**Lecture 4**

**Reflection and Refraction at Boundaries**

We have seen how plane waves propagate in vacuum or in isotropic homogeneous linear media. The next thing Maxwell’s equations can give us is a description of the propagation of light across a boundary between two media described by different values of and.



Clearly, in order to solve this problem we first need to know the **boundary conditions** on the fields at the interface. We consider the special case where no free charges or currents are present near the boundary, so the source-free Maxwell equations may be used (which is the case for most problems in optics). In integral form:

  

 

Consider first the closed surface integrals. Construct a “hockey puck” whose faces lie parallel to, and on opposite sides of, the interface:



=unit vector from i to t

=-dA in region i, and +dA in region t

=>

=>the normal components of D and B are continuous across the interface.

Now consider the line integrals. Construct a loop across the interface as follows:



As dh->0, the flux $Φ\_{B},Φ\_{D}$->0.

=>

Thus the tangential components of E and H must be continuous across the surface.

**Kinematic Conditions**

(see Guenther p65~7 for further discussion)

Suppose we have a monochromatic field incident on an interface, so the wave is of the form cos(t-**k·r**). In order for the above field continuity conditions to apply, it is clear that the phase of the wave must be continuous across the interface. This is illustrated for plane waves in the following picture:



The continuity of the phase also implies that **ki**, **kt (**and the reflected wave **kr**) must lie in the same plane. In homework 1, you will prove that translational invariables along a certain direction **s** also implies that the **s** component of three wave vector **k** is conserved i.e. is constant across the interface; this also implies that **ki kr** and **kt** lie in the same plane (which is termed the “plane of incidence”).

The phase continuity condition is usually characterized by wave vector diagrams.

Consider a monochromatic wave:



Where the index l can be i, r or t for the incident, reflected, or transmitted wave.

In the region i, the possible wave vectors which can propagate lie in the surface of a sphere of radius:



Let’s now suppose that the wave vector lies in the x-z plane so we can write:



Which is a form we will find very useful later when we consider optical waveguides?

Note that: 

If the interface lies in the x-y plane, we have the following configuration: 

The locus of possible wave vectors are thus a circle in the kx-kz plane.

Consider the case  (so )

The wave vector diagram is thus the following:



Note that the projection of  along the x-axis is the same for all 3 waves <-> conservation of momentum along x.

At z=0,, so continuous phase implies 

For the three waves:



So 

This gives us the kinematic condition at the interface:

 (Snell’s Law)

Thus the directions of the reflected and refracted waves are entirely determined by the phase continuity condition.

However, to get the amplitudes of the reflected and transmitted waves, one must apply the electromagnetic boundary conditions. The results will depend on the polarization of the incident field.

(1)S polarization (also calledorpolarization)

 ⊥ “plane of incidence” (plane defined by ki and kt)

Our convention:

 Material interface=x-y plane of z=0

 Plane of polarization=x-z plane



(2) P polarization (also called π polarization)

 ∥plane of polarization



Of course, for an arbitrary input polarization, one just considers the appropriate linear combination of these two cases.

Mnemonic: S-polarization =>  “skips” the materials interface

 P-polarization=> ”pokes” the interface

(1) S-polarization:

Incident:

 

  

Reflected:



 

Transmitted:

 

 

The boundary condition that the normal component of D is continuous is trivially satisfied, since there is no normal component for S-polarization light.

In fact, the field is entirely tangential, so we require:



The normal and tangential B-field conditions lead to :

 (Normal)

And

 (Tangential)

The normal equation above is actually redundant, by Snell’s law and.

Eliminating  from the tangential equ, we have



 

This allows the definition of the field reflection coefficient:



Or, using (\*), above becomes:





Thus yields:



And

 

These are the Fresnel refection and transmission coefficients for S-polarization. Of course, there is really only one independent variable, namely, since  is fixed by Snell’s Law.

In most cases in optics, a much simpler form can be used, since quite frequently  for both materials. Then a little algebra yields:



 

(2)P-polarization

The fields now have the form:

Incident:

 



Reflected:

 

 

Transmitted:

 

Now the normal B continuity is trivially satisfied.

The other conditions are:

Transverse E: 

Normal D: 

Transverse H: 

By Snell’s Law, it is clear that the latter two equations are equivalent. Applying the same algebraic steps as before yields the field reflection and transmission coefficents as:



And:



**Power reflection+transmission**

You can’t just square the field values to get the power reflection+transmission coefficients!(That’s because the calculation is for plane waves, which are infinite in extent. You have to consider the power per unit area which is incident on the interface-i.e. use Poynting’s vector.)

There are two options:

1. work out the field coefficients for the magnetic field, following the same procedure carried out above for the electric fields, use Poynting’s vector.
2. Consider the geometry+ modify S accordingly, as is done on the next page.



Clearly, all the power in area Ai covers an area Ao=Ai/cos in the plane of the interface. The power transmitted is in At, with Ao=At/cos.

Thus what is required is the normal component of the Poynting vector(which is the power flow across the boundary)

 for incident wave

 for transmitted wave

 for reflected wave

=>Power reflectivity



The power transmissivity is:



Plots of the reflection coefficients are shown in Guenther figs 3-5 and 3-6.

There are several interesting angles to be considered in particular.

1. Normal incidence: 0

Clearly, exactly at normal incidents, it is no longer possible to distinguish between S and P polarization. For  near 0, we can find the coefficients by taking  of the expressions for both S and P. We find:

P-polarization: 

S-polarization: 

(Trivial note: “index-matching”<-> nt=ni =>r=0 t=1)

The sign flip between the two cases is merely a result of the choice in sign conventions in setting up the geometries on page 27,and are not, therefore, physically significant. Usually sign convention (in most texts) is that for S-polarization (=>180°phase shift on reflection for nt>ni).

In either case, the electric field flips direction.

Note that, while r may be positive or negative, it is always real, which means the plane shift is either 0 or  (nothing in between).

(b)Brewster’s angle

 Looking at the expression for P-polarization:  when 

From snell:

 

Or when  , ,.

In other words, at Brewster’s angle, all P-polarized light is transmitted through the interface.

Note that the reflectivity never goes to zero for S-polarization light. If is on that basis one can make a cheap, if inefficient, polarizer from a simple piece of glass. A beam of arbitrary polarization is incident on the glass at Brewster’s angle, and all reflected light is then S-polarized.(the transmitted beam may still be unpolarized, but will have a greater degree of P-polarization.)

