 11)

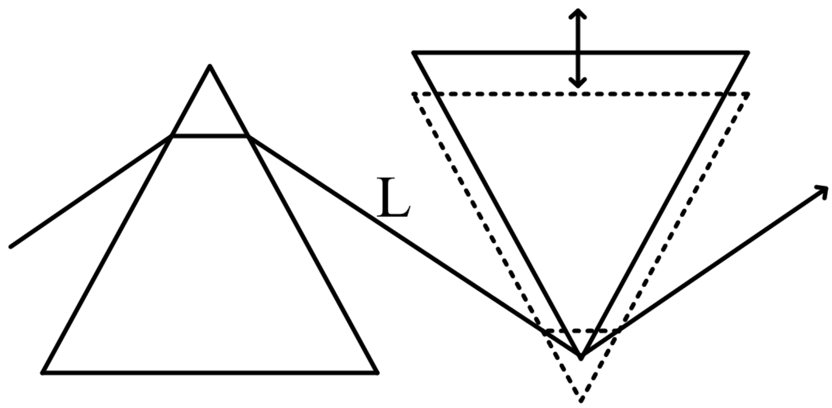

Figure 11: adjustable GDD by simply varying the amount of material d.

2. Note also that prism dispersion is generally much less than that of gratings. Here are some numbers for comparison. (Table 1) (SQ1= quartz)

Table 1: dispersion of prism and grating

device	λ_l (nm)	$\omega_l\,(fs^{-1})$	$\varphi''\,(fs^{-2})$	$\varphi'''\,(fs^{-3})$
f	620	3.04	550	240
	800	2.36	362	280
Brewster prism pair	620	3.04	-760	-1300
SQ1				
($L = 50$ cm)	800	2.36	-523	613
grating pair	620	3.04	-8.2 $\times 10^4$	1.1×10^5
$b = 20$ cm; $\beta = 0°$				
$d = 1.2\,\mu m$	800	2.36	-3×10^6	6.8×10^6

3. Note that the prisms must be **sufficiently far apart** to give negative dispersion. The minimum material path length d is something on the order of the beam diameter. (i.e. typically a few mm) in order to avoid vignetting; this establishes a minimum L in practice.

For reference, the following table shows prism dispersions for various materials; this is for a **single pass** through a **pair** of prisms at **800 nm.**

Table 2: dispersion of a pair of prisms

	Pair of prisms (L and d in cm)			
	$\varphi''\,(fs^{-2})$		$\varphi'''\,(fs^{-3})$	
prism media	d	L	d	L
titane-sapphire	1020.99	-26.00	753.32	-32.62
silica	683.56	-10.81	522.88	-12.14
SF10 glass	2850.13	-89.04	2020.50	-222.14
SF14 glass	3667.41	-122.66	3418.22	-356.04
LaK31 glass	1259.20	-31.39	897.59	-47.16
LaFN28 glass	1632.31	-44.82	1107.33	-81.10

In fact, for femtosecond oscillators, intracavity dispersion compensation is generally accomplished with prisms and not gratings for three reasons:

(i) lower dispersion

(ii) essentially no insertion loss

(iii) φ''' can be negative for prisms, while it is always positive for gratings and material. Thus a cavity compensated to 3rd order requires prisms.

0.2 Gratings again

The dispersion of gratings can be calculated in the same way, via

$$\varphi'' = -\frac{L\omega_0}{c}(\frac{d\alpha}{d\omega})^2_{\omega_0}$$

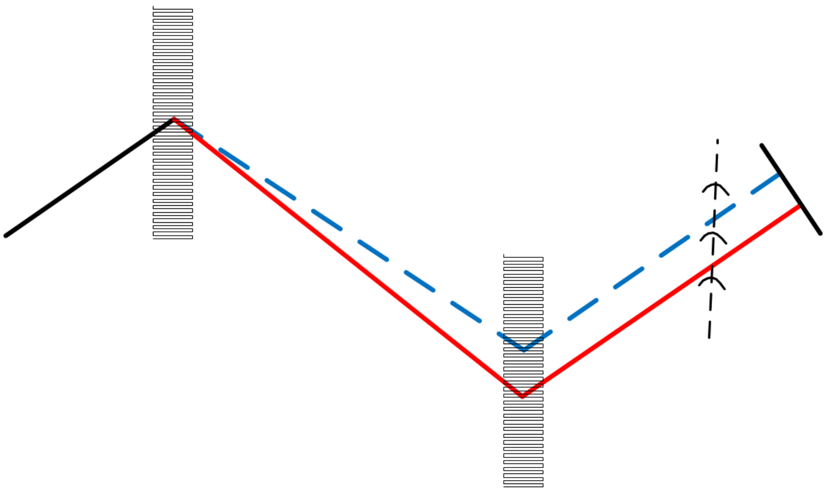

Figure 12: The dispersion of gratings.

From the diagram(Figure 12), $L = \frac{G}{\cos\theta}$.

grating eqn.

$$\sin\gamma = \sin\theta + \frac{\lambda_0}{d} = \sin\theta + \frac{2\pi c}{\omega_0 d}$$

and

$$\sin\gamma = \sin(\theta + \alpha) + \frac{2\pi c}{\omega d}$$

$$\sin(\theta + \alpha) = \sin\gamma - \frac{2\pi c}{\omega d}$$

$$\cos(\theta + \alpha)\frac{d\alpha}{d\omega} = \frac{2\pi c}{\omega^2 d}$$

$$\frac{d\alpha}{d\omega} = \frac{2\pi c}{\omega^2 d\cos(\theta+\alpha)}$$

We can evaluate this at $\alpha = 0$ since we are interested in φ'' at ω_0.

$$\frac{d\alpha}{d\omega}|_{\omega_0} = \frac{2\pi c}{\omega^2 d\cos\theta}$$

$\Rightarrow$

$$\varphi'' = -\frac{L\omega_0}{c}\frac{4\pi^2 c^2}{\omega_0^4 d^2\cos^2\theta}$$

$$= -\tfrac{4\pi^2 c}{\omega_0^3 d^2}\tfrac{G}{\cos^3\theta}\ (L = \tfrac{G}{\cos\theta})$$

$$(\times 2 \text{ for double pass})$$

$$= -\tfrac{GN^2\lambda^3}{\pi c^2\cos^3\theta}\ \text{(exactly as we found last time)}$$