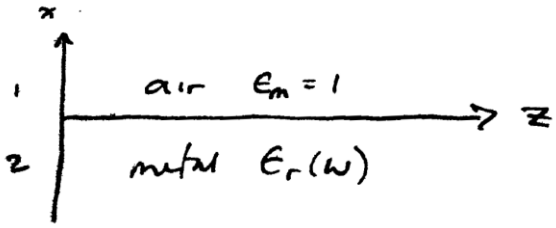
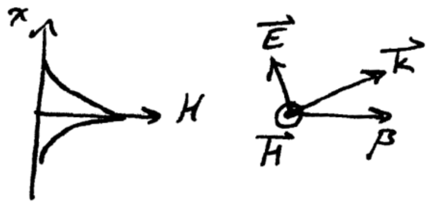
Lecture 15

Surface Plasmons

Surface plasmons are a good example of a surface or interface wave, in which the electromagnetic excitation is tightly confined to the interface.
It is evanescent in both regions and propagates along the interface. Surface plasmons are TM waves.





TM waves: $\vec{H}(x,z) = \hat{y}H(x)e^{i(\omega t-\beta z)}$

The Helomholtz wave eqn. is just as we used in waveguide problems:

$$\frac{\partial^2}{\partial x^2}H(x) + \left[\varepsilon(\vec{r})\frac{\omega^2}{c^2} - \beta^2\right]H(x) = 0$$

For a wave confined to the interface

$$H_1(x) = H_0 e^{-\gamma_1 x}, H_2(x) = H_0 e^{\gamma_2 x}$$

$\gamma_1, \gamma_2 > 0$ => Evanescent waves in both regions

$$\frac{\partial^2 H_1}{\partial x^2} = \gamma_1{}^2 H_1, \frac{\partial^2 H_2}{\partial x^2} = \gamma_2{}^2 H_2$$

$$\gamma_1{}^2 + \varepsilon_1\frac{\omega^2}{c^2} - \beta^2 = 0, \gamma_2{}^2 + \varepsilon_2\frac{\omega^2}{c^2} - \beta^2 = 0$$

$$\gamma_1 = \sqrt{\beta^2 - \varepsilon_1\frac{\omega^2}{c^2}}, \gamma_2 = \sqrt{\beta^2 - \varepsilon_2\frac{\omega^2}{c^2}}$$

Now apply the boundary conditions

1. transverse H continuous at boundary

$$H_1(0) = H_2(0) = H_0 \ \checkmark$$

2. normal D continuous at boundary

$$\nabla \times \vec{H} = +\frac{\partial D}{\partial t} = +i\omega\vec{D} = +i\omega(\underbrace{D_x\hat{x}}_{\text{Normal component is } D_x} + D_z\hat{z})$$

$$\nabla \times \vec{H}\big|_x = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}_x = \hat{x}(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z})$$

$$\vec{H} = H\hat{y} \Rightarrow H_y \neq 0, H_z = 0$$

⇨ require $\frac{\partial H_y}{\partial z}$ continuous

$$\frac{\partial H_j}{\partial z} = -i\beta H_j \quad \text{(j=1, 2)}$$

⇨ Require β same in both regions (obvious!)

This gives one relation between γ_1, γ_2:

$$\beta^2 = \gamma_1{}^2 + \varepsilon_1 \frac{\omega^2}{c^2} = \gamma_2{}^2 + \varepsilon_2 \frac{\omega^2}{c^2}$$

For $\varepsilon_1 = \varepsilon_m = 1$ (air) and

$$\varepsilon_2 = \varepsilon_r(\omega) = \varepsilon_L(1 - \frac{\omega_p^2}{\omega^2})$$

plasmon dielectric constants neglecting damping

(a)
$$\gamma_1{}^2 + \frac{\omega^2}{c^2} = \gamma_2{}^2 + \varepsilon_r(\omega)\frac{\omega^2}{c^2}$$

3. Transverse E continuous

$$\nabla \times \vec{H} = +i\omega\vec{D} = +i\omega\varepsilon_r\varepsilon_0\vec{E}$$

$$\vec{E} = \frac{i}{\omega\varepsilon_r\varepsilon_0}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H & 0 \end{vmatrix} = \frac{i}{\omega\varepsilon_r\varepsilon_0}(-\frac{\partial H}{\partial z}\hat{x} + \frac{\partial H}{\partial x}\hat{z}) \quad \text{since } \vec{H} = H\hat{y}$$

$$\frac{\partial H}{\partial z} = -i\beta H(x)e^{i(\omega t - \beta z)} = -i\beta H_0 e^{-\gamma_j x}e^{i(\omega t - \beta z)}$$
$$\frac{\partial H}{\partial x} = -\gamma_j H_0 e^{-\gamma_j x}e^{i(\omega t - \beta z)}$$
$$\gamma_{j=1} = \gamma_1; \gamma_{j=2} = -\gamma_2; \gamma_1, \gamma_2 > 0$$

Transverse E continuous => E_z continuous

=> $-\frac{\gamma_1}{\varepsilon_1} = \frac{\gamma_2}{\varepsilon_2}$ (bad notation, ε_2 is ε_r in region 2, not the imaginary part as previously!)

For $\varepsilon_1 = \varepsilon_m = 1$ (air) and $\varepsilon_2 = \varepsilon_r(\omega)$ = plasmon response

(b) $$\boxed{\varepsilon_r(\omega) = -\frac{\gamma_2}{\gamma_1}}$$

This could be inserted into (a) to relate γ_1 and γ_2.

Our primary interest here is to find the dispersion relation ω vs. β

$$\varepsilon_r(\omega) = -\frac{\sqrt{\beta^2 - \varepsilon_r(\omega)\frac{\omega^2}{c^2}}}{\sqrt{\beta^2 - \frac{\omega^2}{c^2}}}$$

$$\varepsilon_r{}^2(\beta^2 - \frac{\omega^2}{c^2}) = \beta^2 - \varepsilon_r \frac{\omega^2}{c^2}$$

$$\beta^2(\varepsilon_r{}^2 - 1) = \frac{\omega^2}{c^2}(\varepsilon_r{}^2 - \varepsilon_r) = \frac{\omega^2}{c^2}\varepsilon_r(\varepsilon_r - 1)$$

$$\beta^2(\varepsilon_r + 1)(\varepsilon_r - 1) = \frac{\omega^2}{c^2}\varepsilon_r(\varepsilon_r - 1)$$

$$\omega^2 = c^2\beta^2 \frac{(\varepsilon_r + 1)}{\varepsilon_r}$$

$$\boxed{\omega = c\beta\sqrt{\frac{(\varepsilon_r(\omega) + 1)}{\varepsilon_r(\omega)}}}$$

This is easier to plot if we solve for β:

$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon_r(\omega)}{\varepsilon_r(\omega) + 1}}$$

Using $\varepsilon_r(\omega) = \varepsilon_L(1 - \frac{\omega_p^2}{\omega^2})$, we have

$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon_L(1 - \frac{\omega_p^2}{\omega^2})}{1 + \varepsilon_L(1 - \frac{\omega_p^2}{\omega^2})}}$$

In the limit $\omega \ll \omega_p$,

$$\beta \simeq \frac{\omega}{c}\sqrt{\frac{\varepsilon_L(-\frac{\omega_p^2}{\omega^2})}{\varepsilon_L(-\frac{\omega_p^2}{\omega^2})}} = \frac{\omega}{c}$$

Which is the vacuum dispersion relation.

Note that β has a pole when

$$1+\varepsilon_r(\omega)=0 \quad \text{or} \quad \varepsilon_r(\omega)=-1 \quad \text{(generally true)}$$

$$\varepsilon_r(1-\frac{\omega_p^2}{\omega^2})=-1$$

⇨ after algebra

$$\boxed{\omega = \frac{\omega_p}{\sqrt{1+\frac{1}{\varepsilon_L}}}}$$

In many treatments of surface plasmons, one assumes $\varepsilon_L = 1$

So $$\boxed{\omega = \frac{\omega_p}{\sqrt{2}}}$$

Coupling light to surface plasmons:

Consider region 1, with $\varepsilon_m = 1$

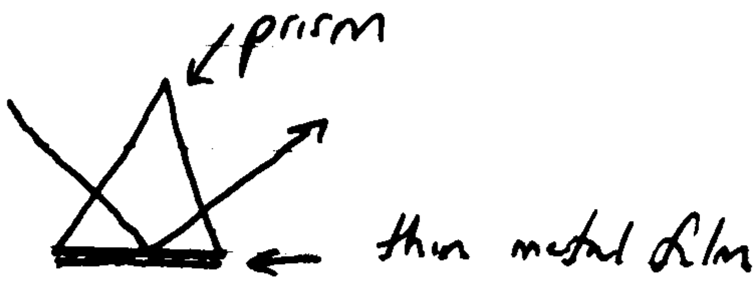
$$\gamma_1 > 0 \Rightarrow \beta^2 - \frac{\omega^2}{c^2} > 0 \Rightarrow \beta > \frac{\omega}{c}$$

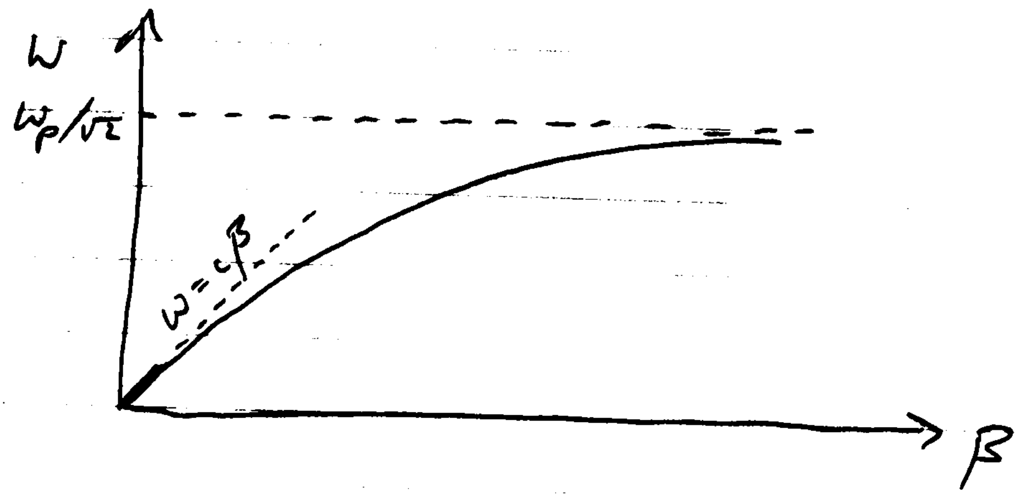
Thus one cannot couple to a surface plasmon wave from free space. One needs to use either of

(1) grating: couples to a specific $\beta = 2\pi/\Lambda$ where Λ = grating periodicity

(2) total internal reflection

recall $\beta = k_z > \frac{\omega}{c}$ in T.I.R.





ω
ω_p/√2
ω = cβ
β