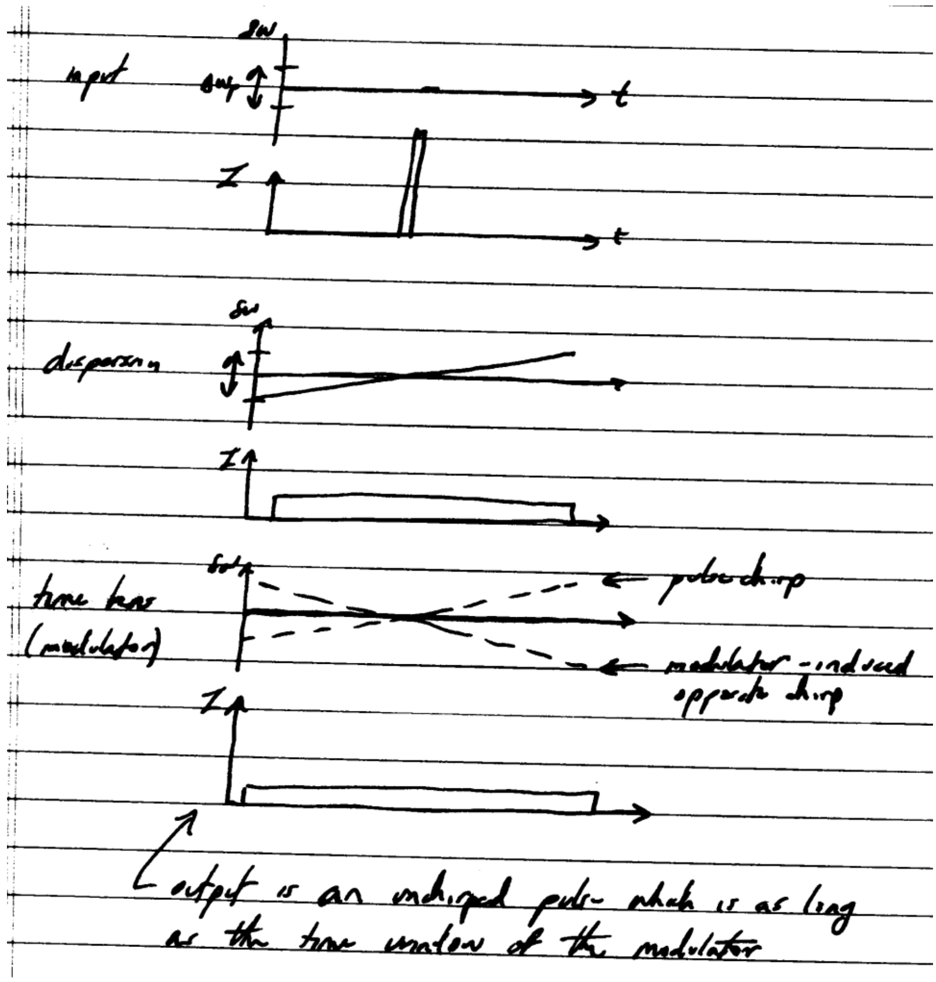
Temporal microscope



The analysis of the imaging system can now proceed since we know the transfer functions of each of the three stages in the system.

TABLE II
TIME- AND FREQUENCY-DOMAIN FUNCTIONS CORRESPONDING TO THE PHENOMENON OF DISPERSION AND THE ACTION OF A TIME LENS IN A TEMPORAL-IMAGING SYSTEM

	TIME	FREQUENCY	
INPUT DISPERSION	$G_1(\xi_1,\tau) = \frac{1}{\sqrt{4\pi i a}} \exp\left(i\frac{\tau^2}{4a}\right)$	$G_1(\xi_1,\omega) = \exp(-ia\omega^2)$	$a = \frac{\xi_1}{2}\frac{d^2\beta_1}{d\omega^2}$
OUTPUT DISPERSION	$G_2(\xi_2,\tau) = \frac{1}{\sqrt{4\pi i b}} \exp\left(i\frac{\tau^2}{4b}\right)$	$G_2(\xi_2,\omega) = \exp(-ib\omega^2)$	$b = \frac{\xi_2}{2}\frac{d^2\beta_2}{d\omega^2}$
TIME LENS	$H(\tau) = \exp\left(i\frac{\tau^2}{4c}\right)$	$\mathcal{H}(\omega) = \sqrt{4\pi i c}\exp(-ic\omega^2)$	$c = \frac{f_T}{2\omega_0}$

Again, we assume we are given the source field:

$$\mathrm{E}(\tau,0) = F^{-1}\{A(\omega,0)\}$$

At the end of the 1st stage of dispersion, the field is

$$\mathrm{E}(\tau,\xi_1) = F^{-1}\{A(\omega,0)g_1(\omega,\xi_1)\}$$

Where $g_1 = e^{-i\frac{\xi_1}{2}\beta_1''\omega^2} = e^{-ia\omega^2}$

Immediately after the time lens, the field is

$$\mathrm{E}(\tau,\xi_1+\epsilon) = F^{-1}\{A(\omega,0)g_1(\omega,\xi_1)\}\cdot H(\tau)$$

Where $\mathrm{H}(\tau) = e^{-i\omega_0\tau^2/2f_\tau} = e^{i\tau^2/4c}$

$$\mathrm{E}(\tau,\xi_1+\epsilon) = \frac{1}{2\pi}F^{-1}\{A(\omega,0)g_1(\omega,\xi_1)\} * h(\omega)$$

Finally, to get the output, we just propagate the frequency-domain field through the 2nd stage of dispersion

$$\mathrm{E}(\tau,\xi_2) = \frac{1}{2\pi}F^{-1}\{\left[\left(A(\omega,0)g_1(\omega,\xi_1)\right) * h(\omega)\right]g_2(\omega,\xi_2)\}$$

Carrying this through explicitly:

A $(\omega,0)\, g_1(\omega,\xi_1) * h(\omega) = \int A(\omega',0)g_1(\omega',\xi_1)h(\omega-\omega')d\omega'\cdot$

=>

$$\mathrm{E}(\tau,\xi_2) = \frac{1}{2\pi}\int d\omega e^{i\omega\tau}g_2(\omega,\xi_2)\,\frac{1}{2\pi}\int d\omega' A(\omega',0)g_1(\omega',\xi_1)h(\omega-\omega')\cdot$$

$$= \frac{1}{2\pi}\int d\omega' A(\omega',0)g_1(\omega',\xi_1)\,\frac{1}{2\pi}\int d\omega e^{i\omega\tau}g_2(\omega,\xi_1)h(\omega-\omega')\cdot$$

$$g_2 = e^{-ib\omega^2} \quad ,b = \frac{\xi_2}{2}\beta_2''$$

$$\mathrm{h}(\omega-\omega') = \sqrt{4\pi ic}\,\mathrm{e}^{-\mathrm{i}b\omega^2}\mathrm{e}^{-\mathrm{i}c(\omega-\omega')^2},\ \mathrm{c}=\frac{f_T}{2\omega_0}$$

Ignoring the unimportant constants at front, integral II is

$\mathrm{II} = \int d\omega\, e^{i\omega\tau}e^{-ib\omega^2}e^{-ic(\omega-\omega')^2}$

$= \mathrm{e}^{-\mathrm{i}c\omega'^2}\int d\omega\, e^{-i(b+c)\omega^2}e^{i\omega(\tau+2c\omega')}$

$= \mathrm{e}^{-\mathrm{i}c\omega'^2}e^{i(\tau+2c\omega')^2/4(b+c)}$

$=> \mathrm{E}(\tau,\xi_2) = \int d\omega' A(\omega',0)e^{-ia\omega'^2}e^{-ic\omega'^2}e^{i(\tau+2c\omega')^2/4(b+c)}\,\cdot$

$$= e^{i\tau^2/4(b+c)}\int d\omega' A(\omega',0)e^{-i(a+c)\omega'^2} * e^{i}(4c\tau\omega' + 4c^2\omega'^2)/4(b+c)\cdot$$

$$= e^{i\tau^2/4(b+c)}\int d\omega' A(\omega',0)e^{-i\left(a+c-\frac{c^2}{b+c}\right)\omega'^2} * e^{i(\frac{c\tau}{b+c})\omega'}\cdot$$

Two things to note:

(1) As usual, the output is the Fourier transform of an input spectrum which has been multiplied by a spectral quadratic phase
(2) The time faction in the Fourier transform has been scaled by c/(b+c)

Imaging condition

We can say that the output pulse is an "image" of the input pulse if the two waveforms are identical, except for a possible change in time scale (i.e. magnification or demagnification). This will be the case if the quadratic phase in the integrand vanishes (remember that it's the quadratic phase term that describes dispersion and hence changes in pulse shape).

If the quadratic phase vanishes, then

$$\epsilon(\tau, \xi_2) = e^{\frac{i\tau^2}{4(b+c)}} \int d\omega' \, A(\omega', 0) \, e^{i\tau'\omega'}$$

$e^{\frac{i\tau^2}{4(b+c)}}$ overall quadratic phase

$\int d\omega' \, A(\omega', 0) \, e^{i\tau'\omega'}$ F.T. of input field spectrum but timescale is shifted (magnification)

Quadratic phase term=0:

$$a + c - \frac{c^2}{b+c} = 0$$

$$ab+ac+bc+c^2 - c^2 = 0$$

$$ac+bc=-ab$$

$$\frac{1}{b} + \frac{1}{a} = -\frac{1}{c}$$

$$\frac{1}{\frac{\xi_1}{2}\beta_1''} + \frac{1}{\frac{\xi_2}{2}\beta_2''} = -\frac{1}{f_T/2\omega_0}$$

$$\frac{1}{\xi_1\beta_1''} + \frac{1}{\xi_2\beta_2''} = \frac{-\omega_0}{f_T}$$

$$\boxed{\frac{1}{\xi_1\beta_1''\omega_0} + \frac{1}{\xi_2\beta_2''\omega_0} = \frac{-1}{f_T}}$$

Note the strong resemblance to the spatial imaging condition:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

(Of course, unlike spatial diffraction, we can control the rate of pulse spreading via β_2'',so we can adjust both $\xi_1 and\ \beta_1''$ to adjust the imaging condition.)

Alternative form:

$$\frac{1}{\varphi_1''} + \frac{1}{\varphi_2''} = -\frac{\omega_0}{f_T}$$

One difference: the minus sign

Spatial diffraction: high frequencies diffract (i.e. spread out) more rapidly with propagation than low spatial frequencies

Normal dispersion: high frequencies travel more slowly with propagation than low frequencies

- ⇨ We might say that propagation leads to negative chirp for diffraction
  Positive chirp for normally dispersive propagation

To get real images, the lens must provide phase modulation in the opposite sense than the dispersion propagation.

- ⇨ A positive space lens has the opposite phase curvature from a “positive” time lens

We noted earlier that the output waveform is equal to the input waveform apart from an overall phase factor and, more importantly, a modified time scale.
=>define magnification

$$\mathrm{M} = \frac{\tau}{\tau'} = \frac{b+c}{c} = -\frac{b}{a} = -\frac{\xi_2 \beta_2^{"}}{\xi_1 \beta_1^{"}} = -\frac{\varphi_2^{"}}{\varphi_1^{"}}$$

This actually works! D. Bloom’s group at Stanford has huilt a number of time lenses using various kinds of electro-optic modulators. Some of their results are given in A. Godil et al. Appl. Phys. Lett. 62, 1047(1993). They used

$$\omega_m = 2\pi * 5.2\, GHz, \qquad \lambda_0 = 1.06\mu m$$

$$\Gamma_0 = 12\, radius \text{ (=A in their notation)}$$

$$\text{Aperture } \tau_a = 31 ps$$

- ⇨ 45 ps pulses focused (i.e. compressed, i.e. demagnified) to 6.7 ps

$$\Gamma_0 = 45\, rad$$

- ⇨ 45 ps -> 1.9 ps

<u>Resolution</u>

Recall in spatial imaging systems the spatial resolution is limited by diffraction by whatever limits the aperture D of the system (i. e. the pupil function)

One finds a resolution that looks something like

$$\Delta x \sim \lambda f *= \lambda \frac{f}{D}$$

(apart from a proportionality constant of order unity).

In general, one can express the output field as a convolution of the input field with the <u>impulse response</u> of the system:

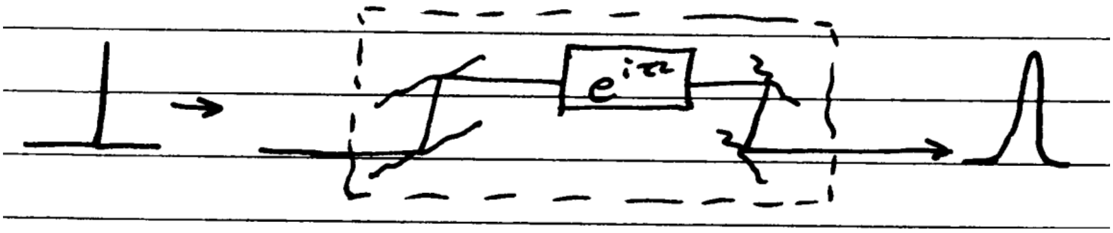
$$\mathrm{E}(\tau, \xi) = \int h_s(\tau - \tau^{'}) E(\tau^{'}, 0) d\tau'$$

If the aperture of the system is infinitely wide (as we assumed in our imaging calculation), then the impulse response is just a delta-function (the waveform is undistorted). Kolner has shown that, with no aperture the impulse response of the temporal imaging system is

$$h_s(\tau - \tau') = \frac{1}{\sqrt{M}} e^{\frac{i\omega_0 \tau^2}{2M f_T}} \delta(\tau - \tau')$$

Introduction of a finite aperture broadens the δ-function, thus limiting the temporal scale of the fastest features on the pulse.

Note: if $E(\tau',0) = E_0 h(\tau)$



The temporal microscope is a special case

$\frac{1}{d_i} + \frac{1}{d_0} = \frac{1}{f}$ => $f = d_0$

For this case, Koler has shown that the duration of the minimum resoluble feature on the input pulse is

$$\delta\tau_m = T_0 f_T^*,\ T_0 = \frac{2\pi}{\omega_0} = optical\ period$$

This looks just like the spatial case, with the resolution determined by the f-numberand the fundamental period (temporal and spatial) of the carrier wave.

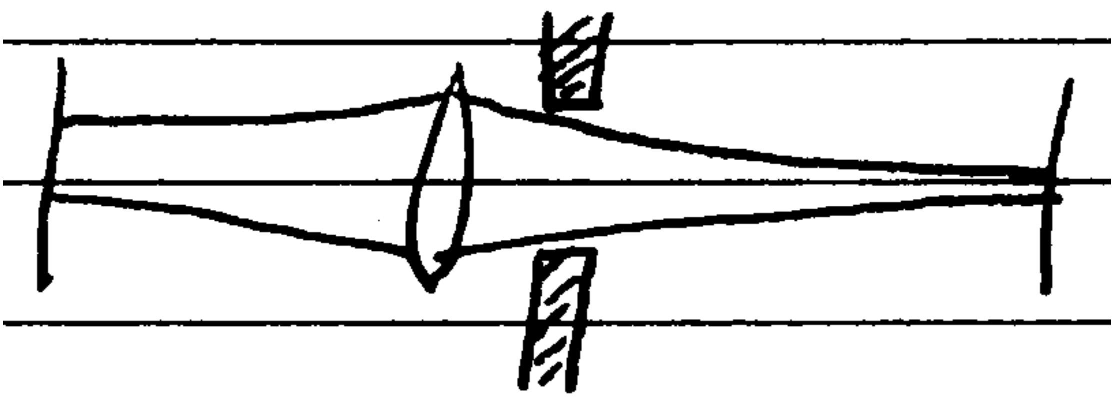
It can be shown that(again, for the temporal microscope)

$$\delta\tau_m = \frac{1}{\Gamma_0 f_m} = \frac{1}{\Delta f},$$

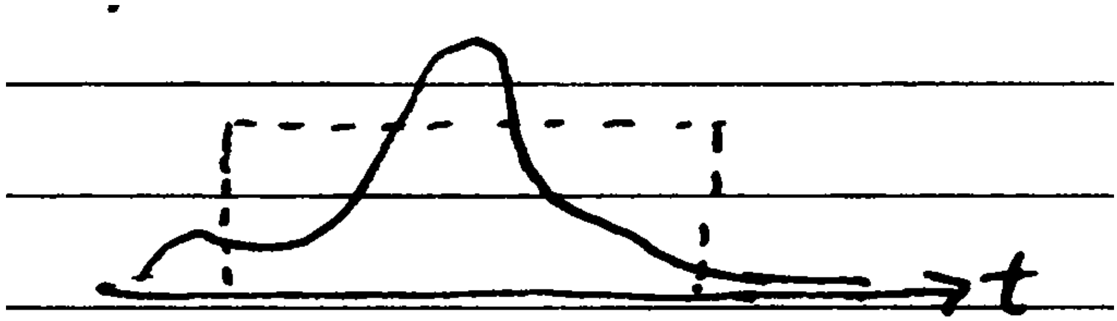
where Δf is just the new bandwidth of the chirped spectrum imposed by the modulator, which makes good intuitive sense.

One important difference between the spatial and temporal imaging systems:

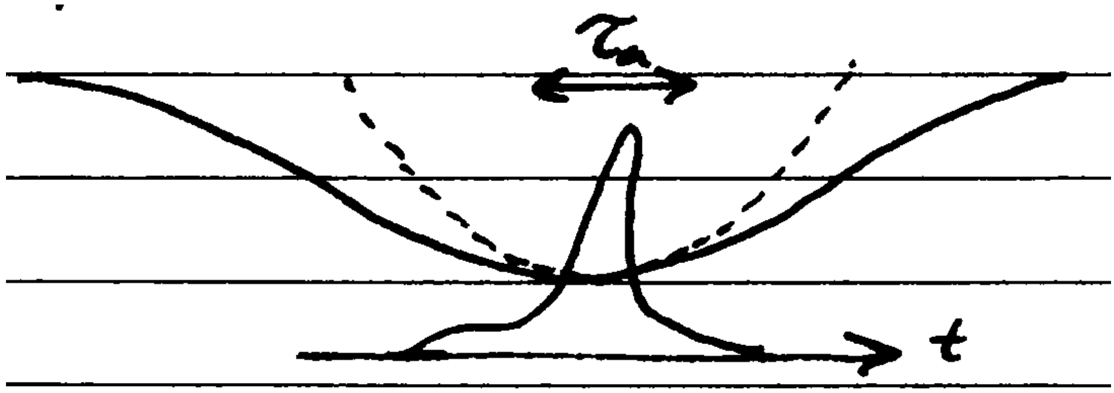
Typically, for a good spatial imaging system, the pupil function is a hard aperture



For a temporal imaging system, this is generally not the case. The aperture is not really something that prevents light from getting through if it’s not within the aperture



Rather, it’s just a measure of the pulse width of the primarily quadratic part of the phase of the lens:



Thus components of the pulse outside τ_m will not be neglected, but will be distorted in time(i.e. higher-order phase errors, leading to wings on the pulse)