**Lecture 24**

A complete discussion of image formation using various lens combination may be found in Hecht 5.2.

The procedure is quite straight forward:

1. Start with the object and just the first lens
2. Find the image formed by the first lens (may be real or virtual )
3. Consider the image formed by the first lens to be the object of the second , and find that image
4. Repeat seq uentially for all the lenses in the system.

Ex. compound microscope(not to scale!)

 

L- “tube length “ = 160mm usually

= 60 “diopters” (q diopter = 1)

Ex. two lenses of focal lengths and, spaced by a distance d <

This is considerably trickier –see Hecht fig.5.29 for an accurate drawing.

The procedure here is to first ignore the second lens and find the image produced by the first lens. Then trace a ray back through the center it is undeviated, and is thus a real ray through both lenses. Then ray 3 on the figure, which is parallel to the axis, goes through the focus of the second lens, and the intersection of rays 2 and 3 gives the location of the final image.

**Paraxial Ray Tracing and Matrix Optics**

Lipson 3.5

Guenther chap.5 pp.138-144 and 182-190

Siegmen’s lasers chap. 15 sections 1 and 2 only (on CTOOLS)

We consider only paraxial rays, propagating at small angles with respect to the z-axis.



 small =>　

Clearly, we need to specify only two parameters to completely characterize rays in the meridional plane , namely y and .

Sign convention: y and  are positive as shown

 (i.e. y>0 = “up” and  in ccw direction )

We shall not concern ourselves with the extensions to the theory required to describe skew rays; it will be sufficient for our purposes to consider only meridional rays.

Terminology: we will speak of the ray parameters y and  at “reference planes” corresponding to z = correct.

1. Propagation in free space



As the ray goes from reference plane  to , remains unchanged, and only y changes .

 

Define d =  = distance between two reference planes paraxial approximation: 

*  “ ray transfer equations “

These can be written in the following form:



The quantity  is called a ray vector. (note alternative conviction : ; this yields slightly different ray matrix from ours )

The matrix  is the ray transfer matrix of free space.

Our program in ray tracing will follow this same line of treatment. For every optical element, we will determine a ray transfer matrix which will be of the general form

 

(often called the “ABCD ” matrix of the system)

The usefulness of this approach should be obvious:





In other words, we can find the ray transfer matrix of a complicated optical system by simple matrix multiplication , yielding

 

Our first task, then, is to determine the transfer matrices for all the relevant optical elements.

1. Refraction at Planar Interface



Snell: 

Paraxial: 

 

* 
1. Refraction at spherical surface

First, we need a sign convention. We follow Guenther +Lipson R>0 if the center of curvature is to the right of the vertex , as in the following drawing (Guenther figs 5-9,10):



(vertex of origin O)

Note that the angler  and  have been greatly exaggerated for clarity. As drawn, they clearly would not satisfy the paraxial condition!

We use Snell’s law at the interface:



Now our job is to write  and  in terms of the ray vector angles and.

 

  (since  is negative for the ray pointing down )

 

The ray intersects the sphere at height y1, so

 (Paraxial approx. again)

=>Snell becomes



And 

The ray transfer matrix for a single spherical surface is therefore just

 

Digression: we are following a convention in Guenther and many other texts (e.g. Saled+Teich, Photonics), where the ratio of the indices of refraction  appears explicitly in the ray matrix. Note that for the refraction matrices, we have

 

In fact, the determinant of the free space matrix also satisfies this, since!

Theorem:  for any paraxial system, where  and  are the indices of refraction of the entrance and exit reference planes of the system.

Siegman’s Layers and many other texts define the ray matrix in such a way that  always. In their convention

****ray vector = 

It is important in consulting any text or publication to first determinate their convention!

Note: the planar surface result follows from the spherical surface matrix simply by taking!

1. Thin lens

A “thin” lens is one in which we can neglect the thickness (I.e. we don’t have to include the propagation in the glass – only the refraction at the surfaces.

The most general case is easily considered, in which the index of refraction may be different on each side of the lens: 

But for our purposes it will be sufficient to consider a lens in air, so that.

The thin lens matrix is obtained by multiplication



Where = radius of curvature of second surface

 = radius of curvature of first surface



Note that  and  can be positive or negative

Eg. 

Plano-convex biconvex biconcave meniscus

   

   

Imaging with one lens

In order to see the significance of the form of  above, consider the imaging problem

 

* We need a matrix taking us from the object to the image plane

Recall we have 

For an image to be formed at reference plane, all rays coming from appoint  must end up at a point, independent of .That means we must have B=0 for an image to be formed.

B=0 =>



This looks just like the thin lens imaging eqn.

 

If we identify the focal length as

 

This result is sometimes called the “lens-makers” equation, since it tells you what combinations of refractive index and radii of curvature will produce a lens of a desired focal length.

Back to the ray matrix for a thin lens:

 Given the above “lens-makers” equations, we can simplify the form of the matrix  on P.205:



Recall that we assumed n=1 at both the object and image planes. This yields



Example: 2 immediately adjacent thin lenses

 

The matrix of the combination is



Where the focal length of the combination is given by

 

Note that without the matrix formalism, the calculation of this would be a mess (see Hecht)! Here it is completely trivial.

Back to the one-lens imaging problem.

We now know that we can rewrite the object-image matrix  in the form

 S’= image distance

 S= object distance (<0)

Now we want to consider the physical meaning of the off-diagonal elements A and D.