

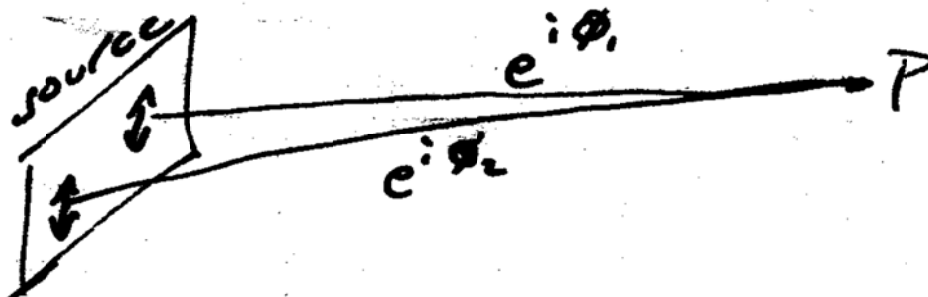
## Lecture 39

### Coherence

So far in our discussion of interference, we have assumed that the incident wave is a perfect harmonic wave  $\cos \omega t \rightarrow e^{i\omega t}$  (complex analytic signal).

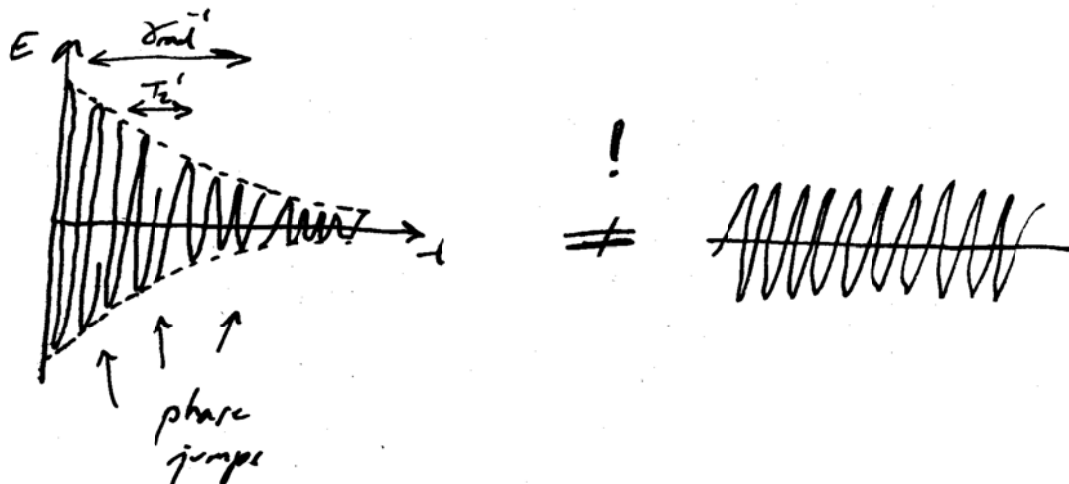
We have also assumed either that this wave emanated from a true point source (spherical wave) or that it is a plane wave (esp. a spherical wave far from the point source).

Reality. Most sources are not point sources => no plane waves (although a laser operating on a "single mode" is a pretty good approximation to one). Light from thermal or fluorescent sources is emitted by many atoms, so the light emitted from different points on the source will have no definite phase relationship.



- Generally  $\phi_1 - \phi_2 = \text{random variable}$

Reality. Even a single atom does not emit light with a definite wave! (Again, in a laser there is an "almost" definition phase, but we'll consider only "classical" sources.) We saw this earlier in our discussion of dephasing. It takes a classical dipole a time  $\gamma_{rad}^{-1}$  to radiate away its power, but the phase kept changing on a time scale of  $T_2'$  (and usually  $T_2' < \gamma_{rad}^{-1}$ ). Also, the amplitude decreases with a time constant  $\gamma_{rad}^{-1}$ . The net result is that light emitted by even a single atom looks nothing like our ideal harmonic source  $\cos \omega t$



Thus we must consider what happens to interference with real light sources, and what is the frequency spectrum of light with this complicated (and generally chaotic) time dependence?

In order to simplify the discussion, we will break our treatment down into questions:

- ① If you sit at one point in space, and you know the phase +amplitude at some time  $t$ , how well can you predict the phase and amplitude at a later time  $t + \tau$ ? This is called the temporal coherence problem.
- ② If you freeze a wave at one point in time, and you know the phase and amplitude at one position in space, how well can you predict the phase and amplitude at some other point in space? In other words – how well do you know the wavefront? This is called the spatial coherence problem.

A nice simple picture of what we mean by these questions is shown in Lipson fig. 11.1(see next page).

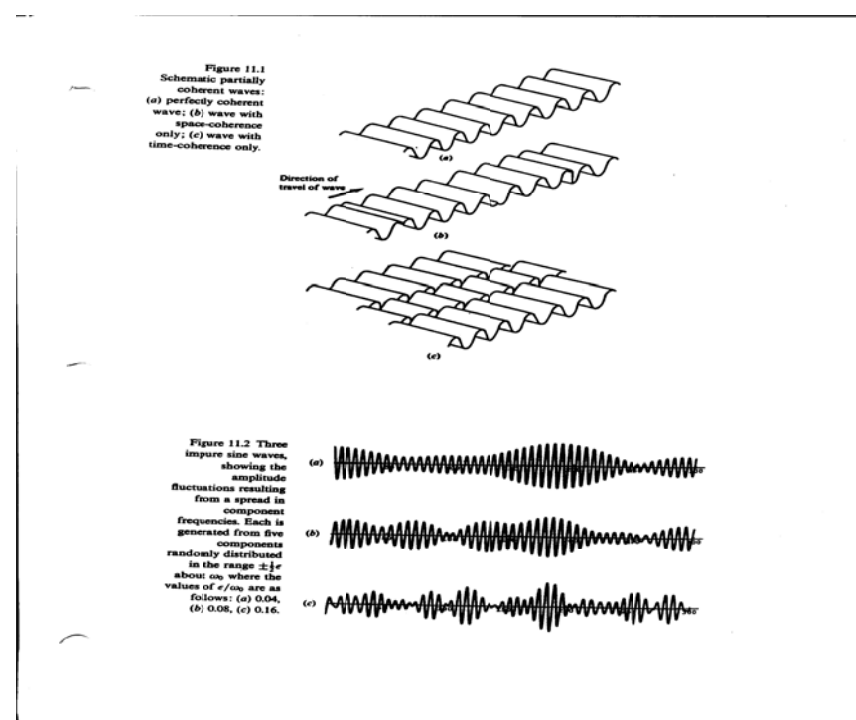
### Temporal Coherence

We begin with a discussion of the temporal properties of light which is not perfectly coherent. An intuitive picture of such waves is provided by Lipson Fig.11.2. We expect (and shall show below) that such complicated waveforms in the time domain must be made up of many frequency components in the frequency domain.

We already know the Fourier transform relation of the complex analytic signal

$$\mathbb{F}\{E(t)\} = E(\omega) = \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt$$

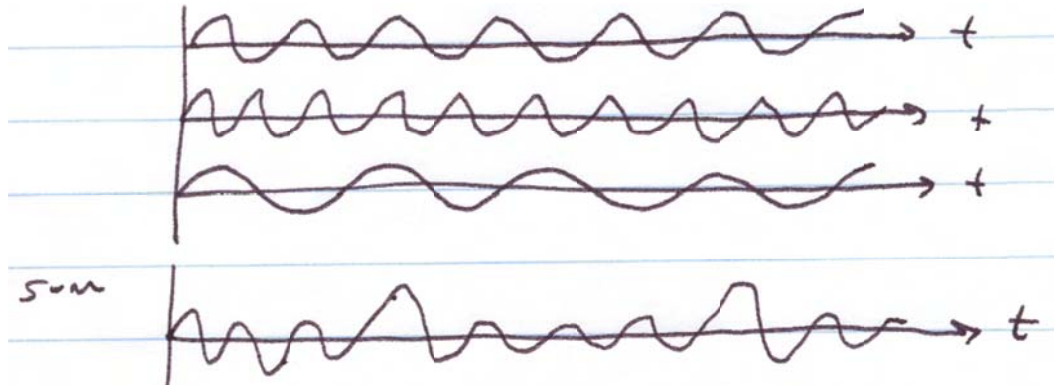
However, we also know that we can never directly measure the field amplitude in optics: our detectors are simply much too slow. We need to see how the lack of coherence manifests itself in real experiments.



## Temporal Coherence: intuitive ideas

1. Real sources  $\Leftrightarrow$  many atoms emitting independently with random phases + slightly differing frequencies

e.g.3 ideal sources with different  $\omega_0$



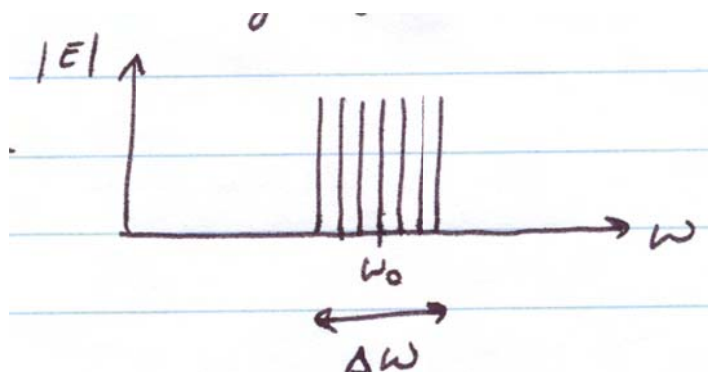
beats  $\rightarrow$  random amplitude fluctuations when  $N$  become large

now add time-dep. Amplitude + fluctuating phase to each atom  $\Rightarrow$  randomly fluctuating

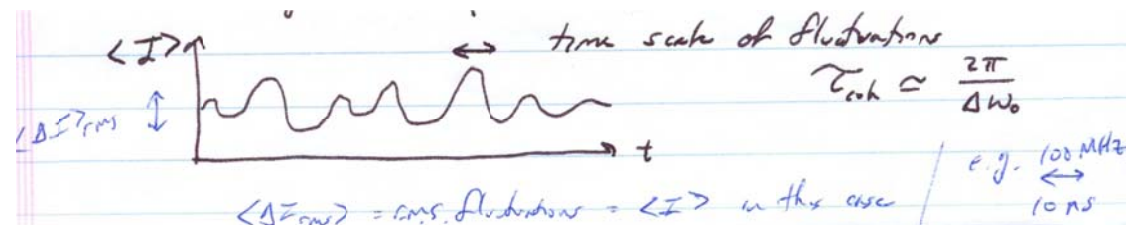
field  $E_{tot}(t)$

2. models of fluctuating light sources:

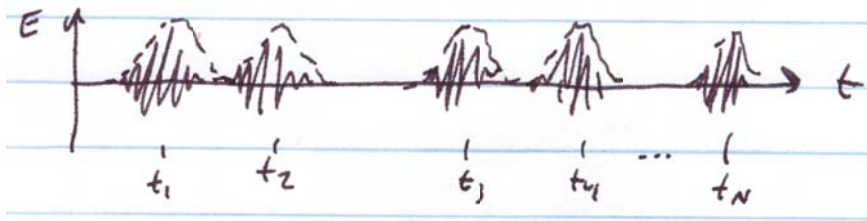
- (i) Spectrum random phases



time-averaged intensity (slow detector)



- (ii) Random series of wave groups

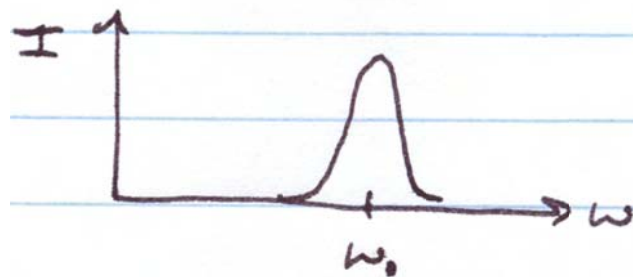


Arrival times  $t_N$  random

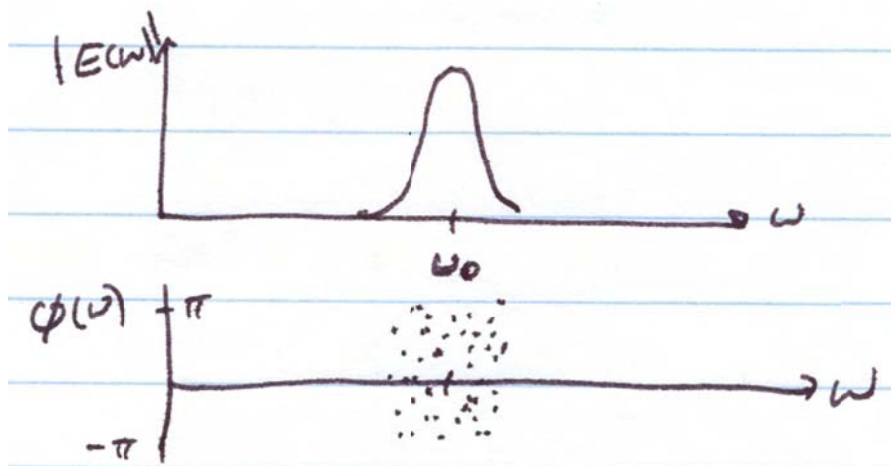
$$\text{Spectrum } E(\omega) = \mathbb{F}[E(t)] = \sum_n \mathbb{F}[E(t-t_n)]$$

Fourier transforms: displacement in time domain = phase shift in freq. domain

⇒ Power spectrum same as for 1 wave group

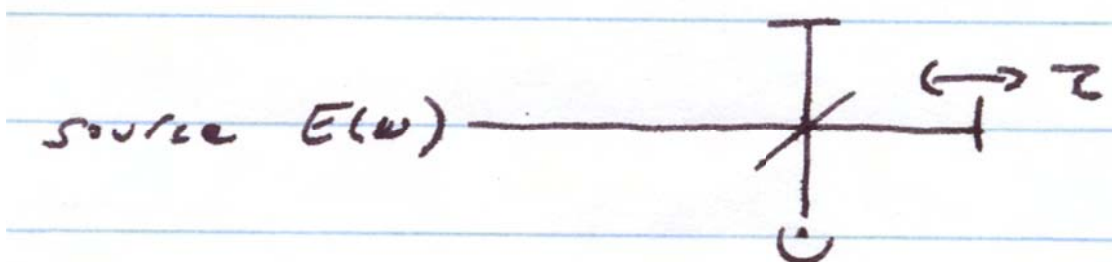


But phases in freq. domain are random

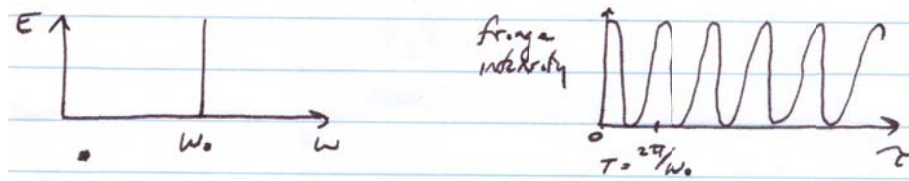


How to measure temporal coherence?

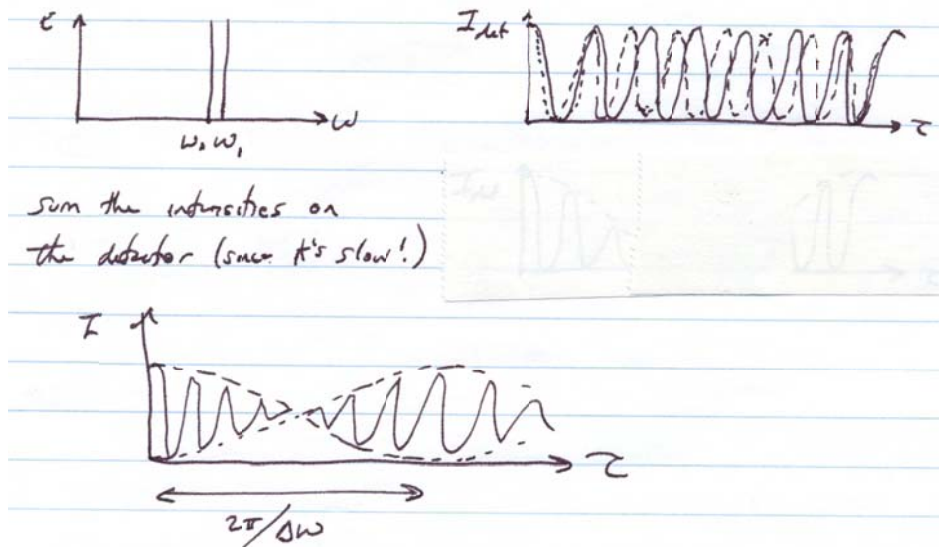
Consider a (balanced) Michelson interferometer with a slow deflector (i.e. so slow that it averages over all possible temporal beats between different frequency components.)



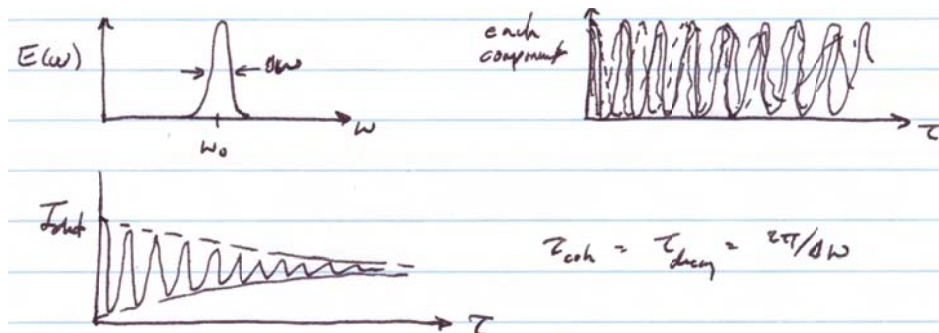
- (i) If  $E(\omega) = E_0 \delta(\omega - \omega_0)$  (perfectly coherent source)



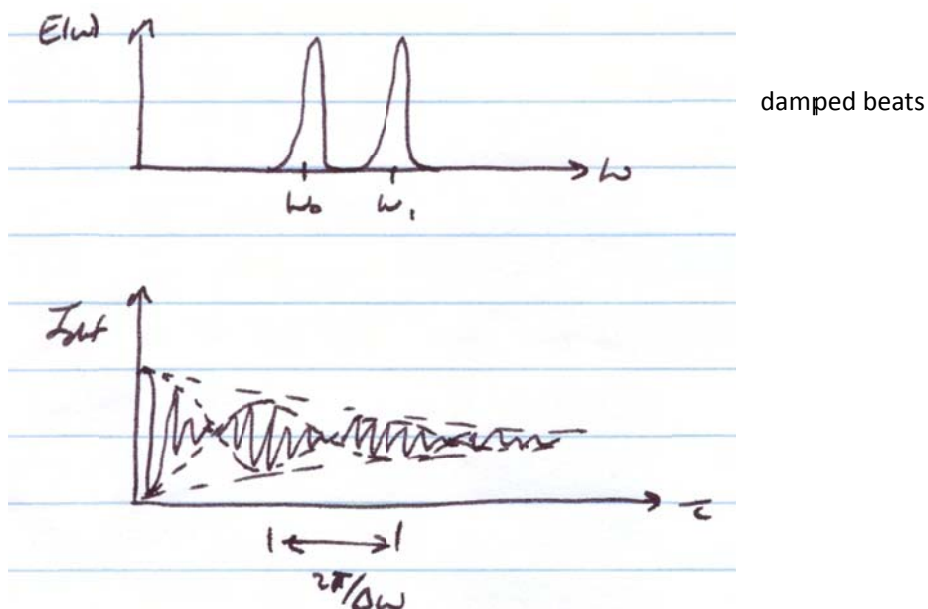
- (ii)  $E(\omega) = E_0 \delta(\omega - \omega_0) + E_0 \delta(\omega - \omega_1)$



- (iii)



- (iv)

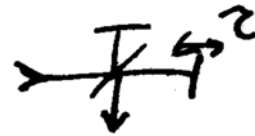


### Fringe Visibility, Correlation Functions, and Degree of Coherence

It is extremely valuable to take a slightly different perspective on the interference pattern produced by a Michelson interferometers; we want to think about the fringes in terms of the fields directly. We go all the way back to the basic expression for the detector signal. We take an input wave, split it, delay one arm, and recombine the fields

$$E_{tot} = E(t)/2 + E(t+\tau)/2$$

$$\begin{aligned} \langle I(t) \rangle &= \left[ \langle |E(t)|^2 \rangle + \langle |E(t+\tau)|^2 \rangle + \langle (E^*(t))E(t+\tau) + c.c. \rangle \right] \frac{1}{4} \\ &= \left[ I_1 + I_2 + 2 \operatorname{Re} \langle E^*(t)E(t+\tau) \rangle \right] \frac{1}{4} \end{aligned}$$



( $I_1$  = intensity in one arm)

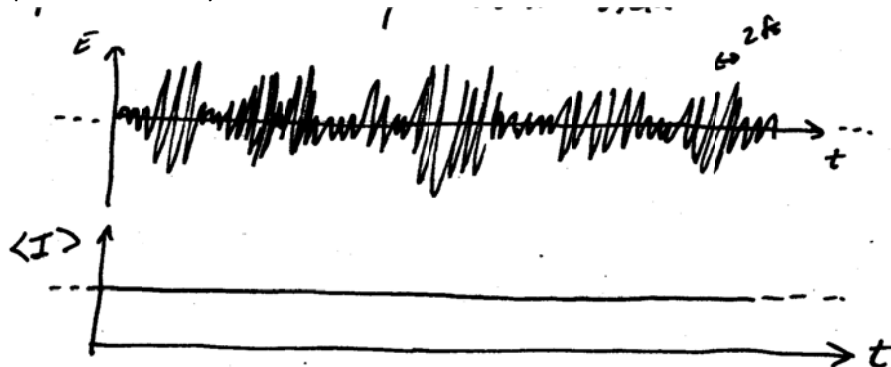
$$\text{def. } \Gamma(\tau) = \langle E^*(t)E(t+\tau) \rangle = \frac{1}{T} \int_t^{t+\tau} E^*(t)E(t+\tau) dt$$

assumption: fields are stationary

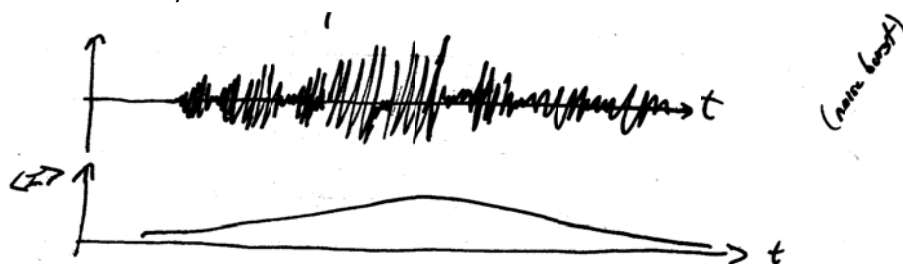
i.e. the time average is independent of the origin of time

(the overall average intensity and the r.m.s. fluctuations are constant in time)

example of a stationary random field :



Ex. of a nonstationary random field



We will consider only stationary random fields.

Then  $\Gamma(\tau)$  depends only on the time delay, and not on what particular time interval is used to measure it.

$\Gamma(\tau) = \langle E^*(t)E(t+\tau) \rangle$  is called the autocorrelation function (or sometimes the

self-coherence function) of the field E.

Now  $\Gamma(\tau)$  has the units of intensity. It is frequently convenient to define a new dimensionless quantity

$$\tilde{\gamma}(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)} = \frac{\langle E^*(t)E(t+\tau) \rangle}{\langle |E(t)|^2 \rangle}$$

Nothing that  $\Gamma(0) = \langle E^*(t)E(t+0) \rangle = \langle |E(t)|^2 \rangle = I_1$ ,

we can write

$$\langle I(\tau) \rangle = \frac{2I_1}{4} [1 + \text{Re}(\tilde{\gamma}(\tau))] = \frac{I_0}{2} [1 + \text{Re}(\tilde{\gamma}(\tau))]$$

Where  $I_1$  = the intensity in each arm of the Michelson interferometer, and  $I_0$  = the incident intensity.

#### Heuristic argument

On P.305, by considering the interference in the frequency domain, we obtained

$$\langle I(\tau) \rangle = \frac{I_0}{2} [1 + \gamma(\tau)]$$

where  $\gamma(\tau) = \frac{1}{I_0} \int_0^\infty I(\omega) \cos \omega \tau d\omega = \int_0^\infty S(\omega) \cos \omega \tau d\omega$

where  $S(\omega)$  = spectral density of the input wave .

Now, in the time domain, we have obtained  $\langle I(\tau) \rangle = \frac{I_0}{2} [1 + \text{Re} \tilde{\gamma}(\tau)]$ , so we have might expect that we can equate the two expressions:

$$\text{Re} \tilde{\gamma}(\tau) = \text{Re} \frac{\langle E^*(t)E(t+\tau) \rangle}{I_0/2} = \frac{1}{I_0} \int_0^\infty I(\omega) \cos \omega \tau d\omega$$

Thus we might expect to find

$$\Gamma(\tau) = \langle E^*(t)E(t+\tau) \rangle = \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} d\omega$$

i.e. that the field autocorrelation and spectral density form a Fourier-transform pair. Indeed this does turns out to be the case, and it is called the Weiner-Kintchine theorem.

An in-depth discussion of when the W-K theorem holds may be found in Mandel+ Wolf see 2.4. We emphasize again the main physical requirements is that the fields be stationary. The basic form of the “proof” is as follows.

Consider the quantity  $E^*(\omega)E(\omega')$ , and introduce the Fourier transforms:

$$E^*(\omega)E(\omega') = \iint_{-\infty}^{\infty} E^*(t) e^{i\omega t} E(t') e^{-i\omega' t'} dt dt' = \iint E^*(t) E(t') e^{-i(\omega' t' - \omega t)} dt dt'$$

If use time-average in the usual way over an interval T, then

$$\langle E^*(\omega)E(\omega') \rangle = \iint_{-\infty}^{\infty} \langle E^*(t)E(t') \rangle e^{-i(\omega' t' - \omega t)} dt dt'$$

Now if the field is stationary, so that the autocorrelation is independent of time origin, it must be of the form

$$\langle E^*(t)E(t') \rangle = \Gamma(t' - t)$$

(i.e. it depends only on the time difference  $\tau = t' - t$ ).

$$\langle E^*(\omega)E(\omega') \rangle = \int_{-\infty}^{\infty} dt e^{-i(\omega' - \omega)t} \int_{-\infty}^{\infty} dt' \Gamma(t' - t) e^{-i\omega'(t' - t)}$$

$$(\text{where we used } \omega' t' - \omega t = \omega'(t' - t) + \omega' t - \omega t = \omega'(t' - t) + (\omega' - \omega)t)$$

Now  $\tau = t' - t \Rightarrow dt' = d\tau$

$$\langle E^*(\omega)E(\omega') \rangle = \int_{-\infty}^{\infty} dt e^{-i(\omega' - \omega)t} \int_{-\infty}^{\infty} d\tau \Gamma(\tau) e^{-i\omega'\tau} = \delta(\omega' - \omega) \int_{-\infty}^{\infty} d\tau \Gamma(\tau) e^{-i\omega'\tau}$$

Thus we have

$$S(\omega) = \langle E^*(\omega)E(\omega) \rangle = \int_{-\infty}^{\infty} d\tau \Gamma(\tau) e^{-i\omega'\tau} \quad \text{Weiner-Khinchin theorem}$$

Thus measurement of the field autocorrelation, which is what the Michelson fringe pattern gives directly, yields the power spectrum by Fourier transform. Conversely, if you know the spectrum, then the field autocorrelation is determined.

**Physical significance of  $\gamma(\tau)$ :**

$$I = \frac{I_0}{2} [1 + \text{Re } \tilde{\gamma}(\tau)] = \frac{I_0}{2} [1 + \gamma(\tau)]$$

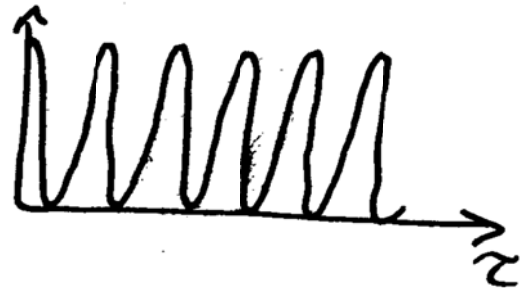
(i) for a perfectly coherent wave  $\delta(\omega) = \delta(\omega - \omega_0) \Rightarrow \gamma(\tau) = \cos \omega_0 \tau$

$$I_{\max} = \frac{I_0}{2} [1 + 1] = I_0$$

$$I_{\min} = \frac{I_0}{2} [1 - 1] = 0$$

Fringe visibility

$$V \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_0}{I_0} = 1$$



(ii) More generally,  $\gamma(\tau)$  is of the form

$$|\gamma(\tau)| \cos \omega_0 \tau, \text{ i.e. a damped oscillatory function}$$



$$I_{\max} = \frac{I_0}{2} [1 + |\gamma(\tau)|]$$

$$I_{\min} = \frac{I_0}{2} [1 - |\gamma(\tau)|]$$

$$\Rightarrow V = \frac{I_0 [1 + |\gamma(\tau)|] - I_0 [1 - |\gamma(\tau)|]}{I_0 [1 + |\gamma(\tau)|] + I_0 [1 - |\gamma(\tau)|]}$$

$$\boxed{V = |\gamma(\tau)|} \quad \text{The coherence function is just the fringe visibility!!}$$

$$|\gamma(\tau)| = 1 \Rightarrow \text{complete coherence}$$

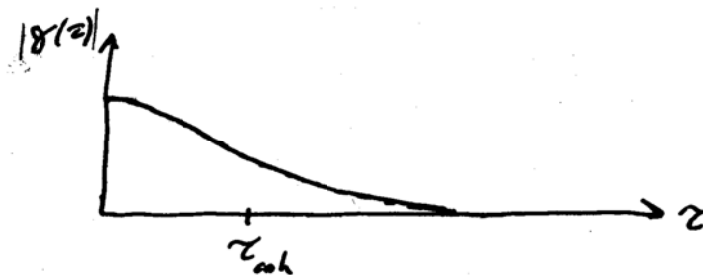
$$|\gamma(\tau)| = 0 \Rightarrow \text{complete incoherence}$$

$$0 < |\gamma(\tau)| < 1 \Rightarrow \text{partial coherence}$$

$$\tilde{\gamma}(\tau) = \frac{\langle E^*(t) E(t+\tau) \rangle}{\langle |E|^2 \rangle}$$

An autocorrelation (suitably time-averaged) is also intuitively just a measure of how well you can predict the value of the field at one time  $t + \tau$  given the field at  $t$ . Complete coherence means you can predict it with certainty. Partial coherence means that there is some correlation between the fields at two different times, but the correlation is not perfect; there may have been some phase shifts or amplitude fluctuations in between.

The time over which a field is strongly coherent with itself, i.e. where the autocorrelation has a significant amplitude, is called the coherence time.



The quantity  $l_c = c\tau_{coh}$  is often called the (longitudinal) coherence length.

The quantitative characterization of the coherence time is a little bit arbitrary. There are two principal ways of defining the coherence time in the literature.

(i) Given  $\Gamma(\tau) = \langle E^*(t) E(t+\tau) \rangle$

The coherence time may be defined as the root mean square width of  $|\Gamma|^2$ :

$$\tau_c^2 = \frac{\int_{-\infty}^{\infty} \tau^2 |\Gamma(\tau)|^2 d\tau}{\int_{-\infty}^{\infty} |\Gamma(\tau)|^2 d\tau}$$

Similarly, the effective spectral width of the light may be defined as

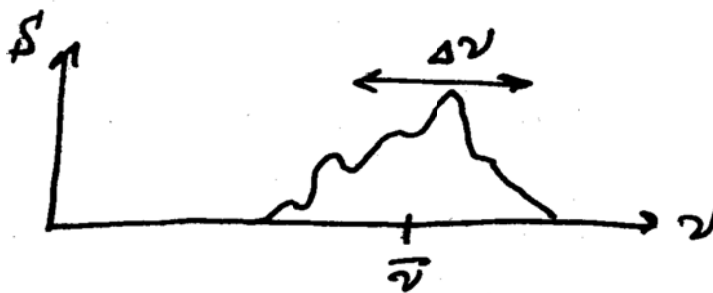
$$(\Delta\nu)^2 = \frac{\int_0^{\infty} (\nu - \bar{\nu})^2 S^2(\nu) d\nu}{\int_0^{\infty} S(\nu)^2 d\nu} \quad S(\nu) = \int_{-\infty}^{\infty} \Gamma(\tau) e^{-2\pi i \nu \tau} d\tau$$

Where

$$\bar{\nu} = \frac{\int_0^{\infty} \nu S^2(\nu) d\nu}{\int_0^{\infty} S(\nu)^2 d\nu} = \text{mean frequency}$$

And  $S(\nu)$  is the spectral density vs. freq.  $\nu = \frac{\omega}{2\pi}$ .

e.g.



It is possible to show using standard theorems on Fourier transform relations (see Mandel + Wolf

P4.3.3) that these satisfy  $\tau_c \cdot \Delta\nu \geq \frac{1}{4\pi}$ .

Equality is obtained only if the spectrum is a Gaussian. This definition of the coherence time is useful generally when the light is quasi-monochromatic and the spectrum has a "reasonably well-defined peak"

(ii) Another common definition is to use the normalized degree of coherence

$$\tilde{\gamma}(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)} = \frac{\langle E^*(t) E(t+\tau) \rangle}{\langle |E|^2 \rangle}$$

And define the coherence time as

$$\tau_c = \int_{-\infty}^{\infty} |\tilde{\gamma}(\tau)|^2 d\tau$$

It turns out (see Mandel + Wolf 4.3.3 for proof) that the width can be written as

$$\Delta\nu = \frac{|\Gamma(0)|^2}{\int_0^{\infty} S(\nu)^2 d\nu}$$

With these definitions  $\tau_c \cdot \Delta\nu = 1$  always.

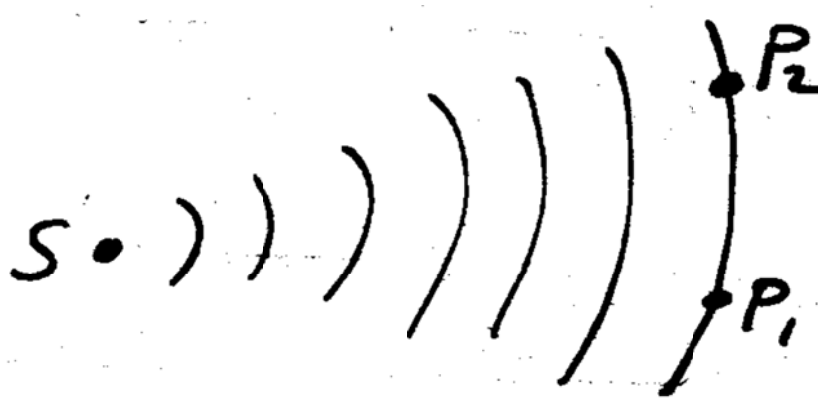
It should be noted that the above two definitions of the coherence time give roughly similar numbers for quasi-monochromatic light, but can give significantly different result for complicated, broad-band light.

### Spatial Coherence

The idea of spatial coherence of a light wave is closely analogous to the concept of temporal coherence. The fundamental issue is: at a frozen instant in time, given a field at one point in space, how well can you predict what it will be in another point in space?

example: complete spatial coherence

- Consider emission from a point source (which may emit light with a randomly fluctuating field!)



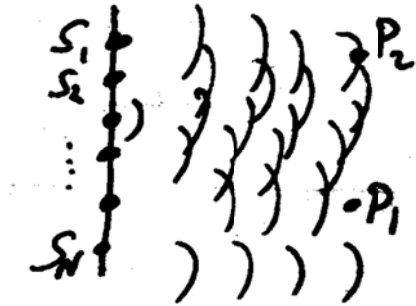
Clearly,  $P_1$  and  $P_2$  are on the same wavefront, so there is complete spatial coherence between these two points.

- Even if S is temporally incoherent :



Example: complete spatial incoherence

- Consider emission from an extended source of dipoles which oscillate with random phases and/or amplitudes (uncorrelated dipoles)
- Look at the wave near the source



Clearly there is no correlation between the waves at  $P_1$  and  $P_2 \Rightarrow$  complete incoherence.

Note that, just as we did for the case of temporal coherence, we phrase the question of spatial coherence in terms of the presence (or absence) of correlations between fields, which can be quantified by the means of correlation functions.

We thus extend the definition of  $\Gamma$  to:

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau) = \langle E^*(\vec{r}_1, t) E(\vec{r}_2, t + \tau) \rangle$$

(Again, for stationary fields this is independent of  $t$ .)

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau) = \text{mutual coherence function.}$$

Similarly, we can extend  $\tilde{\gamma}(z)$  to

$$\tilde{\gamma}(\vec{r}_1, \vec{r}_2, \tau) = \frac{\Gamma(\vec{r}_1, \vec{r}_2, \tau)}{\sqrt{|\Gamma(\vec{r}_1, \vec{r}_1, 0)| |\Gamma(\vec{r}_2, \vec{r}_2, 0)|}} = \text{"complex degree of coherence"}$$

Clearly, temporal and spatial coherence are connected by propagation, but we can consider spatial coherence to be measured by

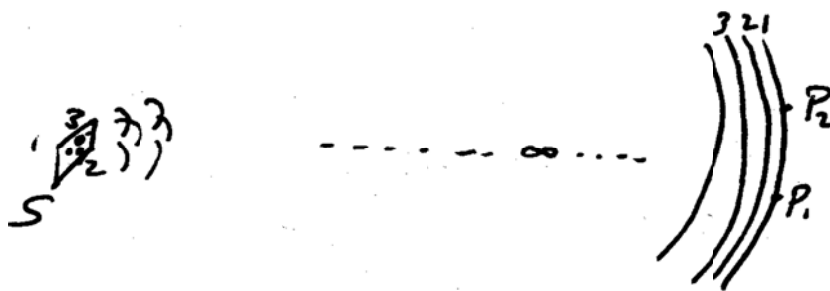
$$\Gamma(\vec{r}_1, \vec{r}_2, 0) \text{ or } \tilde{\gamma}(\vec{r}_1, \vec{r}_2, 0).$$

Note that in general  $0 \leq |\tilde{\gamma}| \leq 1$ ,  $0 =$  complete incoherence, and  $1 =$  complete coherence.

Now, it might seem, given our picture on the previous page, that spatial coherence is always negligible when the source is extended and consists of a large number of uncorrelated dipole emitters. Life turns out to be more interesting than that, however.

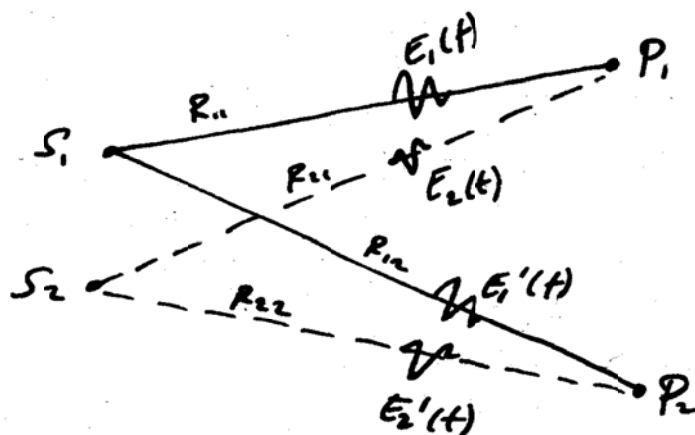
Surprise: the spatial coherence of light increases with propagation.

A very naive argument can make this statement at least sound plausible: "at a large enough distance, any source looks like a point source!"



At  $P_1$  and  $P_2$ , the multiple sources clearly give rise to temporal incoherence, but since all the waves look like spherical waves centered on  $S$  (which looks negligibly small), the wave has acquired spatial coherence!

A somewhat more sophisticated argument is given in Mandel + Wolf P 4.2.2. Consider the emission from two uncorrelated point sources  $S_1$  and  $S_2$ , and look at the net field at two observation points  $P_1$  and  $P_2$ .



If  $|R_{11} - R_{12}| < c\tau_c$ , then

$$E'_1(t) = E_1(t)e^{i\phi_1} \quad (\phi_1 = 0 \text{ if they are on the same wave front})$$

Similarly  $|R_{21} - R_{22}| < c\tau_c \Rightarrow E'_2(t) = E_2(t)e^{i\phi_2}$

(the phases  $\phi$  are just fixed by geometry – the positions of  $P_1$  and  $P_2$  – and are not fluctuating variables). ( $\phi_1 = |R_{11} - R_{12}| \cdot k$ )

$$\begin{aligned} E(P_1) &= E_1(t) + E_2(t) \\ \text{Now} \\ E(P_2) &= E'_1(t) + E'_2(t) = E_1(t)e^{i\phi_1} + E_2(t)e^{i\phi_2} \end{aligned}$$

$S_1$  and  $S_2$  uncorrelated  $\Rightarrow E_1$  and  $E_2$  uncorrelated

$$\langle E_1^*(t) E_2(t) \rangle = 0$$

However, the total fields at  $P_1$  and  $P_2$  are correlated, since the sum of the two waves at each point looks nearly the same.

$$\begin{aligned} \Gamma(P_1, P_2, 0) &= \langle [E_1^* + E_2^*][E_1 e^{i\phi_1} + E_2 e^{i\phi_2}] \rangle \\ &= \langle E_1^* E_1 e^{i\phi_1} \rangle + \langle E_2^* E_2 e^{i\phi_2} \rangle \quad \leftarrow \text{NOT } = 0! \\ &\quad + \underbrace{\langle E_2^* E_1 e^{i\phi_2} \rangle}_0 + \underbrace{\langle E_1^* E_2 e^{i\phi_1} \rangle}_0 \end{aligned}$$

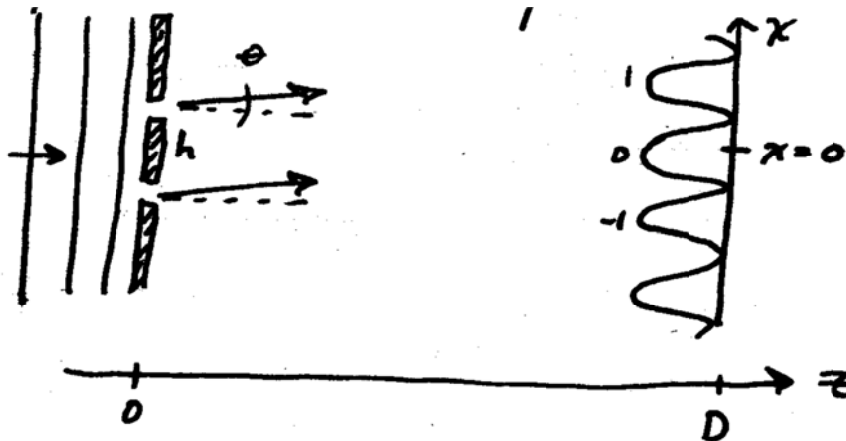
Because spatial coherence arises from extended sources on propagation, it is intimately connected with the theory of diffraction. The propagation of spatial correlations is described by the Van Cittert-Zernike theorem. We don't have the tools yet to develop this theory any further, but we can gain considerable insight with a simple calculation.

Q: how to measure spatial coherence?

A: just like we did with temporal coherence – with an interferometer.

Of course, if we want to measure spatial correlations, we should use an interferometer which divides a wavefront at different points in space. This is just what Young's double-slit arrangement does.

- Recall plane wave normally incident on a double slit:



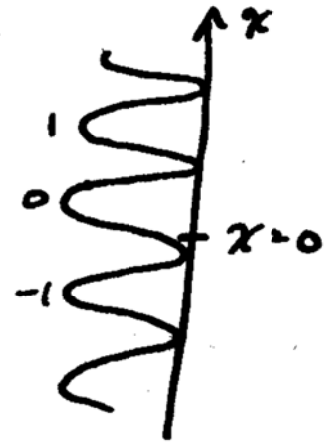
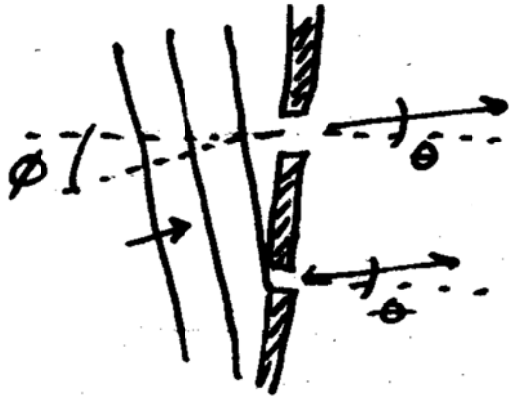
- Zero-order maximum at  $\chi = 0$  ( $\theta = 0$ )

- First-order maximum when  $OPD = \lambda$

$$h \sin \theta \approx h \theta = \lambda \Rightarrow \theta = \frac{\lambda}{h}$$

$$(\text{position on screen: } \tan \theta \approx \theta = \frac{\chi}{D} \Rightarrow \chi = D \theta = D \frac{\lambda}{h}.)$$

Now consider a plane wave incident at an angle  $\phi$ :



There is now a phase difference  $\delta = -\frac{\omega}{c} h \sin \phi$  between the two slit, so the zero-order max. now occurs when

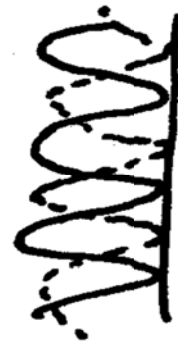
$$-h \sin \phi + h \sin \theta = 0 \quad \text{or} \quad \theta = +\phi$$

$$1^{\text{st}} \text{ order: } -h\phi + h\theta = \lambda \Rightarrow \theta = \frac{\lambda}{h} + \phi$$

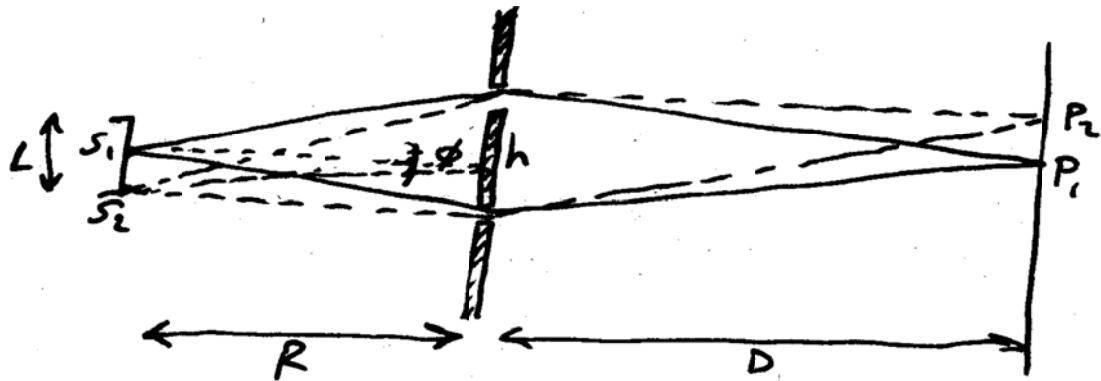
Note that the maxima of the tilted fringe pattern exactly overlap the minima of the  $\phi = 0$  (normal) fringe pattern when

$$\phi = \pm \frac{\lambda}{2h}$$

When this occurs, the interference pattern disappears!



Now we can consider light from two source points on an extended source:



$S_1 \Rightarrow$  fringe pattern centered on  $P_1$

$S_2 \Rightarrow$  fringe pattern centered on  $P_2$

$$\phi = \frac{\lambda}{2h} \Rightarrow \text{pattern disappears}$$

Geometry :  $\tan \phi \approx \phi = \frac{L/2}{R} = \frac{L}{2R}$

$\Rightarrow$  Pattern disappears when the source size is equal or larger than

$$L = 2R\phi = 2R \cdot \frac{\lambda}{2h} = \boxed{\frac{\lambda R}{h} = L}$$

There are other ways of phrasing this which are perhaps more general:

(i)  $\phi = \pm \frac{\lambda}{2h} \Rightarrow$  fringes disappear  $\Rightarrow$  no spatial coherence at the two slits

$\Rightarrow$  Source is coherent if it subtends an angle smaller than  $\Delta\phi = \frac{\lambda}{h}$

(ii) The converse of this is perhaps even more useful. If the source subtends an angle  $\Delta\phi$ , then the transverse coherence length is given by

$$\boxed{l_t = h = \frac{\lambda}{\Delta\phi}} \quad (\text{"coherence area"} \sim l_t^2)$$

Since this is the slit separation at which the interference pattern would disappear, where we consider  $h$  now to be a variable.

Note as  $\Delta\phi \rightarrow 0$ , source looks more +more like a point source, and the transverse coherence length becomes very large, as expected.

This is the main result of our discussion.

### Stellar Interferometer

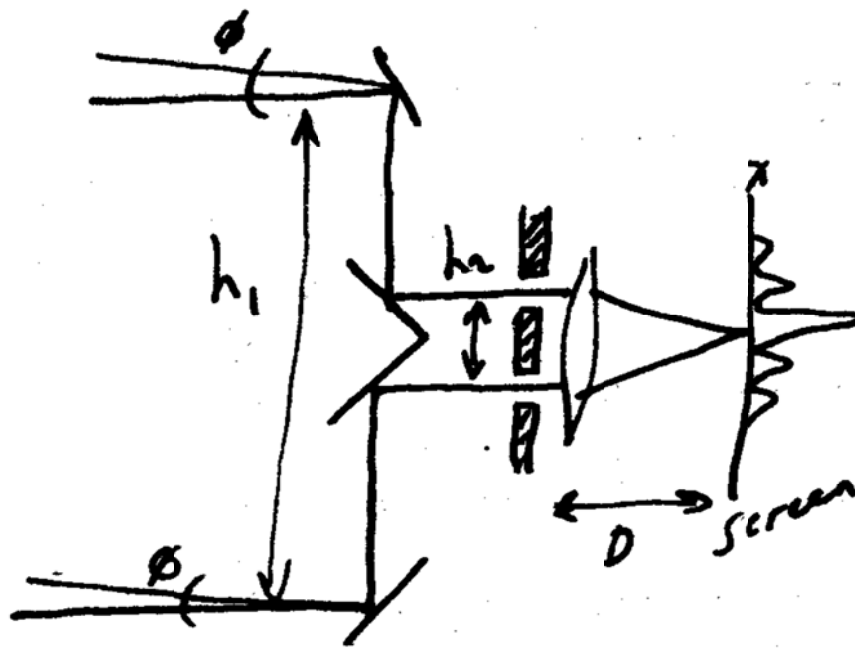
Michelson realized that the above relation could be used to measure the angular size of stars. The fundamental idea is to vary  $h$ , the distance between two sampled points on the wavefronts, and see at what value of  $h$  the fringe disappear.

The naïve approach would be to just use Young's arrangement; the problem is that, for large  $h$ , the fringe spacing would get too small, and the amount of light near the zero order would also be too small.

Michelson's trick: build an interferometer with a fixed slit \* separation  $h_2$  (\*or pinhole),

But which can sample a wavefront with separation  $h = h_1$





- Fringe spacing determined by  $h_2$
- Fringe contrast determined by  $h_1$
- See Guenther for derivation of the intensity on the screen:

$$I_p \approx 2I_0 \left[ 2 - kh_1 \Delta\phi \sin(kh_2 \frac{x}{D}) \right]$$

Ex, star Betelgeuse (red giant in Orion)

$\Delta\phi \sim 0.047$  sec. of arc corresponds to  $h_2$  on order of 2.5m

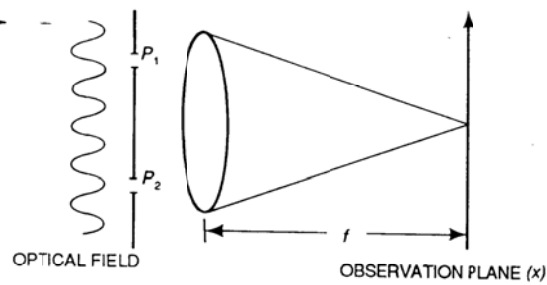


Fig. 10.1 Schematic diagram of the experimental arrangement.

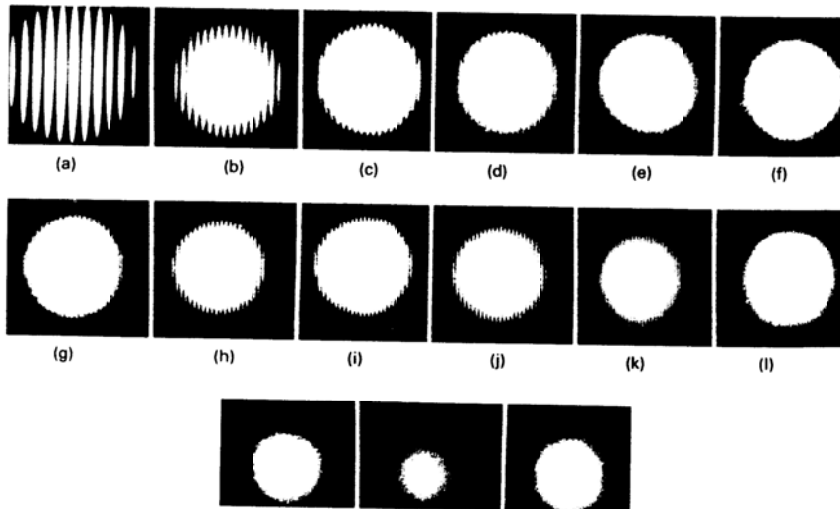
with the diffraction pattern of a circular aperture. We assume that by the use of a microdensitometer we show that it has the form [see Eq. (2.9)]:

$$\left| \frac{2J_1(x)}{x} \right|^2, \quad (10.3)$$

where  $x$  is a normalized radial coordinate. We can, therefore, conclude that:

1. The amplitude distribution across the aperture  $P_1$  is uniform.
2. The radiation across the aperture is essentially coherent.

The second aperture,  $P_2$ , alone gives a similar result. Now when the two apertures are opened together at their closest separation, two-beam interference fringes are observed that are formed by the division of the incident wavefront by the two apertures. At this closest separation, the fringes are extremely sharp [see Fig. 10.2(a)]. As the separation of the apertures increases, the photographic



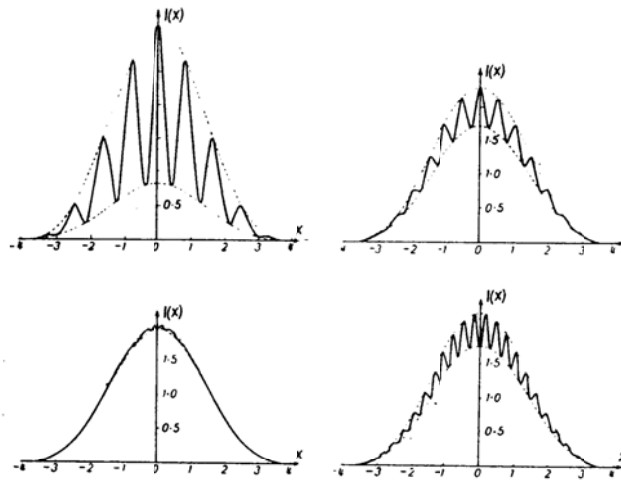


Fig. 10.3 Intensity plots of typical results of Fig. 10.2.

record looks like the results shown in Figs. 10.2(a) through (o). The fringes essentially disappear at (f) only to reappear faintly in (g) through (l), only to fade again at (m), and reappear very faintly at (n) and (o). Intensity plots corresponding to a typical sample of these photographic records are shown in Fig. 10.3. From the results of Fig. 10.2, the following facts are recorded. As the separation of  $P_1$  and  $P_2$  increases,

1. the fringe spacing decreases,
2. the minima are never zero,
3. the relative heights of the maxima above the minima steadily decrease until (f) where they start to increase,
4. the absolute heights of the maxima decrease and the heights of the minima increase until (f),
5. eventually the fringes disappear, at which point the resultant intensity is just twice the intensity observed with one aperture alone, and
6. the fringes reappear with increasing separation but the fringes contain a central minimum not a central maximum.

Items 1 through 5 may be summarized by defining a visibility  $V$  [first introduced by Michelson for this very purpose and previously introduced in Chapter 7 as Eq. (7.2)]:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}. \quad (10.4)$$

If this visibility function is plotted against the separation of the apertures  $P_1$  and  $P_2$  for the example given in Fig. 10.2, a curve similar to that shown in Fig. 10.4