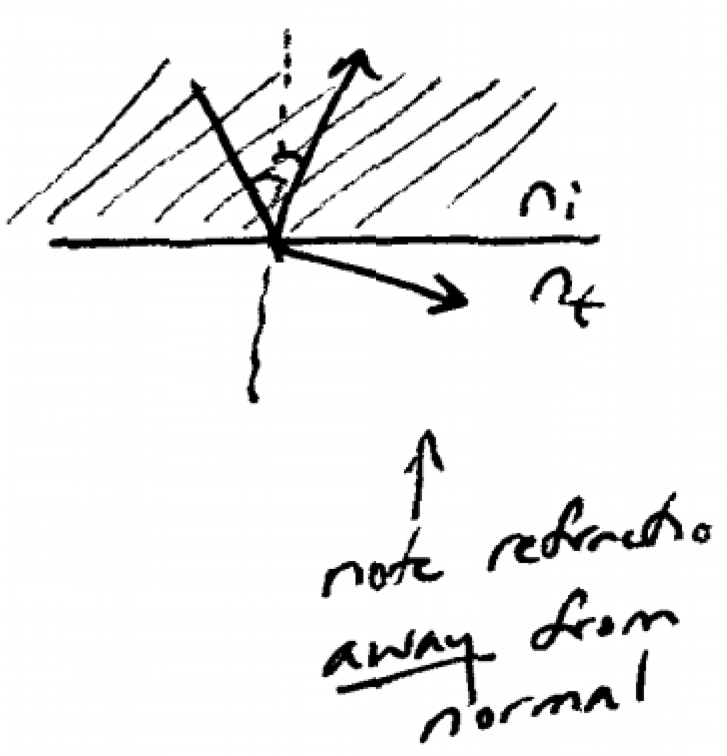
# Lecture 5 Total Internal Reflection

Consider propagation of light from an optically dense medium into a less dense medium, i.e. $n_i > n_t$ (e.g. light going from glass into air)




Snell:

$$n_i \sin\theta i = n_t \sin\theta t$$

$$\theta_t = \sin^{-1}\left(\frac{n_i}{n_t}\sin\theta i\right)$$

Clearly, we cannot have $\frac{n_i}{n_t}\sin\theta i > 1$ and have a reflected beam.

Def. critical angle by $\frac{n_i}{n_t}\sin\theta_c = 1$

=> $\theta_c = \sin^{-1}\frac{n_t}{n_i}$

At the critical angle of incidence, the refracted beam has $\theta_t = 90°$, i.e. the beam runs exactly along the interface,

Writing down the general expression for the phase of transmitted wave

$$\vec{E}_t \sim e^{-i\vec{k}_t\cdot\vec{r}} = e^{-ik_t(x\sin\theta_t + z\cos\theta_t)}$$

We can rewrite Snell as

$$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i = \frac{\sin\theta_i}{\sin\theta_c}$$

And also

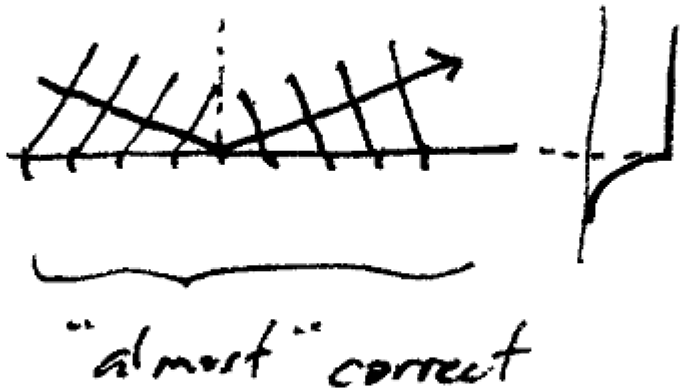
$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\left(\frac{\sin\theta i}{\sin\theta c}\right)^2}$$

Now when $\theta_i > \theta_c$, $\sin\theta_i > \sin\theta_c$ =>the cosine becomes <u>purely imaginary</u>

$$\cos\theta_t = i\sqrt{\left(\frac{\sin\theta i}{\sin\theta c}\right)^2 - 1} \equiv -ia \quad =>$$

$$E_t \sim e^{-ik_t\left[x\left(\sin\theta i/\sin\theta c\right)-iaz\right]} \sim e^{-ak_t z}e^{-ik_t x\left(\sin\theta i/\sin\theta c\right)}$$

Which is the form of a wave running along the interface with <u>amplitude decaying exponentially</u> away from the interface, such a damped wave is called an <u>evanescent wave.</u>
Except when the incident wave is very close to the critical angle, the evanescent wave decays away within a few wavelengths of the interface.




<u>Phase shift in TIR:</u>
Let's consider just the case of S-polarization. The phase shift is easiest to calculate, if we use the form of $r_s$ given on page 29 (with $\mu_i = \mu_t$ ):

$$r_s = \frac{E_r}{E_i} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{\cos\theta_i + i\frac{n_t}{n_i}a}{\cos\theta_i - i\frac{n_t}{n_i}a} = \frac{p-iq}{p+iq}$$

This can be written in polar form

$$r_s = e^{-i\alpha_s} = e^{-i\left[2\tan^{-1}\left(\frac{q}{p}\right)\right]}$$

⇨ $|r_s| = 1$ as it must be for TIR , and there is a phase shift for S-polarized light

$$\alpha_s = \left[2\tan^{-1}\left(\frac{-n_t a}{n_i\cos\theta_i}\right)\right]$$

Recall that near normal incidence, the phase shift was always either 0 or π. In TIR, a continuum of phase shifts are possible. Physically the origin may be thought of as being due to the nonzero propagation (or penetration) of the wave into the evanescent region,

"Goos-Hanchen shift" correct picture: 

Since the wave amplitude decays with z, <u>no net power</u> must be transferred beyond the interface; i.e. there is no energy flow even though a field does exist on the other side of the boundary. This may be confirmed by calculating the normal component of the Poynting vector, as we did before (p.33).

For complex fields $\vec{E}$ and $\vec{H}$, the time-averaged Poynting vector is

$$\langle \vec{S} \rangle = \frac{1}{2}\,\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)$$

$$\Rightarrow \quad \langle \vec{S} \rangle \cdot \hat{n} = \frac{1}{2}\,\mathrm{Re}\left[\left(\vec{E}\times\vec{H}^*\right)\cdot\hat{n}\right] = \frac{1}{2}\,\mathrm{Re}\left[\left\{\vec{E}_t\times\left(\frac{c}{\mu_t\omega}\vec{k}_t\times\vec{E}_t^*\right)\right\}\cdot\hat{n}\right] = \frac{c}{\mu_t\omega}\,\mathrm{Re}\left[|\vec{E}_t|^2\left(\vec{k}_t\cdot\hat{n}\right)\right]$$

$\vec{k}_t\cdot\hat{n} = k_t\cos\theta_t = -i\alpha k_t$ purely imaginary

=> $\langle \vec{S} \rangle \cdot \hat{n} = 0$

=> For all $\theta_i \geq \theta_c$, all power is reflected at the interface

$R_{S,P} = 1, T_{S,P} = 0$

Example: high-power laser mirror

--use TIR in right-angle prism to reflect beam

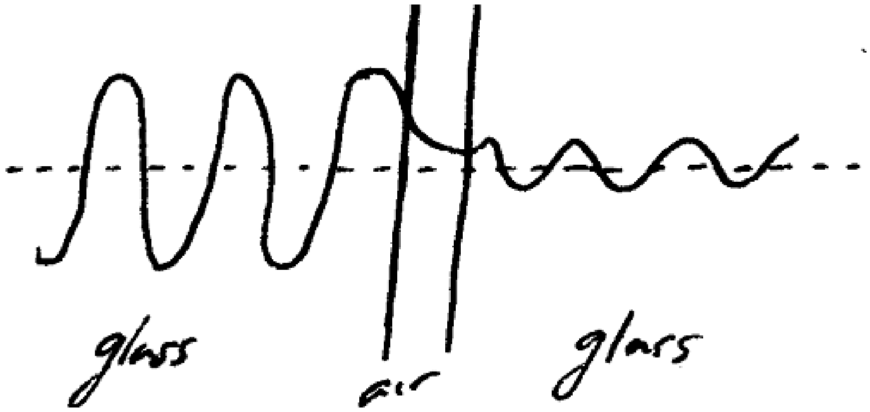
--avoids use of any dielectric or metal coating on the mirror which might be susceptible to damage from the high power

--often used to direct the output of Q-switched lasers

Can the evanescent field be measured? Yes.

The most famous method to observe this field is through the phenomenon of frustrated TIR, in which a second dielectric is brought into close proximity of the first, in order to couple some of the evanescent field out:

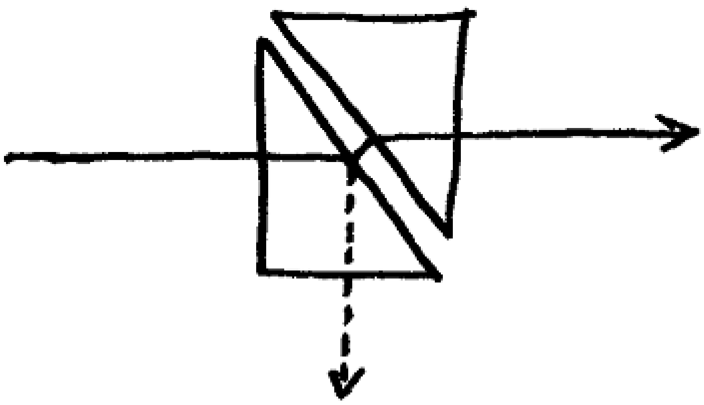
e.g.



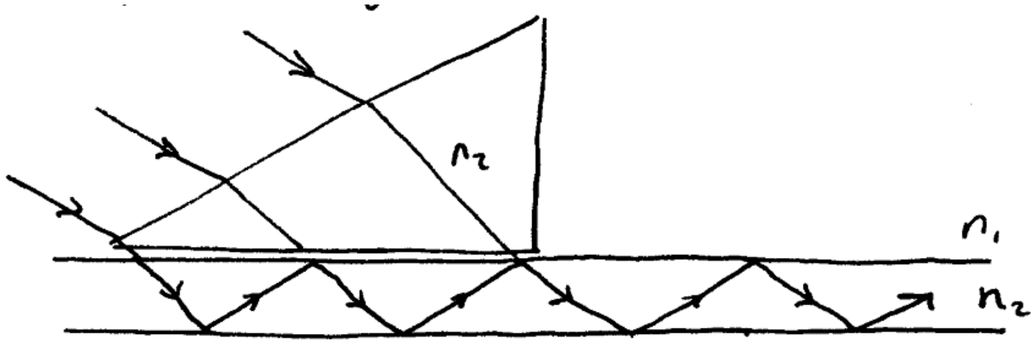

(note analogy to “tunneling” in quantum mechanics!)

Clearly, varying the air spacing will vary the amount of light coupled through (exponentially, in fact).

Example: variable attenuator



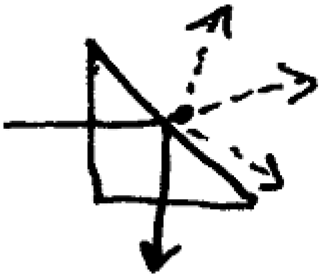
Example: optical waveguide coupler




$n_2 > n_1$

An optical waveguide confines the light by TIR, so there are only two ways of coupling light in: (i) through the end, or (ii)via frustrated TIR using a prism.

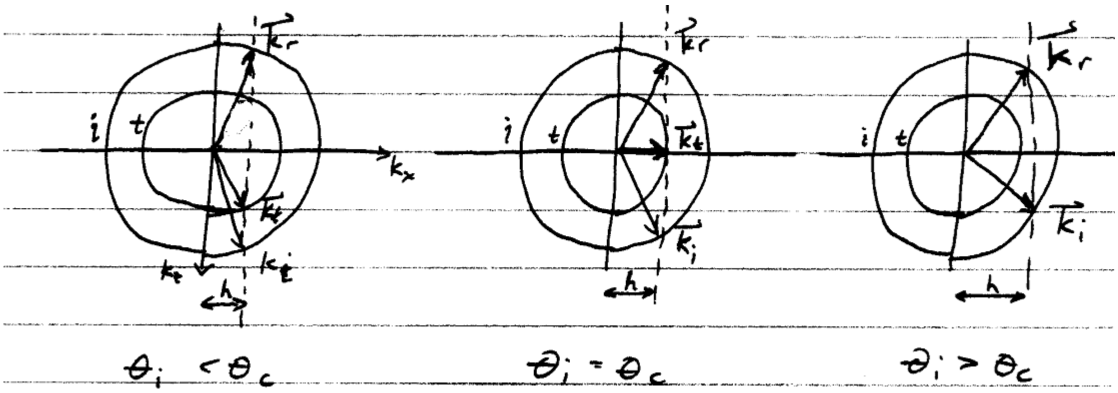
A second proof of the existence of the evanescent field is to an atom or molecule near the surface, which can absorb energy from the field +subsequently fluoresces.



see, e.g. C.K.Carniglin,L.Mandel,and K.H.Drexhage,J.Opt.Soc.Am.62,479(1972).

In fact, Prof. Axelred in Biophysics here at UM makes images of cells (esp. surfaces) using evanescent excitation of fluorescent dyes. The evanescence field gives excellent depth resolution!

The wave vector diagram for various angles of incidence appears as follows




Note that at the critical angle $\vec{k}_t$ is parallel to the interface, i.e. the transmitted wave runs along the interface. For $\theta_i > \theta_c$, a real $\vec{k}_t$ cannot exist, so all the light is reflected into $\vec{k}_r$.

At $\theta_i = \theta_c$ , $\beta_t = 0, n_t^2 k_o^2 = h^2$

=> $h = n_t k_o$

When $\theta_i > \theta_c, h > n_t k_o$

=> $\beta_t^2 = n_t^2 k_o^2 - h^2 < 0$

=> $\beta_t$ is purely imaginary: let $\beta_t = i\gamma_t$

=> $\vec{E}_t = \vec{E}_{to} e^{i(wt-hx)} e^{-\gamma_t z}$

$e^{i(wt-hx)}$ Wave running along interface

$e^{-\gamma_t z}$ Evanescent wave into z direction